Characterization of Stiffness Coefficients of Silicon 
Versus Temperature using “Poisson’s Ratio” 
Measurements

Chun-Hyung Cho¹*, Ho-Young Cha², and Hyuk-Kee Sung²

Abstract—The elastic material constants, stiffness constants (c_{11}, c_{12}, and c_{44}), are three unique coefficients that establish the relation between stress and strain. Accurate knowledge of mechanical properties and the stiffness coefficients for silicon is required for design of Micro-Electro-Mechanical Systems (MEMS) devices for proper modeling of stress and strain in electronic packaging. In this work, the stiffness coefficients for silicon as a function of temperature from -150°C to +25°C have been extracted by using the experimental measurements of Poisson’s ratio (ν) of silicon in several directions.

Index Terms—Stiffness coefficients, compliance coefficients, Poisson’s ratio

I. INTRODUCTION

The mechanical and material properties of silicon, E (Young’s modulus) and ν (Poisson’s ratio), are important for proper modeling of stress and strain in electronic packaging [1]. Also, the structures of Micro-Electro-Mechanical Systems (MEMS) devices are based on the fabrication of mechanical devices in silicon [2], and accurate knowledge of mechanical properties of silicon based upon the values of compliance coefficients (or stiffness coefficients) is required for design of MEMS devices. The stiffness constants (c_{11}, c_{12}, and c_{44}), are three independent coefficients depend upon the symmetry of the crystal and the direction on the crystal surface because of the anisotropic mechanical properties of the material.

Silicon shows an anisotropic characteristic, which is, often neglected in previous works. Wortman, et al. [3] analytically calculated and plotted the graphs of E as a function of crystal direction for orientations in the (100) and (110) planes. Kang [4] used a strain gage technique to measure E of silicon with a four-point bending fixture in which the gages are mounted on the surface of specimen strips.

In this work, we have investigated the temperature dependent characteristics of the stiffness coefficients in silicon by measurements of ν using the Micro-testers. We picked (001) and (111) silicon because the vast majority of silicon devices are now fabricated using (001) silicon wafers and (111) wafers.

The experimental values of ν at room temperature are observed to be in agreement with analytic calculations based upon literature values of the stiffness coefficients [3]. This work presents experimental results for ν from -150°C to +25°C for the (001) and (111) surfaces. Finally, combining the results for ν versus temperature yields the temperature dependent stiffness coefficients over the temperature range -150°C to +25°C.

*Corresponding Author

¹Department of Electronic & Electrical Engineering, College of Science and Technology, Hongik University, Sejong, 339-701, Republic of Korea
²School of Electronic & Electrical Engineering, College of Engineering, Hongik University, 72-1, Sangsu-dong, Seoul, 121-791, Republic of Korea
E-mail : chcho@hongik.ac.kr
II. Theory

The geometries of (001) and (111) silicon wafers are given in Fig. 1.

Silicon exhibits a linear elastic material behavior described by Hooke’s Law:

\[ \sigma_{ij} = C_{ijkl} e_{kl} , \]  
(1)

where \( \sigma_{ij} \) and \( e_{ij} \) are the stress and the strain components, and \( C_{ijkl} \) are the components of the stiffness tensor. Inverting Eq. (1) gives

\[ e_{ij} = S_{ijkl} \sigma_{kl} , \]  
(2)

where \( S_{ijkl} \) are the compliance components. Also, the transformation relations for the reduced index stress and strain components are given by [4]

\[ \sigma'_a = T_{a\beta}^{-1} \sigma'_\beta , \]  
(3)

\[ e'_a = T_{a\beta}^{-1} e'_\beta , \]  
(4)

where the coefficients \( T_{a\beta} \) are elements of a six-by-six transformation matrix related to the direction cosines between an arbitrary coordinate system and the crystallographic coordinate system for the silicon wafer. Also, \( \sigma'_a \) and \( e'_a \) are the stress and the strain tensor components in the unprimed system, respectively, whereas \( \sigma'_{\beta} \) and \( e'_{\beta} \) are those components in a rotated primed coordinate system. Inverting Eq. (4) and combining with Eqs. (2, 3) yields the relations between stress and strain in a rotated coordinate system as follows:

\[ \epsilon' = [T']^{-1} S T^{-1} \sigma' , \]  
(5)

where

\[
[T_{a\beta}]=
\begin{bmatrix}
  \ell_1 & m_1 & n_1 & 2\ell_1 a_1 & 2m_1 a_1 & 2n_1 a_1 \\
  \ell_2 & m_2 & n_2 & 2\ell_2 a_2 & 2m_2 a_2 & 2n_2 a_2 \\
  \ell_3 & m_3 & n_3 & 2\ell_3 a_3 & 2m_3 a_3 & 2n_3 a_3 \\
  l_1 & m_1 & n_1 & l_1 a_1 & m_1 a_1 & n_1 a_1 \\
  l_2 & m_2 & n_2 & l_2 a_2 & m_2 a_2 & n_2 a_2 \\
  l_3 & m_3 & n_3 & l_3 a_3 & m_3 a_3 & n_3 a_3
\end{bmatrix}
\]

Note that \( l, m, \) and \( n \) are the direction cosines with respect to the reference axes. Then, using the relation \([T']^{-1} = [T^{-1}]' \) in Eq. (5) leads to

\[ \epsilon' = [T^{-1}'] S T^{-1} \sigma' . \]  
(6)

Then, using \([S] = [C]^{-1} \) leads to

\[ \sigma' = TS^{-1}T' \epsilon', \quad \text{or} \quad \sigma' = TCP' \epsilon' \]  
(7)

Only limited temperature-dependent data have been reported on silicon. However, room-temperature values of the three independent stiffness coefficients of silicon were measured by several researchers [3, 6-8]. For instance, Table 1 presents the typical literature values for the stiffness and compliance coefficients of silicon obtained by Wortman and Evans [3]. McSkimin, et al. [6] obtained those constants by using ultrasonic measurement techniques, which yielded values for the velocities of wave propagation and the elastic constants, and Hall [7] determined the stiffness constants from sound-velocity measurements by using a pulse-echo technique. Also, Hall [7] investigated the temperature dependence of the stiffness constants for pure and doped silicon over the temperature range of 4.2K to 310K. Chen, et al. [8] measured the stiffness constants below 100K by

Table 1. Typical literature values for the stiffness/compliance coefficients of silicon [3]

<table>
<thead>
<tr>
<th>( c_{ij} )</th>
<th>( s_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>165.7 GPa</td>
<td>7.68 x 10^-12 Pa^-1</td>
</tr>
<tr>
<td>63.9 GPa</td>
<td>-2.14 x 10^-12 Pa^-1</td>
</tr>
<tr>
<td>79.6 GPa</td>
<td>12.8 x 10^-12 Pa^-1</td>
</tr>
<tr>
<td>79.6 GPa</td>
<td>12.8 x 10^-12 Pa^-1</td>
</tr>
</tbody>
</table>
using ultrasonic methods.

Based on the values in Table 1, the elastic modulus and Poisson’s ratio can be evaluated by using Eq. (7).

For any crystallographic direction of silicon, \( \nu \) can be expressed by using the compliance coefficients \( s_{ij} \) as

\[
\nu = \frac{s_{12} + (s_{11} - s_{12} - \frac{1}{2}s_{44})(l^2m^2 + m^2n^2 + n^2s^2)}{s_{11} - 2(s_{11} - s_{12} - \frac{1}{2}s_{44})(l^2m^2 + m^2n^2 + n^2s^2)} \tag{8}
\]

Note that \( l_1, m_1, \) and \( n_1 \) are the direction cosines for the primary orientation whereas \( l_2, m_2, \) and \( n_2 \) are the direction cosines for the normal orientation. In Eq. (8), it can be seen that \( \nu \) exhibit anisotropy along different crystallographic directions.

1. (001) Plane

For the (001) plane, the direction cosines are

\[
\begin{bmatrix}
  l_1 & m_1 & n_1 \\
  l_2 & m_2 & n_2 \\
  l_3 & m_3 & n_3
\end{bmatrix} =
\begin{bmatrix}
  \cos(\phi - \frac{\pi}{4}) & \sin(\phi - \frac{\pi}{4}) & 0 \\
  -\sin(\phi - \frac{\pi}{4}) & \cos(\phi - \frac{\pi}{4}) & 0 \\
  0 & 0 & 1
\end{bmatrix}
\tag{9}
\]

where \( \phi \) is the angle of counterclockwise rotation from the \( x_1 \) axis [110] in Fig. 3. By Eq. (8), \( \nu \) is expressed as

\[
\nu = \frac{2\cos^2(2\phi)s_{12} + 2[1 + \sin^2(2\phi)]s_{11} - \cos^2(2\phi)s_{44}}{2[1 + \sin^2(2\phi)]s_{11} + 2\cos^2(2\phi)s_{12} + \cos^2(2\phi)s_{44}}. \tag{10}
\]

Poisson’s ratio \( (\nu) \) for (001) silicon at varying angular locations with respect to the [110] axis are plotted in Fig. 2 based upon the literature values of compliance coefficients from Wortman and Evans [3].

2. (111) Plane

On the (111) silicon plane, the direction cosines are

\[
\begin{bmatrix}
  l_1 & m_1 & n_1 \\
  l_2 & m_2 & n_2 \\
  l_3 & m_3 & n_3
\end{bmatrix} =
\begin{bmatrix}
  \sin\phi & \cos\phi & \sin\phi & \cos\phi & -2\sin\phi \\
  \frac{\sin\phi}{\sqrt{6}} & \frac{\cos\phi}{\sqrt{6}} & \frac{\sin\phi}{\sqrt{6}} & \frac{\cos\phi}{\sqrt{6}} & -\frac{2\sin\phi}{\sqrt{6}} \\
  \frac{\sin\phi}{\sqrt{6}} & \frac{\cos\phi}{\sqrt{6}} & \frac{\sin\phi}{\sqrt{6}} & \frac{\cos\phi}{\sqrt{6}} & \frac{2\sin\phi}{\sqrt{6}} \\
  1 & 1 & 1 & 1 & 1
\end{bmatrix}
\tag{11}
\]

where \( \phi \) is the angle of counterclockwise rotation from the \( x_1 \) axis [110] in Fig. 3. By Eq. (8), \( \nu \) is given by

\[
\nu = \frac{2s_{11} + 10s_{12} - s_{44}}{6s_{11} + 6s_{12} + 3s_{44}} = \frac{2(c_{11} + 4c_{12})c_{44} - (c_{11} - c_{12})(c_{11} + 2c_{12})}{3(c_{11} - c_{12})(c_{11} + 2c_{12}) + 6c_{11}c_{44}}. \tag{12}
\]

As presented in Eq. (12), \( \nu \) exhibit isotropic characteristics on the (111) silicon plane where the elastic property \( (\nu) \) is independent of direction \( (\nu = 0.262) \).

III. EXPERIMENTS AND ANALYSIS

The expressions of \( \nu \) for each direction are summarized in Table 2 in which analytic values of \( \nu \) for room temperature, based upon literature values of the compliance/stiffness coefficients, are compared with experimental values.

A strain gage technique was used to measure Poisson’s ratio for the [100] and [010] directions of silicon. In this technique, strain gages were mounted on the surface of silicon strips cut from (001) silicon wafers along the [100] and [010] directions and a uniaxial tensile stress was applied to the silicon test strips as presented in Fig. 3. Poisson’s ratio is determined from the slope of the plot of transverse gage reading (lateral strain) over longitudinal
Table 2. for each direction of silicon

<table>
<thead>
<tr>
<th>Direction</th>
<th>$\nu$</th>
<th>(Calc.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[100]</td>
<td>$\frac{s_{12}}{s_{11}}$</td>
<td>0.278</td>
</tr>
<tr>
<td>[010]</td>
<td>$\frac{s_{12}}{s_{11}}$</td>
<td>0.278</td>
</tr>
<tr>
<td>[110]</td>
<td>$\frac{(2s_{11} + 2s_{12} - s_{44})}{(2s_{11} + 2s_{12} + s_{44})}$</td>
<td>0.062</td>
</tr>
<tr>
<td>(001):[110]</td>
<td>$\frac{(2s_{11} + 2s_{12} - s_{44})}{(2s_{11} + 2s_{12} + s_{44})}$</td>
<td>0.062</td>
</tr>
<tr>
<td>(111):[110]</td>
<td>$\frac{(2s_{11} + 10s_{12} - s_{44})}{(6s_{11} + 6s_{12} + 3s_{44})}$</td>
<td>0.262</td>
</tr>
<tr>
<td>[112]</td>
<td>$\frac{(2s_{11} + 10s_{12} - s_{44})}{(6s_{11} + 6s_{12} + 3s_{44})}$</td>
<td>0.262</td>
</tr>
</tbody>
</table>

Fig. 3. Strain gages on silicon test strips in a micro-tester apparatus.

gage reading (axial strain) versus applied load as shown in Fig. 4. However, this method has some limitations for measuring $\nu$ of stiff materials such as silicon. The test setups for measuring Poisson’s ratio at low temperatures are shown in Fig. 5.

The measured values of Poisson’s ratio ($\nu$) for the [100] and [001] direction are 0.275 and 0.291, respectively, which are in agreement with analytic calculations based upon literature values of the compliance/stiffness coefficients. We measured at least 10 times for each data point.

We needed three independent relations from Eqs. (10, 12), for a complete set of the compliance/stiffness coefficients. However, we have only one relation from Eq. (12) because of the isotropic characteristic of (111) silicon. Therefore, we need two more independent relations from Eq. (10). First, we tried the principal directions such as [010], [100], and [110]. However, the analytic expressions from [010] and [100] are the same, and unfortunately, the value from [110] direction is too small to be measured. Finally, we have three independent equations as below:

For $\phi = 0^\circ$ (001) silicon,

$$\nu_0 = \frac{2s_{11} + 2s_{12} - s_{44}}{2s_{11} + 2s_{12} + s_{44}}$$  \hspace{1cm} (13)

Similarly, for $\phi = 45^\circ$ (001) silicon,

$$\nu_{45} = \frac{s_{12}}{s_{11}}$$  \hspace{1cm} (14)

For $\phi = 0^\circ$ on (111) silicon,
As we measure each $\nu$ from Eqs. (13-15) versus temperature, three temperature-dependent compliance (and/or stiffness) coefficients can be obtained. It is observed that $\nu$ goes up with increasing temperatures. A complete set of temperature dependent compliance and stiffness coefficients for silicon can be calculated by combining the experimental results for $\nu$ in Eqs. (13-15), and the plots of the coefficients for silicon versus temperature are shown in Fig. 6.

IV. CONCLUSIONS

In this work, $\nu$ (Poisson’s ratio) are observed to be dependent upon the direction on the silicon surface. Also, $\nu$ may be expressed by compliance coefficients ($s_{11}$, $s_{12}$, and $s_{44}$) for any crystallographic direction of silicon. And $\nu$ of silicon has been measured using strain gages in a micro-tester apparatus. The measured results are in agreement with the calculations from analytical expressions. A strain gage technique was used to measure Poisson’s ratio for 30° and 45° directions of (001) silicon and 0° direction of (111) silicon. Combining the results for $\nu$ (Poisson’s ratio) versus temperature yields the temperature dependent stiffness coefficients and the extracted values show that the magnitude of stiffness coefficients for silicon decrease monotonically versus temperature over the temperature range -150°C to +25°C.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the
Alabama Microelectronics Science and Technology Center (AMSTC) and the NSF Center for Advanced Vehicle Electronics (CAVE). This work (Grants No. C0193818) was supported by Business for Academic-industrial Cooperative establishments funded Korea Small and Medium Business Administration in 2015. Also, this work was supported by 2015 Hongik University Research Fund.

REFERENCES


Chun-Hyung Cho received the B.S. degree in Electrical Engineering from the Seoul National University, Seoul, South Korea, in 1997, and the M.S. and Ph.D. degrees in Electrical and Computer Engineering from Auburn University, Auburn, AL, in 2001 and 2007, respectively. In 2009, he joined Hongik University, Sejong where he is currently an Associate Professor in the Department of Electronic & Electrical engineering. His research interests include the application of analytical and experimental methods of piezoresistive sensors.

Ho-Young Cha received the B.S. and M.S. degrees in electrical engineering from from Seoul National University, Seoul, Korea, in 1996 and 1999, respectively, and the Ph.D. degree in electrical and computer engineering from Cornell University, Ithaca, NY, in 2004. He was a Postdoctoral Research Associate with Cornell University until 2005, where he focused on the design and fabrication of SiC and GaN electronic devices and GaN nanowires. He was with the General Electric Global Research Center, Niskayuna, NY, from 2005 to 2007, developing wide-bandgap semiconductor sensors and high power devices. Since 2007, he has been with Hongik University, Seoul, where he is currently an Associated Professor in the School of Electronic and Electrical Engineering. His research interests include wide bandgap semiconductor devices. He has authored over 60 publications in his research area.

Hyuk-Kee Sung received the B.S. and M.S. degrees in electrical and electronic engineering from Yonsei University, Seoul, Korea, in 1999 and 2001, respectively, and Ph.D. degree in electrical engineering and computer sciences from the University of California, Berkeley, in 2006. He was a Postdoctoral Researcher with the University of California, Berkeley. He is now with the School of Electronic and Electrical Engineering, Hongik University, Seoul, Korea. His research interests are in the area of optoelectronic devices, optical injection locking of semiconductor lasers, and optoelectronic oscillators.