I. INTRODUCTION

Most media exhibit a normal propagation characteristic called subluminal velocity, where the speed of propagation of individual time-harmonic components is slower than the speed of light, $c$, in vacuum at all frequencies. However, in a specific and narrow frequency band of signal attenuation or at an anomalous dispersion frequency, the group velocity is observed to be greater than $c$. This abnormal wave propagation is called superluminal velocity or negative group velocity [1, 2]. At first, the concept of superluminal velocity seems to defy causality. However, many practical experiments have shown that the concept of superluminal velocity does not violate the definition of a causal system [3–5].

One example of the concept of superluminal velocity is the negative group delay (NGD); this refers to the phenomenon whereby an electromagnetic wave traverses a medium in such a manner that its amplitude envelope is advanced rather than delayed [6]. Recently, many studies have designed negative group delay circuits (NGDCs) and used them in practical applications such as enhancing the efficiency of a feed-forward amplifier [7], shorting delay lines [8], realizing non–Foster reactive elements [9], and minimizing beam-squint problems in series-fed antenna arrays systems [10].

The conventional design method of active and passive NGDCs is based on only single $RLC$ resonators [11–18]. The NGD bandwidth and magnitude flatness should be as wide as possible for applications in RF circuits and systems. However, conventional NGDCs suffer from smaller NGD bandwidth and poor magnitude flatness. To overcome these problems, some studies have attempted to design NGDCs using different methods such as cascading the number of resonators with slightly different center frequencies [16] and cross-coupling between resonators [17].

Abstract

This paper presents the design of a negative group delay circuit (NGDC) using the filter synthesis approach. The proposed design method is based on a frequency transformation from a low-pass filter (LPF) to a bandstop filter (BSF). The predefined negative group delay (NGD) can be obtained by inserting resistors into resonators. To implement a circuit with a distributed transmission line, a circuit conversion technique is employed. Both theoretical and experimental results are provided for validating the proposed approach. For NGD bandwidth and magnitude flatness enhancements, two second-order NGDCs with slightly different center frequencies are cascaded. In the experiment, group delay of 5.9±0.5 ns and insertion loss of 39.95±0.5 dB are obtained in the frequency range of 1.935–2.001 GHz.

Key Words: Bandstop Filter, Frequency Transformation, Filter Approach, Low Pass Filter Prototype, Negative Group Delay.
Fig. 1. (a) Low-pass filter prototype and (b) proposed structure of a negative group delay circuit.

Table 1. Specifications of negative group delay circuits

<table>
<thead>
<tr>
<th>( N )</th>
<th>LPF prototype element values</th>
<th>( f_0 ) (GHz)</th>
<th>( \Delta ) (%)</th>
<th>Maximum GD @ ( f_0 ) (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( g_1 = g_2 = 1.4142 )</td>
<td>1.962</td>
<td>10</td>
<td>-6.0</td>
</tr>
<tr>
<td>3</td>
<td>( g_1 = g_3 = 1 ) ( g_2 = 2 )</td>
<td>1.962</td>
<td>10</td>
<td>-6.0</td>
</tr>
</tbody>
</table>

LPF=low-pass filter, GD=group delay.

In this study, a filter synthesis approach is applied to design an NGDC. In this proposed method, circuit element values are obtained from low-pass filter (LPF) prototypes by applying a frequency transformation.

**II. DESIGN EQUATIONS ANALYSIS**

Fig. 1(a) shows the Butterworth LPF prototype. The element values of the LPF are obtained as follows [19].

\[
g_k = 2 \sin \left( \frac{2k-1}{2N} \pi \right), \quad k = 1, 2, \ldots, N \tag{1}\]

where \( N \) is the order of the filter. Fig. 1(b) shows the structure of the proposed NGDC. The lumped elements of the proposed NGDC are easily obtained from the LPF by applying the following frequency transformation [19].

\[
\frac{1}{\omega} \leftrightarrow A \left( \frac{\omega - \omega_0}{\omega} \right) \tag{2}
\]

where \( A \) and \( \omega_0 \) are the 3-dB fractional bandwidth (FBW) and center frequency of the NGDC, respectively. The elements values of the shunt-series branch \((L_{ssk}, C_{ssk})\) are given as follows.

\[
L_{ssk} = \frac{Z_0}{g_i \Delta \omega_b} \tag{3a}
\]

\[
C_{ssk} = \frac{1}{Z_0 g_i \Delta \omega_b} \tag{3b}
\]

where \( Z_0, g_i \) and \( \omega_0 \) are the port impedance, \( i \)-th prototype element value, and center frequency of the NGDC, respectively. Similarly, the element values of the series-parallel branch \((L_{spk}, C_{spk})\) are given as follows.

\[
L_{spk} = \frac{Z_0 g_i \Delta \omega_b}{\omega_0} \tag{4a}
\]

\[
C_{spk} = \frac{1}{Z_0 g_i \Delta \omega_b} \tag{4b}
\]

The resistors are inserted into the shunt and series resonators to obtain the required NGD. The values of these resistors are obtained by analyzing the shunt-series and series-parallel branch elements [18], respectively. The resistances in the shunt-series branch of the NGDC for the required NGD are given as follows.

\[
R_{ssk} = -\frac{Z_0}{2} \sqrt{\left( \frac{Z_0}{2} \right)^2 - 4 Z_0 L_{ssk} / \alpha \tau} \tag{5}
\]

where \( \tau \) and \( \alpha \) are the GD and correction factor, respectively. Similarly, the resistance in the series-parallel branch of the NGDC is given as follows.

\[
R_{spk} = \frac{-\alpha \tau \sqrt{2 C_{spk} + \left( \alpha \tau / 2 C_{spk} \right)^2 - 4 Z_0 \alpha \tau / C_{spk}}}{2} \tag{6}
\]

A correction factor \( \alpha \) is substituted into (5) and (6) to consider the fact that each resonator contributes to GD and increases the overall GD so that it is higher than the required value for the overall circuit as \( N \) increases. Substituting \( \alpha \) into

Table 2. Element values of negative group delay circuits (NGDCs) with GD= -6 ns

<table>
<thead>
<tr>
<th>( N )</th>
<th>( L_{ss} )</th>
<th>( C_{ss} )</th>
<th>( R_{ss} )</th>
<th>( L_{sp} )</th>
<th>( C_{sp} )</th>
<th>( R_{sp} )</th>
<th>( L_{c} )</th>
<th>( C_{c} )</th>
<th>( R_{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14.34</td>
<td>0.4588</td>
<td>8.18</td>
<td>1.147</td>
<td>5.736</td>
<td>305.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20.28</td>
<td>0.3245</td>
<td>15.14</td>
<td>1.622</td>
<td>4.056</td>
<td>279.2</td>
<td>20.28</td>
<td>0.324</td>
<td>15.14</td>
</tr>
</tbody>
</table>

Unit: \( L_{ss}, L_{sp} \) (nH); \( C_{ss}, C_{sp} \) (pF); \( R_{ss}, R_{sp} \) (Ω).
Fig. 2. Synthesized results of proposed negative group delay circuit (NGDC) with different numbers of stages N: (a) group delay/magnitude and (b) phase characteristics.

Table 3. Calculated NGD BW and SA$_{\text{max}}$ for different numbers of filter stages

<table>
<thead>
<tr>
<th>N</th>
<th>GD$_{\text{max}}$(ns)</th>
<th>NGD (GD &lt; 0) BW (MHz)</th>
<th>SA$_{\text{max}}$(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-6</td>
<td>200</td>
<td>24.48</td>
</tr>
<tr>
<td>3</td>
<td>-6</td>
<td>240</td>
<td>34.96</td>
</tr>
</tbody>
</table>

NGD=negative group delay, BW=bandwidth, GD$_{\text{max}}$=maximum achieved group delay, SA$_{\text{max}}$=maximum signal attenuation.

these equations compensates for the deviation from the required value.

Fig. 2 shows the synthesized results of the NGDC with different numbers of stages N. The specifications and calculated circuit element values of the proposed NGDC are shown in Tables 1 and 2, respectively. As shown in Fig 2(a), the NGD bandwidth increases slightly with N. However, the signal attenuation (magnitude of $S_{21}$) also increases for the same maximum achievable GD, which can be compensated using general purpose gain amplifiers. Therefore, the signal attenuation (SA) of the proposed NGDC should be as small as possible so that the number of gain amplifiers can be reduced. The NGD bandwidth and maximum SA of the simulated circuits are shown in Table 3. From Table 3, it is clear that a trade-off between the NGD, insertion loss (magnitude of $S_{21}$), NGD-bandwidth, and number of stages N should be considered. Fig. 2(b) shows the phase characteristics of the NGDC with N=2 and N=3. As seen from Fig. 2(b), the phase of $S_{21}$ increases with frequency (positive slope) in the case of an NGDC, indicating the presence of NGD.

The proposed NGDC shown in Fig. 1(b) consists of both shunt-series ($R_{sh}$, $L_{sh}$, $C_{sh}$) and series-parallel ($R_{sp}$, $L_{sp}$, $C_{sp}$) lumped elements. A series parallel-resonator is difficult to implement at the microwave frequencies owing to the limited feasibility of the designed lumped elements.

To overcome this problem, the circuit conversion technique can be applied to convert the series-parallel lumped elements ($R_{sp}$, $L_{sp}$, $C_{sp}$) into shunt-series lumped elements ($R_{ssh}$, $L_{ssh}$, $C_{ssh}$) using admittance inverters ($J$-inverters), as shown in Fig. 3. The corresponding conversion relations are given as follows.

Fig. 3. Series-parallel to shunt-series circuit transformation.

![Fig. 4. Proposed negative group delay circuit with $J$-inverters and shunt-series RLC circuits.](image)

![Fig. 5. Structure of cascaded two-stage negative group delay circuit (NGDC) for NGD bandwidth and magnitude flatness enhancements. Physical dimensions: $L_1=22.90$, $L_2=26.10$, $L_3=5$, $L_4=55.2$, $L_5=57.36$, $W_1=3.54$, $W_2=0.32$, $W_3=0.54$, $W_4=0.36$, $W_5=0.58$ (unit: mm).](image)
Theoretically, the value of the $J$-inverter can be chosen arbitrary. However, in practice, $J$-inverters should be chosen carefully because their physical dimensions must be realizable without any difficulties. Fig. 4 shows the overall structure of proposed NGDC; it consists of $J$-inverters and shunt-series $LC$ resonators. The $J$-inverters can be implemented using quarter-wavelength transmission lines. The shunt-series $LC$ resonators can be implemented using either short-circuited half-wavelength or open-circuited quarter-wavelength resonators [19].

\begin{align}
R_{\text{ess}} &= \frac{1}{J_i^2 R_{pk}} \quad (7a) \\
L_{\text{ess}} &= \frac{C_{pk}}{J_i^2} \quad (7b) \\
C_{\text{ess}} &= J_i^2 L_{pk} \quad (7c)
\end{align}

As seen from Table 4, the proposed NGDC provides the

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Ref & GD$_{\text{max}}$ (ns) & SA$_{\text{max}}$ (dB) & GD$_{\text{max}}$ ± 0.5 ns BW (MHz) & FOM \\
\hline
[7] & -9.0 & 64.2 & 30 & 0.089 \\
[13] & -7.9 & 16.9 & 10 & 0.078 \\
[16] & -5.8 & 26.8 & 60 & 0.112 \\
[17] & -1.1 & 29.3 & 350 & 0.083 \\
[18] & -1.7 & 16 & 60 & 0.043 \\
This work & -5.9 & 39.95 & 70 & 0.131 \\
\hline
\end{tabular}
\caption{Performance comparison of the proposed NGDC with those in previous works.} 
\end{table}

\begin{align}
\text{NGDC}= \text{negative group delay circuit, GD}_{\text{max}}= \text{maximum achieved group delay, SA}_{\text{max}}= \text{maximum signal attenuation, BW}= \text{bandwidth, FOM}= \text{figure of merit.}
\end{align}
highest FOM and best performance in terms of GD/magnitude flatness even though the SA is higher than of some previously proposed NGDCs [13, 16-18].

IV. CONCLUSION

In this paper, a filter synthesis approach is applied to design a NGDC. The circuit elements of the proposed filter can be obtained from LPF prototype elements. The proposed circuit was implemented with a distributed transmission line using a circuit conversion technique. An experiment was performed to validate the proposed design method. The measurement results agreed well with the simulation results. The proposed NGDC shows the highest FOM, large negative group delay, good magnitude flatness, and wider negative group delay–bandwidth.

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REFERENCES

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