Comparison between Parametric and Semi-parametric Cox Models in Modeling Transition Rates of a Multi-state Model: Application in Patients with Gastric Cancer Undergoing Surgery at the Iran Cancer Institute

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Abstract

Background: Research on cancers with a high rate of mortality such as those occurring in the stomach requires using models which can provide a closer examination of disease processes and provide researchers with more accurate data. Various models have been designed based on this issue and the present study aimed at evaluating such models. Materials and Methods: Data from 330 patients with gastric cancer undergoing surgery at Iran Cancer Institute from 1995 to 1999 were analyzed. Cox-Snell Residuals and Akaike Information Criterion were used to compare parametric and semi-parametric Cox models in modeling transition rates among different states of a multi-state model. R 2.15.1 software was used for all data analyses. Results: Analysis of Cox-Snell Residuals and Akaike Information Criterion for all probable transitions among different states revealed that parametric models represented a better fitness. Log-logistic, Gompertz and Log-normal models were good choices for modeling transition rate for relapse hazard (state 1→state 2), death hazard without a relapse (state 1→state 3) and death hazard with a relapse (state 2→state 3), respectively. Conclusions: Although the semi-parametric Cox model is often used by most cancer researchers in modeling transition rates of multi-state models, parametric models in similar situations-as they do not need proportional hazards assumption and consider a specific statistical distribution for time to occurrence of next state in case this assumption is not made- are more credible alternatives.

Keywords: Gastric cancer - multi-state model - parametric model - proportional hazards model - transition rate

Introduction

Gastric cancer is one of the most common causes of cancer deaths all over the world. Every year, more than 870 thousand new cases are reported throughout the world and more than 650 thousand people die from this type of cancer (Parkin, 1998). According to the latest statistics of Iran Cancer Research Center, gastric cancer is the most common cancer among Iranian men and the third most common cancer among Iranian women after breast cancer (Mohagheghi et al., 1998; Mohagheghi, 2004). One of the most important objectives specified after the right diagnosis and prompt treatment for the patients with gastric cancer is the survival rate increase especially the 5-year survival rate. Unfortunately, more than 80% of patients with gastric cancer are diagnosed at a stage when common treatments such as gastrectomy, chemotherapy, or radiation therapy are not effective in increasing the patients’ survival (Gunderson and Sosin, 1982; Wisbeck et al., 1986; Sadighi et al., 2005; Samadi et al., 2007; Sadighi et al., 2008; Association, 2011). For this reason, the 5-year survival rate is low in patients with gastric cancer after surgery (Thong-Ngam et al., 2001; Triboulet et al., 2001; Schwarz and Zagala-Nevarez, 2002; Adachi et al., 2003; Ding et al., 2004). The increase in these patients’ survival after surgery requires using models which could provide a closer examination on the behavior of variables so that it will better describe the natural process of the disease and will provide the researchers with more accurate data.

One of the statistical models designed to accomplish this purpose is the multi-state model. According to this model, patients experience different states (save for death event) during the study from the beginning to death event. The time reaching each state and factors affecting its occurrence play a fundamental role in patients’ survival. Considering these states (often called intermediate events) has developed a novel approach in survival studies, because the natural process of disease in such cases can be considered as a stochastic process in which patients can be placed in various states throughout the study.
Most studies on cancer regard the death event as the only probable occurrence for patients, but in many cases, events other than death event occur in patients during the study period which may affect the final results. Relapse is an obvious example for these events. Not only does it affect patients’ survival as a variable but its occurrence is also influenced by different factors.

Standard models of survival are the simplest of multi-state model. In these models, the patient is “Alive” at the beginning of the study and then his/her state may change to “Death”. This is the only transition which is considered for patients during the study. This transition can be illustrated as show in Figure 1.

Models more complex than two-state, provide the patients with the probability of more transition during the study. These models are used when the initial state of the patient, i.e. “being alive”, is itself divided to two or more other states. The number of division depends on the type of disease. In the process of gastric cancer, the survival time of patient is recorded since the patient has undergone surgery. After surgery the patient enters the study and is subjected to death hazard. In these studies, the occurrence of death and relapse are considered as the end point of the study and the intermediate even respectively. This modeling is schematically shown in Figure 2.

The most important practical application of multi-state models is transition rate modeling among states in each transition. There are various statistical methods for this purpose including parametric and semi-parametric models. These models are divided into two main categories: Proportional Hazard model and Accelerated Failure-time model. In the proportional hazard model, modeling is done based on hazard function. In this case, if baseline hazard is considered parametric, one of the exponential, Weibull or Gompertz models will be obtained. If the baseline hazard is considered indefinite, the semi-parametric Cox model will be achieved. In the accelerated failure-time model, modeling is done on time logarithm to the occurrence of next state. The obtained models in this case include: Exponential, Weibull, Log-normal, Log-logistic, Gompertz and Generalized gamma. It should be noted that the Weibull and exponential models are the only ones that have, both, the PH and AFT features.

The semi-parametric Cox model does not need to consider a specific probability distribution for time to the occurrence of next state; therefore, it is the most useful model in modeling transition rates of multi-state models (Hougaard, 1999; Yagi et al., 2000; Andersen and Keiding, 2002; Adachi et al., 2003; Buonadonna et al., 2003; Chau et al., 2004; Zeraati et al., 2005; Dehkordi and Tabatabae, 2007; Biglarian et al., 2009). But this model is severely affected by proportional hazards assumption and, for this reason, is often called Cox proportional hazard model. In cases where the proportional hazard model is not tenable, inferences derived from this model will be flawed and will have a bias (Hougaard, 1999; Collett, 2003; Klein and Moeschberger, 2003; Hosmer Jr et al., 2011). Accelerated failure-time models are particularly important in such circumstances. These models—due to having a specific probability distribution for time to the occurrence of next state — make statistical inference more accurate and cause the standard errors of the estimations to be smaller in ratio with the time when there were no such assumptions. Modeling of transition rates in multi-state models is often identified by Cox proportional hazard model (Klein et al., 1984; Kay, 1986; Andersen, 1988; Hougaard, 1999; Andersen and Keiding, 2002; Klein and Moeschberger, 2003; Putter et al., 2007; De Wreede et al., 2010; Jackson, 2011). Neither have these studies generally used proportional hazards assumption nor did they attempt to identify a suitable parametric model as an alternative to Cox proportional hazard model. The main problem in the application of multi-state models is the need to determine the most appropriate model in modeling transition rates. So, in addition to comparing parametric and Cox semi-parametric models in modeling transition rates among different states, Akaike Information Criterion and Cox-Snell Residuals have been also used in this study to assess these models.

**Materials and Methods**

In this study, 330 patients with gastric cancer with the following data were studied: i) the patients had been hospitalized and had undergone surgery from 1995 to 1999 in surgical wards of Cancer Institute of Iran; ii) they had records in the archives of the hospital, and in their files their addresses and phone numbers were available for subsequent follow-up. The survival time of patients was determined after surgery and those patients who were still alive at the end of study period or the ones whose data were not available after a specific time-period were considered right censored.

Since it is common to use the Cox proportional hazards model in modeling transition rates of multi-state models, there is a risk that if the assumption of proportional hazards is not fulfilled, the results will not be reliable enough. Although some researchers tend to turn a blind eye to this defect in their researches due to the ease of Cox’s model’s application and its interpretations, it is essential to use alternative models with a higher degree of reliability such as parametric models for more precise investigations in such cases. Therefore, in this study to compare parametric models including Exponential, Weibull, Log-normal, Log-logistic, Gompertz, and Generalized gamma as well as
Cox semi-parametric model in modeling transition rates among different states, a multi-state model with three states of patient’s being alive without a relapse (state 1), relapse (state 2) and death (state 3) was considered. Moreover, Cox-Snell Residuals and Akaike Information Criterion were used to assess these models properly. Cox-Snell Residuals is a graphical criterion for assessing the fitness of parametric and semi-parametric models; the less deviation of residuals from the bisector, the more appropriate fitness of model (Weissfeld and Schneider, 1990; Escobar and Meeker Jr, 1992; Collett, 2003). Graphical methods are often associated with optical illusion. For a better judgment, thus, Akaike information criterion can be used along with Cox-Snell residuals. Akaike information criterion is used to measure the goodness of statistical models’ fitness, and the smaller it is, the better it is (Collett, 2003; Klein and Moeschberger, 2003). AIC for the models used in this study has been calculated according to the following equation: \[ \text{AIC} = -2 \times \log(\text{likelihood}) + 2(p+k) \]

In which \( p \) is the number of parameters in the model and \( k \) is a constant coefficient which has been used depending on the type of model. For example, \( k=2 \) for Weibull model (Klein and Moeschberger, 2003). The smaller the AIC is, the better it is (Collett, 2003; Klein and Moeschberger, 2003). AIC for the models used in this study has been calculated according to the following equation: \[ \text{AIC} = -2 \times \log(\text{likelihood}) + 2(p+k) \]

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### Results

Figure analysis of Cox-Snell residuals for parametric models and Cox semi-parametric model in modeling transition rates in three transitions; relapse hazard (state 1–state 2), death hazard without a relapse (state 1–state 3) and death hazard with a relapse (state 2–state 3) represents better fitness of parametric models compared with Cox semi parametric model. The figure of Cox-Snell residuals for relapse hazard (state 1–state 2) shows that among parametric models, Log-logistic model has a better fitness to data (Figure 3). Furthermore, analysis results of these residuals to compare parametric and Cox semi-parametric models in modeling transition rates for death hazard without a relapse (state 1–state 3) also revealed that among parametric models Gompertz proved better fitness to the data (Figure 4). Besides, the analysis of these residuals in modeling transition rates for death hazard with a relapse (state 2–state 3) showed that Log-normal model is a suitable alternative for Cox semi-parametric model in modeling this transition rate (Figure 5). Akaike information criterion confirms these results too. Based on this criterion, Log-logistic model in modeling transition rate state 1–state 2, Gompertz in modeling transition rate state 1–state 3 and Log-normal in modeling transition rate state 2–state 3 are the best models (Table 1).
Table 1. The Comparison Results of the Akaike Information Criterion (AIC) between the Cox Proportional Hazard and Parametric Models

<table>
<thead>
<tr>
<th>Models</th>
<th>state 1→state 2</th>
<th>state 1→state 3</th>
<th>state 2→state 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cox</td>
<td>414.2</td>
<td>1884.5</td>
<td>231</td>
</tr>
<tr>
<td>Exponential</td>
<td>293.9</td>
<td>809.7</td>
<td>163.3</td>
</tr>
<tr>
<td>Weibull</td>
<td>290.4</td>
<td>807.8</td>
<td>162.4</td>
</tr>
<tr>
<td>Log-logistic</td>
<td>285.5</td>
<td>808</td>
<td>161.8</td>
</tr>
<tr>
<td>Log-normal</td>
<td>290</td>
<td>863.9</td>
<td>160.5</td>
</tr>
<tr>
<td>Gompertz</td>
<td>294</td>
<td>805.5</td>
<td>163.8</td>
</tr>
<tr>
<td>Gamma</td>
<td>288.8</td>
<td>809.7</td>
<td>161.3</td>
</tr>
</tbody>
</table>

![Graphs showing Cox-Snell residuals for different distributions](image)

**Figure 5. The Cox-Snell Residuals in the Considered Cox Proportional Hazard and Parametric Models in Modeling Transition Rate state 2→state 3**

**Discussion**

Most cancer researchers tend to use Cox semi-parametric model rather than parametric models in modeling transition rates among different states in a multi-state model. A systematic review on cancer journals indicates that only 5% of studies in which Cox model has been used for modeling transition rates among different states, investigated the required assumptions for this model (Altman et al., 1995). The absence of proportional hazards assumption causes the estimations of transition rates among different states to be unreliable and biased. Moreover, studies conducted in this scope demonstrate that either proportional hazards assumption is made or not, parametric models are more efficient (Orbe et al., 2002; Patel et al., 2006). Therefore, parametric models such as exponential, Weibull, log-normal, log-logistic, Gompertz and gamma can be better choices in such situations. Considering a particular statistical distribution for time to the occurrence of next state and requiring no assumption of proportional hazards (PH), these models provide fitness for data.

A major objective of this paper is to investigate the comparative performance of Cox semi-parametric and parametric survival models in modeling transition rates of multi-state models. So, in this study the results of Cox semi-parametric model and parametric models were compared in modeling transition rates of a multi-state model with three states of patient’s being alive without a relapse (state 1), relapse (state 2) and death (state 3). To assess these models, Akaike information criterion (AIC) and Cox-Snell residuals were used. The analysis of Cox-Snell residuals for all probable transitions among states revealed that parametric models had better fitness. This finding is consistent with the findings obtained from most studies carried out on patients with gastric cancer (Orbe et al., 2002; Nardi and Schemper, 2003; Dehkordi, 2007; Pourhoseingholi et al., 2007). In the meanwhile, Log-logistic, Gompertz, and Log-normal were suitable choices for modeling transition rate state 1→state 2, modeling transition rate state 1→state 3, and modeling transition rate state 2→state 3, respectively. In addition, the analysis of models based on Akaike information criterion also confirmed the results obtained from Cox-Snell residuals (Table 1).

Although most researchers in medical and cancer fields have made use of Cox semi-parametric models in modeling transition rates of a multi-state model into account, results of parametric models have often been more reliable and have had less bias. As parametric models do not need proportional hazards assumption (PH) in similar situations and they consider a specific statistical distribution for time to the occurrence of next state, they have a better fitness. Parametric models will also be credible alternatives to Cox semi-parametric model where proportional hazard assumption is not made. In addition, fully parametric models may offer some advantages. Based on asymptotic results, parametric models lead to more efficient parameter estimates than Cox model. With a decrease in sample sizes, relative efficiencies may further change in favor of parametric models. When empirical information is sufficient, parametric models can provide some insights into the shape of the baseline hazard.

**References**


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