Reaction Dynamics of CH$_3$ + HBr → CH$_4$ + Br at 150-1000 K

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The kinetics of the radical-polar molecule reaction CH$_3$ + HBr → CH$_4$ + Br has been studied at temperatures between 150 and 1000 K using classical dynamics procedures. Potential energy surfaces constructed using analytical forms of inter- and intramolecular interaction energies show a shallow well and barrier in the entrance channel, which affect the collision dynamics at low temperatures. Different collision models are used to distinguish the reaction occurring at low- and high-temperature regions. The reaction proceeds rapidly via a complex-mode mechanism below room temperature showing strong negative temperature dependence, where the effects of molecular attraction, H-atom tunneling and recrossing of collision complexes are found to be important. The temperature dependence of the rate constant between 400 and 1000 K is positive, the values increasing in accordance with the increase of the mean speed of collision. The rate constant varies from 7.6 × 10$^{-12}$ at 150 K to 3.7 × 10$^{-12}$ at 1000 K via a minimum value of 2.5 × 10$^{-12}$ cm$^3$ molecule$^{-1}$ s$^{-1}$ at 400 K.

Key Words: Complex-mode reaction, Tunneling, Trajectory, Rate constant

Introduction

Reactions of alkyl radicals with hydrogen halides have been a subject of active studying for several decades, producing wealth of information on their kinetics and thermochemistry. Interest in these and related reactions is due in part to the usefulness of their data in thermochemical studies and developing further studies involving free radicals. 1-18 The common characteristics of these reactions are the abstraction of a hydrogen atom by a radical in a highly exo-ergic environment with a low or negligible activation energy barrier and rate constants showing weak temperature dependence. 7,10,12-14,17,18 A particularly significant aspect of the temperature dependence is that some simple reactions involving radicals proceed via negative activation energies at low temperatures (< 500 K). 7,12-14,17,18 One such reaction is CH$_3$ + HBr → CH$_4$ + Br, which is a prototypical system for free radical reactions and the subject of the present paper. In recent years, chemical reactions involving bromine and its derivatives such as HBr, BrO, HOB and CH$_3$Br have become an important subject in studying the delicate balance of atmospheric molecules and atoms, in particular those in the Arctic polar region, 19-23 even though their concentrations are low. Bromine atom produced in the CH$_3$ + HBr reaction undergoes many reactions with various molecules in the atmosphere, such as the ozone depletion reaction O$_3$ + Br → BrO + O$_2$, where BrO regenerates Br either by photolysis at ~325 nm or by reaction with NO. Hydrogen bromide enters the atmosphere by a variety of industrial, natural and anthropogenic processes, such as coal-burning, mineral industries, waste combustion, oceanic biological processes and reactions of acidic gases with sea salt. Earlier reports indicate about 700 metric tons of HBr were released to atmosphere in 2002. 24 One of the sources of bromine formation is the CH$_3$ + HBr reaction, where the reactivity can be greatly influenced by the attractive interaction of a free radical with a polar molecule especially at low temperatures prevailing in the atmospheric environment.

In this study, we report the temperature dependence of CH$_3$ + HBr → CH$_4$ + Br with emphasis on the low temperature range (< 300 K) of the atmospheric environment based on a complex-mode model which uses collision trajectories radically different from those of high-temperature gas-phase reactions. We will consider the effects of free radical-polar molecule attraction, H-atom tunneling through a transition-state barrier and recrossing of the nascent complex formed in the interaction of CH$_3$ and HBr in detail. The reaction will be extended to higher temperatures (300-1000 K) to study how the magnitudes and temperature dependence of the rate constants differ from those of the complex-mode rate constants at low temperatures. We use a molecular dynamics approach to solve the equations of motion for the potential energy composed of the analytical expressions of inter- and intramolecular interactions.

Model and Methods

The collision model is similar to that used for CH$_3$ + OH in ref. 25. We recapitulate the essential aspects here defining all coordinates in Figure 1. The center-of-mass (c.m.) of HBr is taken to be the origin of the XYZ frame and planar CH$_3$ approaches at the impact parameter $b$ with the collision energy $E$. The potential energy $U$ used for the CH$_3$ + HBr interaction includes the exponential C-H and C-Br intramolecular terms (Morse-type), H-Br intramolecular interactions, intermolecular terms between H of HBr and three

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hydrogen atoms of the methyl radical (H-H$_m$), the same forms between Br and three H$_m$ atoms, and three C-H$_m$ intramolecular terms in CH$_3$. The electrostatic and induction part includes contributions from charge-dipole, charge-induced dipole, and dipole-induced dipole interactions, which contribute to the approach of the reactants to close separations of each other. The time-evolution of the relative distance R between the centers of mass of CH$_3$ and HBr determines the collision trajectory. The complete form of the interaction potential is given in the Appendix.

A representative case of the potential energy surface (PES) determined from the potential function given in the appendix is shown in Figure 2(a). The entrance valley is relatively flat and shows a barrier near r$_{CH}$ = 1.6 Å and r$_{HBr}$ = 2.0 Å, beyond a basin of depth 0.137 eV (13.2 kJ mol$^{-1}$). The valley drops steeply to a minimum in the exit valley, where the CH bond forms and Br recedes from H. The product CH$_4$ travels through a narrow exit valley with a significant amount of vibrational energy in its nascent CH bond (see the next section). These events follow the minimum energy path shown in Figure 2(b), where the reactants enter the entrance channel which slowly declines to a potential well before rising to a barrier. The path then falls off to $-75.5$ kJ mol$^{-1}$, which is close to the thermochemical value $\Delta H^o = -74$ kJ mol$^{-1}$, in an exo-ergic reaction. (Here the entrance energy is set zero and scaled in the kJ units for easy comparison with the thermochemical data.) The shallow well in the entrance valley can trap low energy reactants in the form of a weakly bound complex H$_3$C···HBr. The complex can reach the product state when H-atom of HBr tunnel through the barrier and binds the radical, the tunneling efficiency increasing when the temperature is lowered. Others can return to the reactant state at a faster rate than going on to the product state, thus reducing the extent of forward events.

Tunneling has been an important part of studying reaction rates involving small particles which can pass through the potential energy barrier.$^{14,20,27-32}$ Although tunneling by atoms heavier than hydrogen are considered to be rare, recent studies show atoms as heavy as carbon can tunnel through the reaction barrier at cryogenic temperatures.$^{33-35}$ The extent of H-tunneling in the present reaction can be determined by
The potential energy can be closely fit to the quadratic form: $U(R) = \frac{1}{k_\text{B} T} \int_0^{\infty} \exp\left[-(2/\hbar)(2m_a)^{1/2} \int_0^{R} \left[V_{TS}(R_{\text{MEP}}) - E_{\text{H}}\right]^{1/2} dR_{\text{MEP}}\right] e^{\hbar/kaT} dE_{\text{H}}$, where $V_{TS}(R_{\text{MEP}})$ is the potential energy of the transition state barrier above the well minimum, $R_1$ and $R_2$ are the inner and outer turning points along the minimum energy path (MEP), respectively, and $E_{\text{H}}$ is the translational energy of the H atom. The upper limit of the outer integration is set as the barrier height and the value of $\chi(T)$ is 1 for $E_{\text{H}} > V_\text{e}$. The potential energy can be closely fitted to the quartic form: $V_{\text{TS}}(R_{\text{MEP}}) = 1.47 - 0.0687R_{\text{MEP}} - 32.62R_{\text{MEP}}^2 - 3.80R_{\text{MEP}}^3 + 0.827 \text{ debye}$.

To follow the time evolution of reactive events, we solve the equations of motion for the Hamiltonian $H$ containing $H_0$ and their conjugated momenta in the kinetic energy terms:

$$\dot{q}_n = \partial H / \partial p_n, \quad \dot{p}_n = -\partial H / \partial q_n$$

where $n$ denotes $R$, $r_{\text{HBr}}$, $\theta$, $\phi$, $\chi_{\text{CH}_3}$, and $\chi_{\text{Br}}$. Here $\rho$ is the distance from the c.m. of CH$_3$ to the Z axis and its initial value is the impact parameter $b$. The angle-averaged potential energy is the angle-averaged potential at the Z axis and its initial value is the bond distance and energy becomes less than 5 Å and the dissociation threshold, respectively.

The initial conditions, those describing the approach of the reactant partners from a long distance are particularly important in the present study as they have different forms depending whether the collision occurs at low or high temperatures. For low collision energies, the initial conditions for the relative motion at a large distance given by $U(R) = D[e^{R/\alpha} - 2e^{R/2\alpha}]$ is

$$U(R) = R_a + a \ln(\rho^2 + \rho) + 2a \ln(\cosh[(E/2\mu)^{1/2} t/\alpha] - [\rho/(1 + \rho)]^{1/2})$$

$$p(t) = 2(E/2\mu)^{1/2} \sinh[(E/2\mu)^{1/2} t/\alpha]\cosh[(E/2\mu)^{1/2} t/\alpha] - [\rho/(1 + \rho)]^{1/2}$$

where $\rho = D/E$ and $\mu$ is the reduced mass of the collision system. The potential parameters $D$ and $R_a$ needed to initialize at low distance are 0.0211 eV and 4.04 Å obtained from the Lenard-Jones constants $\sigma_{\text{CH}_3} = 3.80 \text{ Å, e}_{\text{CH}_3}/k_\text{B} = 144.0 \text{ K}$, and $\sigma_{\text{HBr}} = 3.41 \text{ Å, e}_{\text{HBr}}/k_\text{B} = 417 \text{ K}$. The exponential range parameter $a$ is obtained from $a = \pi^{-1/2}[\Gamma(7/12)/\Gamma(1/12)]^{1/2}(\sigma_{\text{CH}_3} + \sigma_{\text{HBr}})$.

For all other spectroscopic and potential constants are given in Table 1. However, at low temperatures, where the strength of molecular attraction is significant, the reactants can form a collision complex as they travel toward the barrier. Statistically they spend more time along the reaction path where attraction is strong. During the journey through the region of potential well, the intermolecular distance $R$ does not change significantly, so that we can expand the interaction potential around the well bottom $R_\text{c}$:

$$U(R) = U(\theta, \phi) = U(\theta, \phi)_{\text{r}} + \frac{\hbar^2}{2m}\left[U(\theta, \phi)/R_{\text{c}}^3(R - R_\text{c})\right]^2 + \cdots$$

The angle dependence of charge-dipole and dipole-induced dipole interaction energies is simple, but together with the angular dependence of intermolecular atom-atom distances, it controls the motion of reactants in the entrance valley. For the weakly bound complex travelling in the region of the potential well, the potential energy is the angle-averaged quantity

$$<U> = \int_0^{2\pi} \int_0^\pi \{U(\theta, \phi)\exp[-U(\theta, \phi)/k_\text{B} T]\} \sin\theta d\theta d\phi$$

where the Boltzmann weighting factor is introduced as the complex spends more time at orientations where the energy is more attractive. The resulting average up to the second order is $<U> = <U_0> + \frac{1}{2}<U''>(R - R_\text{c})^2$, where the angle-averaged second derivative $<U''>$ is evaluated at $R = R_\text{c}$.

From the solution of the equation of motion for the quadratic function the initial conditions appropriate for the reaction at low temperatures are

$$<U> = \frac{1}{2}\int_0^{2\pi} \int_0^\pi \{U(\theta, \phi)\exp[-U(\theta, \phi)/k_\text{B} T]\} \sin\theta d\theta d\phi$$

Table 1. Spectroscopic and Potential Constants

<table>
<thead>
<tr>
<th></th>
<th>HBr</th>
<th>CH in CH$_4$</th>
<th>Br in CH$_3$Br</th>
<th>CH$_3$ in CH$_3$H</th>
<th>H$_3$H$_2$</th>
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<tr>
<td>$D$ (eV)</td>
<td>3.922$^a$</td>
<td>4.587$^a$</td>
<td>2.94$^{d,e}$</td>
<td>5.10$^{a}$</td>
<td>4.751$^a$</td>
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<tr>
<td>$v$ (cm$^{-1}$)</td>
<td>2649$^f$</td>
<td>2916$^{(d)}$</td>
<td>611$^{(c)}$</td>
<td>3270$^{(d)}$</td>
<td>4401$^f$</td>
</tr>
<tr>
<td>$a$ (Å)</td>
<td>0.276</td>
<td>0.278</td>
<td>0.291</td>
<td>0.264</td>
<td>0.257</td>
</tr>
<tr>
<td>$r_0$ (Å)</td>
<td>1.414$^a$</td>
<td>1.094$^a$</td>
<td>1.939$^{d,e}$</td>
<td>1.079$^a$</td>
<td>0.741$^a$</td>
</tr>
</tbody>
</table>

$D = D_a + \hbar^2/2m$, $a = (D/2k)^{1/2}/\sigma_{\text{HBr}} = 0.827 \text{ debye}$, $\sigma_{\text{HBr}} = 2.222 \text{ Å}$.

$D_a = D_a = 3.51 \text{ Å}$. Ref. 45, Ref. 46, Ref. 47, Ref. 48, Ref. 49, Ref. 50, 2013, Vol. 34, No. 8 2475
Results and Discussion

In this section we first consider the dynamics of the reaction. The time evolution of collision events for a representative trajectory at 300 K is shown in Figure 3(a), where we plot the C-H and H-Br distances and collision trajectory R initiated by Eqs. (3a) and (3b). Reaction time is scaled so that the instant of the first impact is zero. The time-variation of collision trajectory and atom-atom distances is simple and indicates the reaction is a short-time event or a direct-mode mechanism. The well-organized oscillation of C-H distance indicates the formation of a nascent bond immediately after the impact. The H-Br distance diverges (or bond dissociation) on impact, describing the time evolution of the Br position from H which is now bound to the methyl radical. The oscillatory structure of the outgoing H-Br distance is due to the vibration of the nascent CH bond, where the alternation of downward and upward oscillations reflects the rotation of CH\(_4\). Similarly, after the impact, the collision trajectory is now the distance between the outgoing Br and C of CH\(_4\). The time evolution of reactive events at higher temperatures follows the same pattern shown in Figure 3(a).

The time evolution of reactive events at lower temperatures (< 300 K) is markedly different from that shown in Figure 3(a). We take a representative reactive event at 200 K and show its time evolution initiated by the conditions given by Eqs. (5a) and (5b) in Figure 3(b). (Note that both Figures 3(a) and 3(b) are plotted on the same scales.) The collision trajectory now progresses with large amplitudes beyond the initial impact. Reaction time passes more than 0.7 ps after the initial impact before the nascent CH bond forms, the duration representing the lifetime of collision complex. The time dependence of the collision trajectory, C-H and H-Br distances clearly indicates the reactive event follows a complex-mode mechanism, where the lifetime of complex far exceeds the period of either HBr or CH vibration. The lifetime is more than 60 times the \(v_1\) CH vibrational period.

From the standpoint of bond dissociation dynamics, the time dependence of H-Br distance shown in Figure 3(b) is particularly interesting as it illustrates the nature of vibrational excitation in the dynamic process. The amplitude of HBr vibration changes gradually from the instant of the initial impact to ~0.7 ps, indicating the excitation of the ground-state reactant HBr toward the dissociation threshold occurs in a series of steps, the HBr bond gaining a small amount of energy at each step. In the direct-mode mechanism at higher temperatures, where the reaction occurs in a strong collision transferring a large amount of energy to the HBr bond, the dynamics is simple as shown in Figure 3(a).

Figure 4(a) shows the relationship between the vibrational energy and bond distance of CH at the instant when the bond is formed. As the bond energy is calculated with the exponential function given in Eq. (A1), the points in the plot are distributed within an envelope of the Morse-type curve. High concentration of the points at low-lying region indicates the vibrational excitation is low. A clearer presentation of vibrational population distribution is shown in Figure 4(b), where, to mimic the quantum distribution, we have used a binning procedure of assigning quantum number \(v_1\) of
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CH$_4$ corresponding to the computed CH vibrational energy $E_{CH}$ through the relation $\nu_1 = \text{int}[E_{CH}/E_{\text{vib}}(\nu_1)]$, $E_{\text{vib}}(\nu_1)$ being the harmonic-oscillator eigenvalue. The highest distribution appears at $\nu_1 = 1$. The population for $\nu_1 > 4$ is very low, as the reaction system does not have sufficient energy for the production of highly excited CH bond at 300 K.

In Figure 5, we plot the temperature dependence of calculated rate constants over the 150-1000 K range using the complex-mode at $T < 300$ K and the direct-mode trajectory at $T \geq 300$ K. Although emphasis is placed on the low region, we include the higher region to show how the complex-mode model relates to the direct-mode result. The rate constant takes the minimum value of $2.53 \times 10^{-12}$ at 400 K and increases in both directions as the temperature is varied. It increases to $3.66 \times 10^{-12}$ at 1000 K and to as large as $7.58 \times 10^{-12}$ at 150 K. Rate constants are expressed in cm$^3$ molecule$^{-1}$ s$^{-1}$ throughout the paper. We first note that the high-temperature branch is a thermally activated process and displays positive temperature dependence, approximately paralleling the increase of the mean speed of collision. At temperatures below 300 K, the complex-mode mechanism based on the contribution of attractive interaction leads to strongly negative temperature dependence. Other factors contributing to the dependence are the tunneling of H-atom in the trapped state and the nascent complex returning to the reactant state at a faster rate than going on to the products. The tunneling factor is only 1.48 at 300 K, but increases 2.86 at 150 K. Note that it is only 1.10 at 1000 K. Even without tunneling corrections the ratio $k(150K)/k(300 K)$ is 1.4, because of the importance of molecular attraction. The strength of molecular attraction decreases when the temperature increases as thermal motion tends to destroy the preferred orientation of H$_3$C…HBr, which leads to decrease in the reaction rate. Further, as the temperature increases, some of the incoming trajectories with energy above the barrier tend to hit the repulsive wall on the PES and bounce back to the entrance, which also leads to decrease in the reaction rate. Therefore, the three factors, namely intermolecular attraction, H-atom tunneling and barrier recrossing cause the decrease of the rate constant when the temperature is raised in the complex-mode mechanism (i.e., negative temperature dependence). Above 300 K, where the reaction occur in direct-mode interactions, however, thermal effects overcome these contributions and lead to the rate constant increasing with rising temperature. As shown in Figure 5, the complex-mode interaction seriously overestimates the rate at 300 K, whereas the direction-mode interaction underestimates below 300 K; see dashed lines extending the lower and higher branches.

Over the temperature range of 350-150 K, the variation of $\log k$ vs 1/T is nearly linear giving an Arrhenius expression $k_\text{vtfr} = 1.10 \times 10^{-12} \exp(299/T)$ with a negative activation energy of 2.48 kJ mol$^{-1}$. The variation between 500 and 1000 K is positive, giving $k_\text{vtfr} = 5.12 \times 10^{-12} \exp(-366/T)$, $E_{\text{act}} = 3.04$ kJ mol$^{-1}$. Although it is slight, the temperature...
dependence is negative even in the intermediate range 300-500 K, where the Arrhenius expression is $2.19 \times 10^{-12} \exp(66.7/T)$, $E_{\text{ad}} = 0.554$ kJ mol$^{-1}$. The values of $k_{\text{lib}}$ at intermediate temperatures showing negative temperature dependence are in agreement with experimental data by Seakins $et$ $al.$ and Nicovich $et$ $al.$, see Figure 5. We note the highest temperature considered by Seakins $et$ $al.$ is 536 K, whereas that by Nicovich $et$ $al.$ is $2.21 \pm 0.18 \times 10^{-12}$, The upper limit of the latter value is close to the curve calculated in the present study. The Arrhenius fit of the data reported by Seakins $et$ $al.$ is $k_{\text{lib}} = (1.57 \pm 0.26) \times 10^{-12} \exp(1.6 \pm 0.6)$ kJ mol$^{-1}$ over the temperature range 299-536 K, whereas that by Nicovich $et$ $al.$ is $k_{\text{lib}} = (1.36 \pm 0.10) \times 10^{-12}$ [233 $\pm 23$/T] for 257-422 K. Seetula’s expression reported in Ref. 12 is $k_{\text{lib}} = (3.8 \pm 0.7)$, (3.8 + 1.05, 11.2 and 101 bar, respectively, at 297 $\pm 3$ K. All measurements show negative activation energy. Also shown in Figure 5 are the measured values by Krasnoperov and Mehta (2.86 at 150 K) but are insignificant at higher temperatures (2.21 at 600 K, 99.96% of the sample is at room temperature. Even at 600 K, 99.96% of the sample is in the ground state. The vibrational population of HBr is depending on the initial impact, whereas the complex-mode collision model in the quasi-classical trajectory approach shows a negative temperature dependence of rate constants below room temperature. The effects of attractive interaction, H-atom tunneling and re-collision of weakly bound complexes are found to be responsible for the temperature dependence. At intermediate temperatures between 300 and 500 K, the rate constant exhibits a slightly negative temperature dependence. At higher temperatures, thermal effects overcome the attractive interaction and lead to a positive temperature dependence. Rate constants vary from 7.58 $\times 10^{-12}$ at 150 K, to 2.53 $\times 10^{-12}$ at 400 K and then to 3.66 $\times 10^{-12}$ cm$^3$ molecule$^{-1}$ s$^{-1}$ at 1000 K. Tunneling corrections are important at lower temperatures (2.86 at 150 K) but are insignificant at higher temperatures (1.10 at 1000 K).

The population distribution of $v_1$ vibrational energy of the nascent CH bond is peaked at $v_1 = 1$. The population of highly excited states ($v_1 > 4$) is found to be insignificant as the reaction does not produce sufficient energy for such excitation. The direct-mode reaction at high temperatures occurs on the initial impact, whereas the complex-mode reaction at low temperatures occurs after a long period of [H:C$\cdots$HBr intermolecular motion. In the complex-mode reaction, HBr dissociation occurs in a series of excitations toward the dissociation threshold.

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### Appendix

The overall interaction potential energy is

$$U = \sum_{i=1}^{3} D_i \{\exp[(r_{e,i} - r_{i})/a_i] - 2\exp[(r_{e,i} - r_{i})/2a_i]\}$$

$$+ \frac{1}{3} \sum_{i=1}^{3} D_{hh}\{\exp[(r_{e,hh} - r_{hh})/a_{hh}] - 2\exp[(r_{e,hh} - r_{hh})/2a_{hh}]\}$$

$$+ \sum_{i=1}^{3} D_{ch}\{\exp[(r_{e,ch} - r_{ch})/a_{ch}] - 2\exp[(r_{e,ch} - r_{ch})/2a_{ch}]\}$$

$$+ \sum_{i=1}^{3} D_{bh}\{\exp[(r_{e,bh} - r_{bh})/a_{bh}] - 2\exp[(r_{e,bh} - r_{bh})/2a_{bh}]\}$$

$$+ \frac{1}{3} \sum_{i=1}^{3} D_{cb}\{\exp[(r_{e,cb} - r_{cb})/a_{cb}] + 2\exp[(r_{e,cb} - r_{cb})/2a_{cb}]\}$$

$$+ c(e^2/4\pi\varepsilon_0)[1/R + \gamma^2/(r_{e,cb} + r_{cb}) - 1/R - \gamma^2/(r_{e,cb} + r_{cb})]$$

$$(\varepsilon_{cb}/4\pi\varepsilon_0)\cos\theta/R^2 - (e^2\varepsilon_{cb}/8\pi\varepsilon_0)R^4 - (e^2\varepsilon_{cb}/8\pi\varepsilon_0)R^4$$

where the subscript “$e$” stands for the equilibrium distance, $\gamma_{cb} = m_{cb}/(m_{cb} + m_{cb})$, $\mu$ and $\alpha$ are the dipole moment and polarizability of the reactant indicated, respectively. The first sum is for the H-Br, C-H and C-Br interactions. The second sum is for the interactions of H of HBr with each of the methyl hydrogen atoms (three $H_m$). Similarly, the third sum is the interactions of Br of HBr with the methyl H atoms. The fourth sum is for the intramolecular C-Hm interactions of the methyl radical. An anti-bonding function of the C-Br interaction is introduced to facilitate the removal of fragment Br from the reaction zone. The intramolecular interactions between $H_m$ atoms in the methyl group do not contribute to the reaction process and are not included. The parameter $c$ represents a partial ionic character of HBr estimated by $\mu_{cb}/e_{cb}$. The interatomic distances are

$$r_{cb} = \sqrt{R^2 + \left[\gamma_{cb}(r_{e,cb} + r_{cb})\right]^2} + 2\gamma_{cb}(r_{e,cb} + r_{cb})R\cos\theta$$

$$r_{hh} = r_{hh}^2 + [(x_{e,ch} + x_{ch})\cos(2\pi/3)]^2 + 2[(x_{e,ch} + x_{ch})\cos(2\pi/3)]R \sin(\theta - \gamma_{cb})$$

$$+ \gamma_{cb}(r_{e,cb} + r_{cb})R\cos\theta$$

$$r_{ch} = r_{ch}^2 + [(x_{e,ch} + x_{ch})\cos(2\pi/3)]^2 + 2[(x_{e,ch} + x_{ch})\cos(2\pi/3)]R \sin(\theta - \gamma_{cb})$$

$$+ \gamma_{cb}(r_{e,cb} + r_{cb})R\cos\theta$$

and

$$r_{cb} = r_{cb}^2 + [(x_{e,ch} + x_{ch})\cos(2\pi/3)]^2 + 2[(x_{e,ch} + x_{ch})\cos(2\pi/3)]R \sin(\theta - \gamma_{cb})$$

$$+ \gamma_{cb}(r_{e,cb} + r_{cb})R\cos\theta.$$
where \( \chi_{\text{CH}} = \sin^{-1}\left[\sin(r_{\text{CH}} + r_{\text{Br}}) \cos(\theta_{\text{CH}}) \cos(\theta_{\text{Br}})\right] \)
\( \chi_{\text{Br}} = \sin^{-1}\left[\sin(r_{\text{Br}}) \cos(\theta_{\text{Br}})\right] \).
The distances \( r_{\text{CH}} \) and \( r_{\text{Br}} \) take the same expressions as \( r_{\text{CH}} \) and \( r_{\text{Br}} \), respectively, except that \( (r_{\text{Br}} + r_{\text{Br}}) \) is multiplied by \( \sin^2 \). Eqs. (A3) and (A4) are for one of the three methyl hydrogen atoms, say \( \text{H}_m.1 \). For \( \text{H}_m.2 \) and \( \text{H}_m.3 \), the distances can be determined similarly after rotating the CH