GROUP ACTIONS IN A UNIT-REGULAR RING WITH COMMUTING IDEMPOTENTS

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Abstract. Let $R$ be a ring with unity, $X$ the set of all nonzero, nonunits of $R$ and $G$ the group of all units of $R$. We will consider some group actions on $X$ by $G$, the left (resp. right) regular action and the conjugate action. In this paper, by investigating these group actions we can have some results as follows: First, if $E(R)$, the set of all nonzero nonunit idempotents of a unit-regular ring $R$, is commuting, then $o_\ell(x) = o_r(x)$, $o_c(x) = \{x\}$ for all $x \in X$ where $o_\ell(x)$ (resp. $o_r(x)$, $o_c(x)$) is the orbit of $x$ under the left regular (resp. right regular, conjugate) action on $X$ by $G$ and $R$ is abelian regular. Secondly, if $R$ is a unit-regular ring with unity 1 such that $G$ is a cyclic group and $2 = 1 + 1 \in G$, then $G$ is a finite group. Finally, if $R$ is an abelian regular ring such that $G$ is an abelian group, then $R$ is a commutative ring.

1. Introduction and basic definitions

Let $R$ be a ring with unity, $X$ the set of all nonzero, nonunits of $R$ and $G$ the group of all units of $R$. In this paper, we will consider some group actions of $G$ on $X$. We call the action, $((g, x) \rightarrow gx)$ (resp. $((g, x) \rightarrow xg^{-1}), ((g, x) \rightarrow gxg^{-1})$) from $G \times X$ to $X$, left regular (resp. right regular, conjugate) action. If $\phi : G \times X \rightarrow X$ is one of the above group actions, then for each $x \in X$ we define the orbit of $x$ by $o(x) = \{\phi(g, x) : g \in G\}$ and stabilizer of $x$ by $\text{stab}(x) = \{g \in G : \phi(g, x) = x\}$. Recall that $G$ is transitive on $X$ (or $G$ acts transitively on $X$) if there is an $x \in X$ with $o(x) = X$ and the group action on $X$ by $G$ is trivial if $o(x) = \{x\}$ for all $x \in X$.

A ring $R$ is von Neumann regular (or simply regular) (resp. unit-regular) provided that for any $a \in R$ there exists an element $r \in R$ (resp. $u \in G$) such that $a = aru$ (resp. $a = auu$). A ring $R$ is strongly regular provided that for any $a \in R$ there exists an element $r \in R$ such that $a = ra^2$. Also a ring $R$ is abelian provided all idempotents in $R$ are central. It is known [1] that $R$ is

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an abelian regular ring if and only if \(R\) is strongly regular and that an abelian regular ring is unit-regular.

Throughout this paper, unless stated otherwise, \(R\) is a ring with unity 1, \(G\) is the group of all units of \(R\) and \(X\) is the set of all nonzero, nonunits in \(R\). Also for each \(x \in X\), \(o_l(x)\) (resp. \(o_r(x), o_c(x)\)) is considered as the orbit of \(x\) under the left regular (resp. right regular, conjugate) action of \(G\) on \(X\). Let \(E(R)\) be the set of all nonzero, nonunit idempotents of \(R\). Recall that \(E(R)\) is said to be commuting if \(ef = fe\) for all \(e, f \in E(R)\). We use \(||\) to denote the cardinality of a set.

It was shown in [3, Lemma 2.3, Theorem 3.3] that \(R\) is unit-regular if and only if every orbit under the left regular action is \(o_l(e)\) for some idempotent \(e \in X\) and that if \(R\) is a unit-regular ring such that \(G\) is a cyclic group and \(2 = 1 + 1 \in G\), then the orbit \(o_l(e)\) is finite. In Section 2, we show that (1) if \(R\) is a unit-regular ring such that \(E(R)\) is commuting, then (1) \(o_l(x) = o_r(x)\) for all \(x \in X\) and \(R\) is abelian regular ring; (2) if for a ring \(R\) such that \(G\) is a cyclic group and \(2 = 1 + 1 \in G\) there exists an idempotent \(e \in X\) such that \(2e = (1 + 1)e \neq 0\), then \(o_l(1 - e)\) (resp. \(o_r(1 - e)\)) is finite; (3) if \(X \neq \emptyset\) for a unit-regular ring \(R\) such that \(G\) is a cyclic group and \(2 = 1 + 1 \in G\), then \(G\) is finite. We also show that if \(R\) is an abelian regular ring such that \(E(R)\) is finite, then \(R\) is isomorphic to the direct sum of a finite number of division rings.

It was shown in [3, Theorem 3.2] that if \(R\) is a unit-regular ring with unity 1 such that \(G\) is abelian and \(2 = 1 + 1 \in G\), then \(R\) is a commutative ring. In Section 3, we show that if \(R\) is a ring with unity such that \(E(R)\) is commuting, then \(o_l(e) = \{e\}\) for all \(e \in E(R)\), i.e., \(ge = eg\) for all \(g \in G\). By using this result we also show that if \(R\) is an abelian regular ring such that \(G\) is abelian, then \(R\) is a commutative ring.

2. Regular action in unit-regular rings

Recall that a nonzero element \(a\) in a ring \(R\) is said to be a right zero-divisor if there exists a nonzero \(b \in R\) such that \(ba = 0\).

The following theorem has been proved in [2]:

**Theorem 2.1.** Let \(R\) be a ring such that \(X\) is a finite union of orbits under the left regular action on \(X\) by \(G\). Then \(X\) is the set of all right zero-divisors of \(R\). Moreover, if \(X\) is a nonempty finite set, then \(R\) is a finite ring.

**Proof.** Refer [2, Theorem 2.2]. \(\square\)

**Lemma 2.2.** Let \(R\) be a unit-regular ring. Then for all \(x \in X\), \(x\) is a zero-divisor.

**Proof.** Since \(R\) is a unit-regular ring, for \(x \in X\) there exists an element \(g \in G\) with \(x = xgx\), and so \(x(gx - 1) = 0 = (xg - 1)x\). If \(gx - 1 \in G\), then \(x = 0\), which is a contradiction. If \(gx - 1 = 0\), then \(gx = 1\), and so \(x = g^{-1} \in G\),
which is also a contradiction. Thus \( gx - 1 \in X \). Similarly, we have \( xg - 1 \in X \). Hence \( x \) is a zero-divisor. □

**Lemma 2.3.** The ring \( R \) is unit-regular if and only if every orbit under the left regular action is \( \alpha(e) \) for some idempotent \( e \in X \).

**Proof.** Refer [3, Lemma 2.3]. □

**Corollary 2.4.** The ring \( R \) is unit-regular if and only if every orbit under the right regular action is \( \alpha_r(e) \) for some idempotent \( e \in X \).

**Proof.** It follows by an argument similar to that in the proof of [3, Lemma 2.3]. □

**Remark 1.** Note that if \( R \) is a noncommutative ring, then \( \alpha_l(x) \neq \alpha_r(x) \) for some \( x \in X \). For example, let \( R = \left( \begin{array}{cc} \mathbb{Z}_2 & \mathbb{Z}_2 \\ \mathbb{Z}_2 & \mathbb{Z}_2 \end{array} \right) \) be the ring of \( 2 \times 2 \) matrices over \( \mathbb{Z}_2 \), a Galois field of order 2, and take \( x = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) \in X \). Then \( \alpha_l(x) = \left\{ \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right), \left( \begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array} \right), \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right) \right\} \neq \left\{ \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right), \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right) \right\} = \alpha_r(x) \).

**Lemma 2.5.** If \( E(R) \) is commuting, then \( \alpha_l(e) \cap E(R) = \{ e \} \) (resp. \( \alpha_r(e) \cap E(R) = \{ e \} \)) for all \( e \in E(R) \).

**Proof.** Let \( e_1 \in \alpha_l(e) \cap E(R) \). Then \( e_1 = ge \) for some \( g \in G \). Thus \( e_1e = (ge)e = ge = e_1 \). Since \( e = g^{-1}e_1, e = ee_1 \). Since \( E(R) \) is commuting, \( e = ee_1 = e_1e = e_1 \). Hence \( \alpha_l(e) \cap E(R) = \{ e \} \). Similarly, we have \( \alpha_r(e) \cap E(R) = \{ e \} \). □

**Corollary 2.6.** Let \( R \) be a unit-regular ring. If \( E(R) \) is commuting, then for all \( x \in X, \alpha_l(x) \cap E(R) = \{ e \} \) (resp. \( \alpha_r(x) \cap E(R) = \{ f \} \)) for some \( e \in E(R) \) (resp. \( f \in E(R) \)).

**Proof.** It follows from Lemma 2.3 and Lemma 2.5. □

Note that if \( R \) is a unit-regular ring such that \( E(R) \) is commuting, then the number of orbits under the left (resp. right) regular action on \( X \) by \( G \) is equal to the cardinality of \( E(R) \) by Lemma 2.3 and Corollary 2.6.

**Theorem 2.7.** If \( E(R) \) is commuting, then \( \alpha_l(e) = \alpha_r(e) \) for all \( e \in E(R) \).

**Proof.** Let \( e \in E(R) \) be arbitrary. Then \( \alpha_l(e) \subseteq \alpha_r(e) \) for some \( e_1 \in E(R) \). Indeed, if \( y \in \alpha_l(e) \) is arbitrary, then \( y = ge \) for some \( g \in G \). Thus \( e = g^{-1}y = (g^{-1}y)(g^{-1}y) = e^2 \), and then \( y = yg^{-1}y \). Let \( e_1 = yg^{-1} \). Hence \( e_1 \in E(R) \) and \( y = e_1g \in \alpha_r(e) \). Hence \( \alpha_l(e) \subseteq \alpha_r(e_1) \). Similarly, we can have that \( \alpha_r(e_1) \subseteq \alpha_l(e) \) for some \( e_2 \in E(R) \). Thus \( e \in \alpha_l(e) \subseteq \alpha_r(e_1) \subseteq \alpha_l(e_2) \).

Since \( E(R) \) is commuting, \( \alpha_l(e_2) \cap E(R) = \{ e_2 \} \) and so \( e = e_2 \). Therefore, \( \alpha_l(e) \subseteq \alpha_r(e_1) \subseteq \alpha_l(e) \), which implies that \( \alpha_l(e) = \alpha_r(e_1) \), and thus \( e_1 = e \) by Lemma 2.5. Consequently, \( \alpha_l(e) = \alpha_r(e) \) for all \( e \in E(R) \). □
Corollary 2.8. Let $R$ be a unit-regular ring. If $E(R)$ is commuting, then $o_X(x) = o_r(x)$ for all $x \in X$.

Proof. Let $x \in X$ be arbitrary. Then $o_X(x) = o_r(e) = o_r(e)$ for some $e \in E(R)$ by from Lemma 2.3 and Theorem 2.7. Since $x \in o_r(e)$, $o_X(x) = o_r(e)$. Hence we have $o_X(x) = o_r(x)$ for all $x \in X$.

Lemma 2.9. Let $R$ be a unit-regular ring. If $o_X(x) = o_r(x)$ for all $x \in X$, then $R$ is abelian regular.

Proof. By [1, Theorem 3.2], it is enough to show that $R$ has no nonzero nilpotent elements. Assume that there exists a nonzero nilpotent element $x \in R$ such that $x^n = 0 \neq x^{n-1}$ for some positive integer $n$. By Lemma 2.3, $x = ge$ for some idempotent $e \in X$ and some $g \in G$. Since $o_X(x) = o_r(x)$, $0 = x^n = he^n$ for some $h \in G$. Thus $e^n = e = 0$, which is a contradiction.

Corollary 2.10. Let $R$ be a unit-regular ring. Then $E(R)$ is commuting if and only if $R$ is abelian regular.

Proof. If $E(R)$ is commuting, then $R$ is abelian regular by Corollary 2.8 and Lemma 2.9. The converse is clear.

Remark 2. If $R$ is a unit-regular ring in which $X = E(R)$, then $R$ is abelian regular.

Theorem 2.11. Let $R$ be a unit-regular ring. Then the following are equivalent:

1. $X = E(R)$;
2. the left (resp. right) regular group action on $X$ by $G$ is trivial;
3. $R$ is a Boolean ring in which $G = \{1\}$.

Proof. (2) $\Rightarrow$ (1). It follows from Lemma 2.3 and Corollary 2.4. (1) $\Leftrightarrow$ (2). Suppose that $X = E(R)$. Then $o_X(e) = o_r(e) = \{1\}$ for all $e \in E(R)$ by (1) $\Rightarrow$ (2). Assume that $G \neq \{1\}$. Then there exist $g, h \in G$ such that $g \neq h$. Since $ge = e = he$ for any $e \in X = E(R)$, $(g - h)e = 0$. If $g - h \in G$, then $e = 0$, a contradiction. Thus $g - h \in X = E(R)$. Since $o_X(g - h) = o_r(g - h) = \{g - h\}$, we have $g - h = g(g - h) = (g - h)g$, and so $gh = hg$. Also we have $g - h = g(g - h) = (-h)(g - h)$, and so $g^2 = h^2$. Since $g - h \in X = E(R)$, $g - h = (g - h)^2 = g^2 - 2gh + h^2 = 2g^2 - 2gh = 2(g(g - h)) = 2(g - h)$, and then $g - h = 0$, which is a contradiction. Therefore $G = \{1\}$. Since $X = E(R)$ and $G = \{1\}$, $R$ is a Boolean ring.

(3) $\Rightarrow$ (2). Clear.

Example 1. Let $R = \prod_{i=1}^{\infty} \mathbb{Z}_2$ where $\mathbb{Z}_2$ is a galois field of order 2. Then $R$ is a unit-regular ring such that $X = E(R)$, and is equivalently a Boolean ring in which $G = \{1\}$ by Theorem 2.11.
Theorem 2.12. Let $R$ be an abelian regular ring. If $E(R)$ is finite, then $R \cong D_1 \times D_2 \times \cdots \times D_n$ where all $D_i$ are division rings for some positive integer $n$. In fact, $|E(R)| = 2^n$.

Proof. Since $E(R)$ is finite, there exists a finite number of orbits under the left regular action on $X$ by $G$ by Lemma 2.3. Observe that every left ideal of $R$ is $G$-invariant and is a union of orbits under the left regular action. Since there exists a finite number of orbits under the left regular action, every left ideal of $R$ is a union of finite number of orbits under the left regular action. Hence $R$ is a left artinian ring. Since $E(R)$ is central, by the Wedderburn-Artin Theorem we have $R \cong D_1 \times D_2 \times \cdots \times D_n$ where all $D_i$ are division rings for some positive integer $n$ and $|E(R)| = 2^n$. 

Corollary 2.13. Let $R$ be an abelian regular ring. If $E(R)$ is finite, then the following are equivalent:

1. $G$ is finite;
2. $X$ is finite;
3. $R$ is finite.

Proof. (1) $\Rightarrow$ (2). Let $|E(R)| = n$. Then $X$ is the union of $n$ orbits $o(x_1), \ldots, o(x_n)$ for some $x_1, \ldots, x_n \in X$ by Corollary 2.6. Since $G$ is finite, $X$ is clearly finite. (2) $\Rightarrow$ (3). It follows from Theorem 2.1.

(3) $\Rightarrow$ (1). It is clear. 

Theorem 2.14. Let $R$ be a ring such that $G$ is a cyclic group. If $e \in X$ is an idempotent such that $2e \neq 2(= 1 + 1)$, then the orbit $o_1(e)$ (resp. $o_r(e)$) is finite.

Proof. If $o_1(e) = \{e\}$ or $G = \{1\}$ for an idempotent $e \in X$, then $o_r(e) = \{e\}$, and so $o_r(e)$ is finite. Suppose that $o_r(e) \neq \{e\}$ and $G \neq \{1\}$. Then $|o_r(e)| > 1$ and $\text{Stab}(e) = \{g \in G | ge = e\}$ is a proper subgroup of $G$. Let $H = \text{Stab}(e)$ and let $a$ be a generator of $G$. Since $e \in X$ is an idempotent and $2e \neq 2$, $2e - 1(\neq 1) \in G$. Thus $(2e - 1)e = e$ implies that $2e - 1 \in H$ and so $H \neq \{1\}$. Since $H$ is a proper subgroup of $G$, $H$ is generated by $a^s$ for some nonnegative integer $s$ ($s \geq 2$). Since $a^s \in H$, $a^s e = e$. For all $g \in G$, $g = a^m$ for some $m \in \mathbb{Z}$. By the division algorithm for $\mathbb{Z}$, $m = r + qs$ form some $g, r \in \mathbb{Z}$, where $s - 1 \geq r \geq 0$. Thus for all $g \in G$, $ge = a^m e = a^{r + qs} e = a^s e$. Therefore $o_r(e) = \{a^se : 0, 1, \ldots, s - 1\}$ is finite. Similarly, we can show that $o_r(e)$ is finite for an idempotent $e \in X$ such that $2e \neq 2$. 

Corollary 2.15. Let $R$ be a ring such that $G$ is a cyclic group. If $e \in X$ is an idempotent such that $2e = (1 + 1)e \neq 0$, then $o_r(1 - e)$ (resp. $o_r(1 - e)$) is finite.

Proof. Since $2e \neq 0$, $2(1 - e) \neq 2$. Hence it follows from Theorem 2.14. 

Corollary 2.16. Let $R$ be a ring such that $G$ is a cyclic group. If there exists an idempotent $e \in X$ such that $2e = (1 + 1)e \neq 0, 2$, then $G$ is a finite group.
Remark 3. Let $R$ be a unit-regular ring such that $X \neq \emptyset$. If $G$ is a cyclic group and $2 = 1 + 1 \in G$, then $G$ is a finite group.

Proof. It follows from Lemma 2.3 and Corollary 2.16.

Corollary 2.17. Let $R$ be a unit-regular ring such that $X \neq \emptyset$. If $G$ is a cyclic group and $2 = 1 + 1 \in G$, then $G$ is a finite group.

Proof. It follows from Lemma 2.3 and Corollary 2.16.

3. Conjugate action in unit-regular rings

Theorem 3.1. Let $R$ be a ring such that $G$ is a cyclic group. If $e \in X$ is an idempotent such that $2e \neq 2(1 + 1)$, then the orbit $\alpha_e(e)$ (resp. $\alpha_e(1 - e)$) is finite.

Proof. The proof is similar to that of Theorem 2.14. If $\alpha_e(e) = \{e\}$ or $G = \{1\}$ for an idempotent $e \in X$, then $\alpha_e(e) = \{e\}$, and so $\alpha_e(e)$ is finite. Suppose that $\alpha_e(e) \neq \{e\}$ and $G \neq \{1\}$. Then $|\alpha_e(e)| > 1$ and $\text{stab}(e) = \{g \in G | geg^{-1} = e\}$ is a proper subgroup of $G$. Let $H = \text{stab}(e)$ and let $a$ be a generator of $G$. Since $e \in X$ is an idempotent and $2e \neq 2, 2e - 1(\neq 1) \in G$. Thus $(2e - 1)e(2e - 1)^{-1} = (2e - 1)e(2e - 1) = e$ implies that $2e - 1 \in H$ and so $H \neq \{1\}$. Since $H$ is a proper subgroup of $G$, $H$ is generated by $a^s$ for some nonnegative integer $s$ ($s \geq 2$). Since $a^s \in H$, $a^s e = e$. For all $g \in G$, $g = a^m$ for some $m \in \mathbb{Z}$. By the division algorithm for $\mathbb{Z}$, $m = r + qs$ form some $g, r \in \mathbb{Z}$, where $s - 1 \geq r \geq 0$. Thus for all $g \in G$, $g^{-1} = a^m e a^{-m} = a^r q e a^{-r} r + (r +qs) = a^r e a^{-r}$. Therefore $\alpha_e(e) = \{a^s e a^{-r} : 0, 1, \ldots, s - 1\}$ is finite.

Corollary 3.2. Let $R$ be a ring such that $G$ is a cyclic group. If $e \in X$ is an idempotent such that $2e \neq 0$, then the orbit $\alpha_e(e)$ (resp. $\alpha_e(1 - e)$) is finite.

Proof. Since $2e \neq 0, 2(1 - e) \neq 2$. Hence it follows from Theorem 3.1.

Lemma 3.3. If $E(R)$ is commuting, then $\alpha_e(e) \subseteq \alpha_e(e)(= \alpha_e(e))$ for all $e \in E(R)$.

Proof. Since $E(R)$ is commuting, $\alpha_e(e) = \alpha_e(e)$ for all $e \in E(R)$ by Theorem 2.7. Let $geg^{-1} \in \alpha_e(e)$ ($\forall g \in G$) be arbitrary. Since $\alpha_e(e) = \alpha_e(e), eg^{-1} = he$
for some \( h \in G \), and so \( geg^{-1} = (gh)e \in o_\mathcal{E}(e) \). Thus \( o_\mathcal{E}(e) \subseteq o_\mathcal{E}(e)(= o_\mathcal{E}(e)) \) for all \( e \in E(R) \). □

**Lemma 3.4.** If \( E(R) \) is commuting, then \( o_\mathcal{E}(e) = \{ e \} \) for all \( e \in E(R) \), i.e., \( ge = eg \) for all \( g \in G \).

**Proof.** Since \( E(R) \) is commuting, \( o_\mathcal{E}(e) \cap E(R) = \{ e \} \) by Lemma 2.5 and also \( o_\mathcal{E}(e) \subseteq o_\mathcal{E}(e) \) by Lemma 3.3 for all \( e \in E(R) \). Since \( o_\mathcal{E}(e) \subseteq E(R) \), \( o_\mathcal{E}(e) \subseteq o_\mathcal{E}(e) \cap E(R) = \{ e \} \), and so \( o_\mathcal{E}(e) = \{ e \} \). □

**Theorem 3.5.** Let \( R \) be a unit-regular ring in which \( E(R) \) is commuting. If \( G \) is an abelian group, then \( R \) is a commutative ring.

**Proof.** Since \( E(R) \) is commuting, \( ge = eg \) for all \( e \in E(R) \) and all \( g \in G \) by Lemma 3.4. Let \( x \in X \) and \( g \in G \) be arbitrary. Then \( x = hel \) for some \( e_1 \in E(R) \) and some \( h \in G \) by Lemma 2.3. Since \( G \) is abelian, we have \( gx = g(hel) = (gh)e_1 = e_1(gh) = e_1(hg) = (e_1hg)g = xg \). Let \( y \in X \) be arbitrary. Then \( y = ke_2 \) for some \( e_2 \in E(R) \) and some \( k \in G \) by Lemma 2.3. Since \( E(R) \) is commuting, \( xy = (h_1(e_2)) = (k)(e_1e_2) = (kh)(e_2e_1) = (ke_2)(he_1) = yx \). Consequently, \( R \) is commutative. □

**Corollary 3.6.** Let \( R \) be an abelian regular ring. If \( G \) is an abelian group, then \( R \) is a commutative ring.

**Proof.** It follows from Corollary 2.10 and Theorem 3.5. □

**Theorem 3.7.** Let \( R \) be an abelian regular ring such that \( G \) is a torsion group. Then the following are equivalent:

1. The conjugate action on \( X \) by \( G \) is trivial;
2. \( G \) is abelian;
3. \( R \) is commutative.

**Proof.** (1)⇒(2). Let \( g, h \in G \) be arbitrary. Since the order of \( g \) is finite, \( 1 - g \in X \). Since the conjugate action on \( X \) by \( G \) is trivial, the orbit \( o(1 - g) = \{ 1 - g \} \), i.e., \((1 - g)h^{-1} = 1 - g \) and so \( gh = hg \). Hence \( G \) is abelian.

(2)⇒(3). It follows from Corollary 3.6.

(3)⇒(1). It is clear. □

Note that (2)⇒(1) in Theorem 3.7 may not be true in a ring which is not an abelian regular ring by the following example:

**Example 2.** Let \( R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{Z}_2 \right\} \). Then \( R \) is a noncommutative ring but \( G = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\} \) is an abelian group. The orbit of \( \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \) in \( X \) under the conjugate action on \( X \) by \( G \) is equal to \( \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\} \neq \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\} \), and so the conjugate action on \( X \) by \( G \) is not trivial.
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