Abstract - This paper deals with the depth and speed controls of a class of nonlinear large diameter unmanned underwater vehicles (LDUUVs), while maintaining its attitude. The concerned control problem can be viewed as an asymptotic stabilization of the error model in terms of its desired depth, surge speed and attitude. To tackle its nonlinearities, the linear parameter varying (LPV) model is employed. Sufficient linear matrix inequality (LMI) conditions are provided for its asymptotic stabilization. A numerical simulation is provided to demonstrate the effectiveness of the proposed design methodology.

Key Words : Large diameter unmanned underwater vehicles (LDUUV), Depth control, Speed control, Lyapunov, Linear matrix inequality (LMI), Asymptotic stability.

1. Introduction

Depth control is one of important control issues for unmanned underwater vehicles (UUVs). Various design techniques, including the sliding mode control [1], the reduced order output feedback [2], the adaptive nonlinear control [3], the adaptive sliding mode control [4], the robust control [5], and the gain-scheduled output feedback [6] are introduced on this issue. Recently, one of the powerful design approach, a linear matrix inequality (LMI)-based design has been applied in the control problems of UUVs [7], [8]. However, there is a lack of research on the LMI approach to the depth control compared to other techniques.

This paper presents an LMI approach to the control problems of a class of nonlinear large diameter UUVs (LDUUVs) in the depth plane. Differently from the existing LMI technique [8], the proposed approach focuses on the depth control as well as the speed one for more general UUV dynamics. We formulate the concerned problem as an asymptotic stabilization of error dynamics with respect to the desired depth, speed, and attitude. By using the proposed controller scheme and the boundedness, we prove that the depth and the speed control design problems can be handled separately. Based on this property, we derive sufficient LMI conditions for its asymptotic stabilization in the sense of Lyapunov criterion.

Notations: The relation $P > Q$ ($P < Q$) means that the matrix $P−Q$ is positive (negative) definite. $\lambda_{\min}(A)$ ($\lambda_{\max}(A)$) is the maximum (minimum) eigenvalue of matrix $A$. $A(i)$ denotes $i$th row of the matrix $A$. $\text{Sym}[^n]$ is defined as $S^T+S$. $B_k$ indicates the ball $\{\eta : \|\eta\| \leq \lambda_k\}$ with $\lambda_k \in \mathbb{R}_{>0}$. $J_m$ indicates the integer set $\{1, \ldots, m\}$. Symbol $\ast$ denotes a transposed element in a symmetric position.

2. Preliminaries

Assuming the pure depth-plane motion with the body-relative surge velocity $\mathbf{u}$, the heave velocity $\mathbf{w}$, the pitch rate $\dot{\theta}$, the depth $z$, and the pitch angle $\theta$, consider the simplified depth model of LDUUVs

$$\dot{\eta} = J\theta \phi$$

$$F_1 \dot{\phi} + F_2(\phi) \phi = \tau$$

where $\eta = [z \theta]^T \in \mathbb{R}^2$, $\phi = [u w \theta]^T \in \mathbb{R}^3$, $\tau \in \mathbb{R}$ is the actuator torque with control inputs $\xi$ and $\delta$, $\xi, \delta \in \mathcal{J}_F$, $\mathcal{J}_F \subseteq \mathbb{R}_{>0}$ is a frame transformation, $F_1 \in \mathbb{R}^{1 \times 3}$ contains the mass and the hydrodynamic added mass terms, and $F_2(\phi) \phi \in \mathbb{R}^3$ means Coriolis-centripetal matrices including the added mass and the damping matrix (see [9], [10]). In specific, $\tau$, $\mathcal{J}_F$, $F_1$, and $F_2$ in (1) and (2) are represented by...
The following propositions and lemma will be required in the proof of our main result:

\textbf{Proposition 2}: It is true that \( G_i(\chi) = \frac{1}{m_i} \left( X_{x_i} u_i^2 + X_{u_i} u_i^2 \sum_{j=1}^{4} \delta_j \right) \)

\begin{align*}
G_0 &= \frac{1}{m_0} \left( X_{x_0} u_0^2 + X_{u_0} u_0^2 \sum_{j=1}^{4} \delta_j \right) \\
H &= E^{-1} \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} E = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & m_0 & m_0 \\
0 & 0 & m_0 & m_0
\end{bmatrix}
\end{align*}

\textbf{Proof}: It follows from (1), (2), and \( u = e_u + u_q \) that

\begin{align*}
\dot{e}_u &= -u_q \sin(\theta) - \cos(\theta) w - \sin(\theta) e_u \\
\dot{\theta} &= q
\end{align*}

where \( m_i = m_{x_i}, m_2 = m_{u_2}, m_3 = -m_{y} - Z_q, m_4 = -m_{x} - M_q, m_5 = I_{xy}, m_6 = I_{xy}, \) and \( m_7 = I_{xy} \) is the moment of inertia term.

\textbf{Problem 1}: Consider LDUUV (1) and (2). Define \( e_z := z - z_d \) and \( e_u := u - u_q \), where the desired depth \( z_d \in \mathbb{R}_{>0} \) and surge velocity \( u_q \in \mathbb{R}_{>0} \). Then, design \( \xi \) and \( \delta \) in \( J_4 \) such that \( |\xi|, |\delta|, [w], \) and \(|q|\) asymptotically converge to zero.

\section{Main Results}

The following propositions and lemma will be required in the proof of our main result:

\textbf{Proposition 1}: Consider the depth motion of LDUUV (1) and (2). Define. With the change of new state variables \( \chi := [e_i, \theta, \nu, q]^T \in \mathbb{R}^4 \) and \( \delta := [\delta_i, \delta_2, \delta_3, \delta_4]^T \in \mathbb{R}^4 \), an error system can be represented by

\begin{align*}
\begin{bmatrix}
\dot{\chi} \\
\dot{e}_u \\
\dot{\theta}
\end{bmatrix} &=
\begin{bmatrix}
G_1(\chi) \| G_2(\chi, e_u, \nu, \delta) \| \chi \\
G_3(\chi) \| G_4(\chi, e_u) \| \chi \\
G_5(\chi, \theta, \nu, \delta)
\end{bmatrix} + \begin{bmatrix}
H \\
0 \\
0 \end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \end{bmatrix} \frac{1}{m_i} \| \xi \|
\end{align*}

where

\begin{align*}
G_1(\chi) &= \left[ \begin{array}{c}
0 - u_q \sin(\theta) \\
\theta \\
\nu \\
\delta
\end{array} \right] \\
G_2(\chi, e_u, \nu, \delta) &= \left[ \begin{array}{c}
0 \\
Z_q + Z_{w_i} |w| \\
M_q + M_{w_i} |w| \\
- \sin(\theta) \nu
\end{array} \right] \\
G_3(\chi) &= \left[ \begin{array}{c}
0 \\
Z_q + Z_{w_i} |w| \\
M_q + M_{w_i} |w| \\
0
\end{array} \right] \\
G_4(\chi, e_u) &= \left[ \begin{array}{c}
0 \\
Z_q + Z_{w_i} |w| \\
M_q + M_{w_i} |w| \\
0
\end{array} \right] \\
G_5(\chi, \theta, \nu, \delta) &= \left[ \begin{array}{c}
0 \\
- \sin(\theta) \nu \\
- \sin(\theta) \nu \\
0
\end{array} \right]
\end{align*}

\textbf{Lemma 1}: There exists \( \zeta \in \mathbb{R}_{>0} \) such that

\[ \| G_1(\chi, e_u, \nu, \delta) \| \leq \zeta \]

on \( B_5 \times B_6 \times B_7 \).
Proof: From the facts that
\[ \| G \| \leq \| R^{-1} \| \left( \| [10m_1M_{12} \ldots M_{17} ]^T \| + \frac{1}{\mu_\rho} (\Delta_a + 2u_o) ) \| R \| \Delta_5, \]
on \( B \times B \times B \), we see that \( G \) is bounded.

Theorem 1: Consider (4) together with the proposed controllers
\[ \xi = m_1 \dot{x} + m_1 G_1[v, \delta - m_1 G_2] - m_1 \dot{v} \]
\[ \eta = \frac{1}{\mu_\rho} (\Delta_a + 2u_o) \]
If there exist \( P \in \mathcal{P} \), \( Q \in \mathcal{Q} \), \( K \) such that
\[ \begin{align*}
\text{Slm} \{ A_{\delta \xi} \} + P + HK \} + Q < 0 \\
E_a P a^T < \Delta_5 \\
E_u P b^T < \Delta_5 \\
E_1 P^T a_i < \Delta_5 \\
\left[ - \Delta_5 K \right] \leq 0
\end{align*} \]
for all \((i, i_o, i_p, i_t) \in J \times J \times J \times J \), then for
\[ \begin{bmatrix} x \\ v \end{bmatrix} \in \Omega = \{ \begin{bmatrix} x \\ v \end{bmatrix} \in \mathbb{R}^4 : \begin{bmatrix} x^T P + \omega \end{bmatrix} < 1 \}
\]
with \( \omega \in \mathbb{R}^4 \), \( \alpha \) asymptotically stabilizes (4), where \( E_a = [0 1 0 0] \), \( E_v = [0 0 1 0] \), and \( E_u = [0 0 0 1] \). In the feasible case, \( P = \tilde{P}^{-1} \) and \( K = \tilde{K} \tilde{P}^{-1} \).

Proof: By substituting (5) and (6) into (4), the closed-loop system becomes
\[ \begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} G_1 + HK \{ x, \dot{v}, \delta \} \\ 0 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} - \gamma \left[ \begin{bmatrix} x \\ v \end{bmatrix} \right] \]
Consider a Lyapunov function
\[ V(x, v_s) = x^T P + \omega \]
with \( P = P^T > 0 \) and \( \alpha \in \mathbb{R}^+ \). If LMI (7) holds on \( \Omega \), then we see that from Proposition 2, Lemma 1, and Schur complement,
\[ \dot{V} < 0 \iff x^T \text{Slm} \{ F(G + HK) \} x + 2x^T PKv + 2\gamma \omega \]
Fig. 3 The time response of $\theta$.

Fig. 4 The time response of $w$.

Fig. 5 The time response of $q$.

When $(z(0), \theta(0), u(0), w(0), q(0)) = (5.0, 1.0, 0.0, 0.0)$, $z_d = 20$, and $u_d = 3$, simulation results for (1) and (2) under (5) and (6) with $\gamma = 10$ are demonstrated in Figs. 1-5. From these figures, both $z$ and $w$ successfully enters in $z_d$ and $u_d$, respectively, while maintaining its attitude $(\theta, w, q)$.

4. Conclusions

This paper has presented an LMI-based design approach to the depth control of a class of LDUUVs. Unlike the previous result [8], the proposed approach includes the speed and attitude controls of the given depth-plane LDUUV. Our theoretical results has been successfully verified through the given numerical simulation.

Appendix

The hydrodynamic coefficients in the depth motion of LIG Nex1 LDUUV model, obtained by CFD and empirical formulations, are as follows.

When $\gamma = 0, L_{xyz} = 27801.3$, $x_g = 0$, $X_e = -157.7122$, $X_{uw} = -62.0022$, $X_{wuw} = 515.6301$, $X_{uww} = -5.6229 \times 10^4$, $X_{uwu} = -228.0492$, $Z_e = -2.8612 \times 10^3$, $Z_{w} = -2.9887 \times 10^3$, $Z_{uw} = -3.1868 \times 10^3$, $Z_{ww} = -2.7937 \times 10^3$, $Z_{uwu} = 1.3259 \times 10^3$, $Z_{uwk} = 295.5343$, $Z_{uw} = 295.5343$, $Z_{uwu} = 295.5343$, $Z_{uwk} = 295.5343$, $Z_{uw} = 295.5343$, $Z_{uwu} = 295.5343$, $Z_{uwk} = 295.5343$, $Z_{uw} = 295.5343$, $Z_{uwu} = 295.5343$, $Z_{uwk} = 295.5343$.

References


