Constitutive Model for a Confined Concrete Cylinder with an Unbonded External Steel Jacket

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Abstract

Early investigations focused mainly on manipulating the confinement effect to develop a reinforced concrete column with lateral hoops. Based on this legacy model, Li's model incorporated the additional confinement effect of a steel jacket. However, recent experiments on plain concrete cylinders with steel jackets revealed relatively large discrepancies in the estimates of strength enhancement and the post-peak behavior. Here, we describe a modified constitutive law for confined concrete with an unbonded external steel jacket in terms of three regions for the loading stage. We used a two-phase heterogeneous concrete model to simulate the uniaxial compression test of a 150 mm × 300 mm concrete cylinder with three thicknesses of steel jackets: 1.0 mm, 1.5 mm, and 2.0 mm. The proposed constitutive model was verified by a series of finite element analyses using a finite element program. The damaged plasticity model and extended Drucker-Prager model were applied and compared in terms of the level of pressure sensitivity for confinement in 3D. The proposed model yielded results that were in close agreement with the experimental results.

Keywords: Confined concrete, Heterogeneity, Steel Jacketing Method

1. INTRODUCTION

A total of 93 recorded earthquakes (magnitude 2.0~5.0) were reported in the Korean peninsula during 2013. The most recent earthquake (magnitude 5.1) hit the western part of Korea in April 2014. Although some evidence has pointed to increased seismic activity, many civil structures were constructed before the design code incorporated seismic load. This is a latent security issue for the Korean population. According to studies performed in the aftermath of earthquakes in Loma Prieta (1989) and Northridge (1994) in the United States and Kobe (1995) in Japan, bridge failures were generally caused by inadequate lateral reinforcements and insufficient lap splices of reinforcing bars. Considerable research has focused on seismic retrofit techniques for substandard RC columns. The application of fiber-reinforced polymer (FRP) as a jacket for columns vulnerable to seismic loads provides several benefits: high installation speed, low maintenance costs, and a high strength-to-weight ratio. However, the FRP jacketing system involves potential delamination issues because epoxy is used to paste the FRP sheets on RC columns. To overcome this disadvantage, Susantha et al. (1990) introduced a pre-compression method for concrete in a circular steel tube and improved its performance. Xiao and Wu (2000) used partially stiffened steel jackets to retrofit RC columns, to enhance strength and improve the ductility of concrete. Mortazavi et al. (2005) used pre-tension FRPs to strengthen RC columns. Choi and Rhee (2008) used epoxy-glued vertical steel-reinforced FRP strips to improve the lateral resistance of plain gravity bridge columns; these steel-reinforced FRP strips can be mounted automatically by a winding machine. Choi et al. (2009) introduced a new steel jacketing method to retrofit RC columns without grouting and assessed its performance using compressive tests for concrete cylinders.

The present study involved using external mechanical pressure on steel plates around RC columns to attach the steel plates onto the concrete surface. This experimental investigation involved two stages. During the first stage, 45 concrete cylinders (φ 150 mm × 300 mm) were fabricated with varying design concrete strengths (21, 27, and 35 MPa). Two split steel jackets and two vertical strip bands were used to confine each cylinder, as shown in Figure 1(a). Failure occurred at the welding line between the strips and jackets, but this jacketing method did not induce full plastic deformation of the steel jackets, so the failure strains were relatively small. During the second stage, 12 concrete cylinders were constructed in an attempt to improve the performance of the proposed jacketing method. In this test, a whole jacket of stainless steel and lateral strip bands [shown in Figure 1(b)] was used to confine cylinders, instead of using two...
split jackets. These jackets had thicknesses of 1.0, 1.5, and 2.0 mm, and dimensions of 290 mm × 481 mm (H × W). The plain cylinders had an average peak strength of 27 MPa. This method involving whole jackets with lateral strip bands induced full plastic deformation of the steel jackets.

Li et al. (2005) proposed a constitutive model of concrete confined by steel jackets and lateral reinforcement: in the model, the stress-strain relationship has two regions: 'Region I' and 'Region II', as shown in Figure 2(a). The strength at the intersection point, \( f'_{cl} \), is the function of the strength of plain concrete and lateral confining strength. By increasing a thickness of external steel jacket or an area of lateral reinforcements (hoops), corresponding lateral confining strength level would be altered accordingly. The determination of conjugate term, \( \varepsilon_d \) as paired to \( f'_{cl} \) could be drawn from experimental observation. Choi et al. (2009) conducted an experimental investigation using different thickness (1.0mm, 1.5mm, 2.0mm) of steel jacketed concrete cylinders as shown in Figure 2(b). Figure 2(b) depicted the axial compressive stress and strain curves of concrete cylinders (150mm × 300mm). Choi et al. (2009) calibrated the constitutive relations of steel jacketed concrete cylinder by extracting, \( \varepsilon_d \) out of experimental results in Figure 2(b) in terms of different level of confinement. However, Li’s model and Choi’s experiment involved discrepancies in the maximum stress and post-peak ductile behaviors. Li’s model overestimated the ductility of steel jacketed plain concrete columns as shown in Figure 2(b). We investigated Choi’s experimental results using nonlinear finite element modeling, and used the results to modify the equations used in Li’s model.

Concrete material is heterogeneous in nature. It has inclusions as hard aggregates onto a weak matrix (the cement paste). The failure pattern of these kinds of heterogeneous materials can be initiated at the interface zone between the aggregates and matrix and propagated toward other weak zones. To incorporate heterogeneity in the computer model, different sizes of spheres were used and positioned randomly, as shown in Figure 3(a). Aggregates were sized 10 mm (69), 15 mm (133), 20 mm (146), 30 mm (10), 40 mm (6), and 50 mm (3). Virtual sieve analysis revealed that the fineness modulus was 7.286, as shown in Figure 3(b), the maximum aggregate size was 30 mm, and the volume fraction was 27% of the standard cylinder volume, which was 150 mm (D) × 300 mm (H). These randomly positioned aggregates helped model the heterogeneous nature of concrete materials during uniaxial compressive testing.

Using the ABAQUS (SIMULIA, 2010) program, two different types of steel jackets were simulated for comparison with the results of Choi et al. (2009): a two-split strip type and a whole jacket type. The two-split strip type had basically welded lines, so that the welded region was the ‘weak’ interface or HAZ (heat-affected zone), as illustrated in Figure 4(a) and Figure 5(a). The heat during the welding process
and subsequent re-cooling causes changes in mechanical properties, normally resulting in a lower (60%) yield strength compared to the base material (two-split steel jacket). The steel jackets used were mild steel with a yielding strength of 217 MPa and a fracture strain of 207.6 × 10−3, as shown in Figures 5(a) and 5(b). Steel jackets were not as tall as the concrete cylinders, which guaranteed that no compressive force was transferred to the steel jackets. The heterogeneous concrete model had hard aggregates with elastic properties of \( E_a = 10,000 \text{MPa} \), \( \nu_a = 0.18 \) onto a weak cement paste of \( E_c = 7,000 \text{MPa} \), \( \nu_c = 0.217 \). Tetrahedral elements were used for meshing aggregates and the cement paste portion of 150 mm (D) × 300 mm (H) and the rigid loading platens. Shell elements were used for meshing the steel jackets. Surface contact conditions were applied on the rigid top and bottom platens against the concrete cylinder. Tangential behavior (friction, \( \mu = 0.2 \)) and normal behavior (hard contact, separation not allowed after contact) were activated on the contact surfaces. Similarly, the general contact surface was defined between the steel jackets and concrete cylinder laterally using \( \mu = 0.1 \). Separation was allowed after contact for each tangential and normal behavior.

3. UNCONFINED AND CONFINED MODEL

(1) Unconfined Model

Two unconfined concrete models were tested for possible manipulation of test results for plain concrete cylinders, to calibrate the constitutive relationship by Choi et al. (2009). First, the concrete damaged plasticity model (Lee and Fenves, 1998) is a continuum, plasticity-based, damage model for concrete. It assumes that the main two failure mechanisms are tensile cracking and compressive crushing of the concrete material. The evolution of the yield surface is controlled by two hardening variables in tension and compression, \( d_t, d_c \), separately, which are equivalent plastic strains, as depicted in Figure 6(a). Hence, the constitutive relations are governed by two independent scalar damage variables under the elasticity theory expressed by Eq (1).

\[
\sigma = (1-d)D_0^e : (\varepsilon - \varepsilon^p) = D^e : (\varepsilon - \varepsilon^p)
\]  

(1)

where \( d \) is a damage variable, which can be \( d_t \) and \( d_c \) in tension and compression, respectively. \( D_0^e \) is the initial stiffness of the concrete while \( D^e \) is the degraded stiffness. The yield condition of this model was based on Lubliner (1989) and modified by Lee and Fenves (1998). Table 1 summarizes the material properties used for FE analysis for both heterogeneous and homogeneous models for damaged plasticity. \( \sigma_t, G_F \) are the tensile strength and fracture energy (Mode I) for tensile softening behavior. \( \sigma_{tu} / \sigma_{to} \) is the ratio of initial equibiaxial and uniaxial compressive yield stress and is in the range from 1.10–1.16. \( K \) is a constant to reduce the tensile and compressive merit of the yield condition.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Elastic properties</th>
<th>Tensile behavior</th>
<th>Compressive behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E ) MPa</td>
<td>( \nu )</td>
<td>( \sigma_t ) MPa</td>
</tr>
<tr>
<td>1</td>
<td>Cement Paste</td>
<td>7,000</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>Aggregates</td>
<td>10,000</td>
<td>0.18</td>
</tr>
<tr>
<td>3</td>
<td>Concrete</td>
<td>9,000</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Second, since being first introduced by Drucker et al. (1952), the extended Drucker-Prager model (ABAQUS) is used to model frictional materials, which are typically granular-like soils, rock, and concrete. These materials exhibit a pressure-dependent yield. Because the external steel jackets reproduced the lateral confinement in a hydrostatic direction in conjunction with the thickness-diameter (t/D) ratio, the extended Drucker-Prager model might be more appropriate than the previous damage model:

\[
F(p,q) = a \cdot q^b - p - p_t = 0
\]  

(2)

where \( P \) is the hydrostatic pressure stress, \( p = -(1/3) \cdot \text{trace}(\sigma) \) and \( q \) is the von-Mises equivalent stress, \( q = \sqrt{(1/2) \cdot \sigma : \sigma} \), \( \sigma \) are the stress tensor and deviatoric stress tensor respectively.
$a, b$ are material parameters that are independent of the plastic deformation and $P_t$ is the hardening parameter, which represents the hydrostatic tension strength of the material, as shown in Figure 6(b). $P_t$ is defined as $a \cdot (\sigma / 3)$ if hardening is defined by uniaxial compression yield stress, $\sigma_c$. Table 2 summarizes the material properties used for FE analysis for both the heterogeneous and homogeneous models for the extended Drucker-Prager model using an exponential failure surface.

| Table 2. Material properties for the extended DP model (Heterogeneous and Homogeneous models). |
|---|---|---|---|
| Notation | Elastic properties | Compressive behavior |
| | $E$ MPa | $\nu$ | Dilatancy Angle | $a$ | $b$ |
| 1 | Cement Paste | 7,000 | 0.21 | 34 | 1.72 | 1 |
| 2 | Aggregates | 10,000 | 0.18 | 42 | 1.72 | 1 |
| 3 | Concrete | 9,000 | 0.21 | 37 | 1.72 | 1 |

Figures 6(c) and 6(d) illustrate the inelastic stress-strain relationship for the plasticity of compression and damage parameters $d_c, d_t$, to evaluate both tension and compression damage. Two different simulation models were calibrated for analysis in the homogenized concrete model and heterogeneous concrete model with hard aggregates.

![Figure 6(a)](image1)

Figures 6(c) and 6(d) illustrate the inelastic stress-strain relationship for the plasticity of compression and damage parameters $d_c, d_t$, to evaluate both tension and compression damage. Two different simulation models were calibrated for analysis in the homogenized concrete model and heterogeneous concrete model with hard aggregates.

![Figure 6(b)](image2)

Figures 7(a) and 7(b) present the calibration results for 27 MPa unconfined concrete cylinders under uniaxial compression tests for two different material models. The simulation results were in fairly close agreement with the experimental test results. Figures 7(c) and 7(d) illustrate the compression damage with the deformed shape of the damaged plasticity case and the von Mises stress contour with the deformed shape of extended Drucker-Prager model, respectively; both of these figures use a heterogeneous model. Figure 7(c) illustrates a plastic strain contour with the deformed shape of the extended Drucker-Prager model using a homogeneous model. Heterogeneous and homogeneous models yield different failure patterns, as shown in Figure 7, but their simulated stress-strain relationships are very similar. Heterogeneous models can reproduce more realistic behavior in both the failure pattern and the load-displacement curve (stress-strain curve). The data of experiment 1, 2 in Figure 7(a) and (b) were from compressive failure tests of plain concrete cylinder (27MPa) by Choi et al. (2009).
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Hence:

\[
f_{cc} = f_{co} + k_1 f_{I} \
\epsilon_{cc} = \epsilon_{co} \left( 1 + k_2 \frac{f_{I}}{f_{co}} \right)
\]

where \(f_{cc}, f_{co}\) are the confined and unconfined peak concrete strengths, respectively, and \(f_{I}\) is the level of confinement. Similarly, \(\epsilon_{cc}, \epsilon_{co}\) are the corresponding strains at the peaks for both confined and unconfined strength. Because these early investigations focused on confinement issues by lateral reinforcements, the confining stress, \(f_{I}\), was defined as \(0.5 \rho_s f_{sh}\). \(\rho_s\) is the ratio of the volume of transverse confining steel to the volume of the confined concrete core. \(f_{sh}\) is the yield strength of the transverse reinforcement. \(k_1, k_2\) are coefficients that are functions of the concrete mix and the lateral pressure. The average values of coefficients are \(k_1 = 4.1\) and \(k_2 = 5k_1\).

Li et al. (2005) proposed a constitutive model of concrete confined by steel reinforcements and steel jackets. Li’s model modified the format of the previous researchers in that the skeleton of the equation was similar to Eq. (3), but it also included the confinement effect of the steel jacket. The effect of elevation of lateral confinement was described using exponent \(a\) rather than the linear slope of \(k_1\). Hence:

\[
f_{cc} = f_{co} + (f_{II} + f_{ID})^a \
\epsilon_{cc} = \epsilon_{co} \left( 1 + b \frac{(f_{II} + f_{ID})}{f_{co}} \right)
\]

where \(f_{II}\) is \(k_1 f_{I}\) in Eq. (3), which represents lateral reinforcement, and \(f_{ID}\) represents confinement of the steel jacket, defined by \(2f_s(t/D)\). \(f_s\) is a hoop stress of the steel jacket and \(t/D\) is the thickness-diameter ratio of the cylindrical steel jacket. \(a, b\) are the coefficients of the power law equation, which can be obtained experimentally.

(a) Two-split strip type

Two-split steel jackets and two vertical strip bands were used to confine the cylinder, as shown in Figure 1. The measured uniaxial compression strength of the concrete cylinder itself was 33 MPa. The steel jackets had thicknesses of 1.0 mm and 1.5 mm, and dimensions of 230 mm × 290 mm (B/H). A lack of hydrostatic pressure sensitivity and excessive strain softening was observed in the case of the damage plasticity model, while the extended Drucker-Prager model yielded good confinement and the results were in fairly close agreement with experimental results. This confined concrete constitutive model was calibrated as shown in Figure 8(a). The failure strains of Figure 8(b) represent the strains of the jacketed cylinders, which varied from 3 to 6 times those in the plain cylinders. Failure occurred at the welding line between strips and jackets, as illustrated in Figure 8(c), in simulations and experiments. The ductility and post-peak behavior of the混凝土的应力-应变曲线，特别是其在高应变率下的表现，作为典型的地震加载情况。曼德 (Mander) 的研究考虑了受压强度和体积比等因素，以合理地反映混凝土在不同条件下的性能。斯科特等 (Scott et al., 1982) 改进了肯特和帕克 (Kent and Park) 的模型，通过加入未受压混凝土的影响来更好地模拟混凝土的真实性能。这些早期研究揭示了混凝土受压性能的限制，由横向 (箍) 钢筋的水平应力和厚度直径比定义。\(k_1, k_2\) 是混凝土和横向压力的函数。
jacketed cylinder depended on the welding condition between the split jackets. This jacketing method did not induce full plastic deformation of the steel jackets, and the failure strain was relatively small. Therefore, in this jacketing method, failure strain depends on the welding condition.

(a) Whole Jacket Type

Instead of two split jackets, a whole jacket of stainless steel and lateral strip bands was used to confine the cylinder, as shown in Figure 9. In this case, the dimension of the jacket was 290 mm × 481 mm (H×W). The width of the jacket was larger than the perimeter of the cylinder by 10 mm to lap one side over the other. The average peak strength of the plain cylinders was 27 MPa. The average peak strengths with the 1.0, 1.5, and 2.0 mm jackets were 40, 45, and 50 MPa respectively. Figures 9(a) and 9(b) present the confined constitutive models for different t/D ratios and their corresponding simulation results. The simulation results were fairly close to the experimental results. The two-region model (Lī’s model) matched experimental results closely in Region I. However, modestly large discrepancies were observed in Region II, because Lī’s model was based on the models of Mander and Park et al., with lateral reinforcement confinement. Later, Lī et al. (2005) incorporated the presence of an external steel jacket, so the resulting combination effect probably involved more strain hardening behavior than the more traditional model (Mander et al., 1988a). A cylinder confined by a 1.5 mm jacket had a strain of 0.06, which is 12 times greater than the failure strain of the plain concrete. It was not fractured at the yielding line in either simulations or experiments, as illustrated in Figure 9(c). Therefore, a whole jacket with lateral strip bands can produce full plastic deformation of steel jackets.
4. MODIFIED CONSTITUTIVE MODEL FOR STEEL JACKETING CONCRETE

The stress-strain relationship of the model has two regions (Region I and Region II), as shown in Figure 2(a). Choi et al. (2009) compared their experimental results with the results of Li’s model, as shown in Figure 2(b). However, Li’s model involved discrepancies in terms of the intersection point of Region I and II, and Region II hardening behavior. Choi et al. compared Li’s model with experimental results of whole jackets confining a cylinder. Figure 2(b) presents six experimental results for 1.0, 1.5, and 2.0 mm jackets and the corresponding models from Li et al. The comparison revealed that the experimental results are almost perfectly consistent with Li’s model in Region I. Just after Region I, strain hardening can be observed from the experimental curves, i.e., corresponding to the constitutive models. However, after that, strain softening develops in all jacketed cylinders, which is not observed in the modeled results. Hardening is maintained at the yield point of the steel jackets, and softening is observed after that. Li’s model also used reinforcement in the lateral direction, so softening may not be observed. The softening can be attributed to the opening of the welding line: if the stress of the steel jackets is greater than the welding strength, the welding line starts to open; this means that hardening cannot be continued and stiffness degradation appears. Initiation of the opening of the welding line does not indicate a fracture in the jackets, and the whole jacket system yields satisfactory ductile behavior.

Because Li’s model cannot incorporate stiffness softening after opening of the welding line, we propose modifying Li’s two-region model to a three-region model consisting of (1) a concrete cracking region, (2) a transition region, and (3) a steel jacket yielding (softening) region, as shown in Figure 10.

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![Figure 10. Proposed constitutive model of concrete confined by a steel jacket](image)

In the proposed model, two intersection points, $f'_{c1}$, $f'_{c2}$, are introduced in Eq. (5)-(7). The first intersection point and Region I curve are identical to those in Li’s model. The first intersection point, $f'_{c1}$, is a function of the strength of plain concrete $f'_{c0}$ and lateral confinement in terms of the diameter-thickness ratio $t/D$. In Region II, the second intersection point $f'_{c2}$, which is a peak point, can be calibrated by $t/D$ and the corresponding strain $\varepsilon_{c2}$ can be set to two times $\varepsilon_{c1}$ as shown in Eq. (8)-(9). This finding was extracted from Choi et al’s experiments and the transition region to the softening branch. For Region III, we propose an exponential softening function as in Eq. (10). This function of base $A$ was formulated from Choi’s results in terms of the steel jacket thickness and calibrated to the descending slope, i.e., the level of confinement. The nonlinear curve fitting process was used to determine $A$. Figure 11 illustrates the close match between the experimental results and the proposed three-region constitutive model and computer simulation results.

\begin{align}
(1) \text{Region I, } & \, 0 \leq \varepsilon_c \leq \varepsilon_{c1} \quad (\alpha = 1.18, \ \beta = 12.2) \\
& f'_c = f'_{c0} + 1.519 f'_c \alpha [\text{MPa}] \\
& \text{where, } f'_c = 2 \sigma_c(t/D) \\
& \varepsilon_{c1} = \varepsilon_{c0} \left[1 + \beta \frac{f'_c}{f'_{c0}}\right] \quad \text{where, } \varepsilon_{c0} = 0.002 \\
& f'_c = f'_{c1} \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right) \left(n - 1\right) \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right) + \left(2 - n\right) \\
& \text{where, } n = 0.1 + 0.075\left(f'_c/f'_{c0}\right)
\end{align}

\begin{align}
(2) \text{Region II, } & \, \varepsilon_{c1} \leq \varepsilon_c \leq \varepsilon_{c2} \\
& \varepsilon_{c2} = 2 \varepsilon_{c1} \\
& f'_c = f'_{c2} \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^{2n-\sqrt{(t/D)}-0.066} \quad \text{where, } f'_{c2} = f'_c(\varepsilon_{c2})
\end{align}

\begin{align}
(3) \text{Region III, } & \, \varepsilon_{c1} \geq \varepsilon_{c2} \\
& f'_{c} = f'_{c3} A \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right) \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^{-0.925} \\
& \text{where, } A = 1.00116 + 0.57662 \varepsilon^{-205(t/D)}
\end{align}

![Figure 11. Simulation model for a heterogeneous composite of a concrete cylinder with steel jacket: experiment, analysis and proposed constitutive model](image)
5. CONCLUSIONS

An early constitutive model for confined concrete with lateral confinement mainly considered lateral confinements using hoop reinforcements. Later, this model was modified with the presence of an external steel jacket, and has since been widely used by many researchers. However, this model can overestimate post-peak behaviors for confined concrete with no lateral reinforcement. Therefore, we proposed a modified constitutive model for confined concrete with a steel jacket, based on previous experimental observation. We performed numerical simulations with two different material laws: a damaged plasticity model, and an extended Drucker-Prager model. Additionally, we used a two-phase composite model (aggregates, cement matrix) to obtain an accurate steel jacket pattern. The simulated results were in close agreement with experimental results when the extended Drucker-Prager model was used; this model can incorporate pressure sensitivity in terms of a level of lateral confinement, which in the present study was the thickness of the steel jacket and the welding method. In contrast, the damaged plasticity model revealed early softening without respect to the elevation of lateral confinement. Because Li’s model has a different perspective regarding post-peak behavior because of hoop reinforcement, we proposed a three-region model in terms of the thickness-diameter ratio t/D and compared it with Li’s two-region model. This model produced results identical to Li’s for Region I. However, the transition zone (Region II) can exhibit stiffness reduction due to confined concrete fracture. The results for Region III reflect the yielding behavior of welded steel jacket due to excessive lateral deformation. This lateral deformation could be caused by the heterogeneity of concrete composite as well as a dilatant tendency after crushing of the concrete inside the steel jacket. This process leads to post-peak softening behavior depending on the level of confinement, t/D. This newly proposed three-region model yields results that are in close agreement with previous experimental results.

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