We derive a new joint power and rate control rule with which we can minimize the mean transmission delay in CDMA networks for a given mean transmission power. We show that it is optimal to respectively control the power inverse-linearly and the rate linearly to the square root of channel gain while maintaining the signal-to-interference ratio at a constant. We also show that the proposed joint power/rate control rule achieves excellent performance results in terms of the probability of the instantaneous delay being within a target delay against one-dimensional control schemes.

Keywords: CDMA, power control, link adaptation, adaptive transmission, variable rate.

I. Introduction

As the demand for high-speed data service increases in wireless cellular networks, it is very important to achieve high data rate capability. For high-speed data service, not only is the data throughput important but the data transmission delay is also a very important factor in determining the service and the network quality. The research in the literature has solved most of the optimum resource control problems for trying to achieve the maximum data throughput [1]-[8]. Their work was devoted to solving resource control problems, that is, trying to increase data throughput. However, these kinds of approaches may be inefficient in terms of transmission delay because they mainly focus on maximizing the mean data rate. These control schemes may result in serious unevenness in the data rate according to the channel conditions and thus, bad channel conditions will cause a significant transmission delay because of a slow data rate [9]-[14].

On the other hand, a constant rate scheme with power control can provide even service quality at all instances [9]. However, such schemes are inefficient in terms of power consumption, because they rely totally on transmission power control to compensate channel degradation and result in an increase in the total required transmission power [12]. This is an especially critical problem in power limited systems, such as, satellite communication networks where, generally, the dynamic range of transmission power is highly restricted. Consequently, this causes a reduction in the data rate under the transmission power constraint.

The authors in [7] examined the relationship between the average delay and processing gain and searched for the optimum processing gain, i.e., the optimum data rate, in order to minimize the average delay. They assumed that channel gain
is time invariant and did not optimize the transmission power. In this paper, we propose an optimum power and rate joint control scheme for uplink CDMA networks in order to achieve the minimum mean transmission time. In [15], we provided the optimum solution for power and rate in the user domain in order to minimize transmission delay averaged over simultaneous users experiencing independent channel variations in CDMA downlinks. In this paper, we focus on time-domain optimization, which aims to minimize the uplink transmission delay averaged over a time varying channel for an arbitrary user link by employing an approach and derivation techniques similar to those we used in [15]. The proposed scheme efficiently allocates the power and rate in order to adapt slow channel variation for a given mean transmission power. We analyze the performance of the proposed scheme in terms of mean transmission delay and the probability of instantaneous delay being larger than a target delay. As reference control schemes for comparison, we also analyze transmission delay performances of one-dimensional control schemes, i.e., a power control scheme with a fixed data rate and a rate control scheme with a fixed transmission power.

After formulating the packet error rate and the transmission delay as the function of power and rate, we first derive a mean transmission delay equation for a power control scheme with a constant rate and a rate control scheme with a constant power. Then, an optimum power/rate control rule is derived in order to minimize the mean transmission delay, and the numerical results are compared for three different control schemes.

II. System Model and Packet Error Probability

We consider a packet data transmission in CDMA networks where users control their transmission power or data rates to compensate for slowly varying channel gains. Therefore, it is assumed that the channel gain remains constant during a successful packet transmission. Let the transmission power be $P$, then the received symbol energy $E_s$ is given as

$$E_s = gPT,$$

where $g$ is the channel gain and $T$ is the symbol duration. Then, the signal energy to total interference ratio $E_s/N_t$ is given as

$$\frac{E_s}{N_t} = \frac{gP}{RN_t},$$

where $R$ is the symbol rate equal to $1/T$ and $N_t$ is the power spectral density (PSD) of the total interference including background noise and multiple access interference. The transmission power $P$ can be controlled according to channel gain $g$ and the average transmission power is $P_0$ as follows:

$$\bar{P} = P_0.$$  

We use a simple form for symbol error probability as follows in order to ease the derivation in the subsequent analysis:

$$P_s \approx a \exp(-bE_s/N_t),$$

where the parameters $a$ and $b$ are introduced to cover various types of modulation. For example, we set $(a, b) = (0.5, 1)$ for BPSK modulation. Let the number of symbols per packet be $L$, then the packet error probability $P_{PE}$ is given as

$$P_{PE} = 1 - (1 - P_s)^L.$$  

III. Mean Transmission Delay and Optimum Power/Rate Control

Let the number of packet retransmissions, including the first transmission, be $n$, and the transmission delay for a successful packet transmission $t_d$ is given as follows [8], [15]:

$$t_d = \frac{nL}{R},$$

where $L/R$ is the transmission time for a packet when there is no retransmission and we neglect the propagation and processing delay which is negligible compared to $L/R$ [7], [8]. Assuming a stop-and-wait automatic request control for simplicity and an identical packet error probability during the retransmission of a packet, $n$ follows a geometric distribution as

$$\Pr(n = k) = P_{PE}^{k-1}(1 - P_{PE}).$$

Since the mean of $n$, $\bar{n}$, is equal to $1/(1-P_{PE})$, the mean transmission delay is obtained as follows:

$$D = E[t_d] = \frac{L}{R} = \frac{L}{R(1 - P_{PE})},$$

where $E[x]$ denotes the mean of $x$.

Consequently, by using (2), (5), and (8), the mean transmission delay is given as

$$D(g) = \frac{L}{R(1 - ae^{-bgP/RN_t})^2},$$

where we assume that the channel gain, transmission power, and data rate are unchanged during the retransmission of a packet. Accordingly, the mean transmission delay averaged
over the channel gain variation is calculated as follows:

\[
\overline{D} = E_g [D(g)] = E_g \left[ \frac{L}{R(1 - ae^{-bgP/RN_t})} \right].
\]

(10)

Before proceeding to optimum joint power/rate control for minimum mean transmission delay, we analyze the mean transmission delays of one-dimensional control schemes.

1. Power Control with a Constant Rate

For power control with a constant rate, we considered a scheme where the data rate \( R \) is fixed to \( R_0 \) and the transmission power \( P \) is inverse-linearly controlled against channel gain \( g \) to maintain \( E_s/N_t=\gamma_0 \) as follows:

\[
E_s/N_t = \frac{gP}{R_0N_t} = \gamma_0 \Rightarrow P(g) = \frac{\gamma_0N_tR_0}{g}.
\]

(11)

By the constraint for the mean transmission power given in (3), the fixed data rate \( R_0 \) is determined by the following equation:

\[
P(\gamma_0) = \frac{\gamma_0N_tR_0}{g}.
\]

(12)

From (11) and (12), \( R_0 \) is written as

\[
R_0 = \frac{P_0}{\gamma_0N_t(1/g)}.
\]

(13)

By substituting (13) into (9), the packet delay for power control with constant rate \( D_{PC} \) is given as

\[
D_{PC} = \frac{LN_t(1/g)\gamma_0}{P_0(1 - ae^{-bg_0})^L}.
\]

(14)

where we note that the packet delay is constant irrespective of the channel gain since the data rate and the SIR are kept constant irrespective of the channel gain. We note from (13) and (14) that as the desired \( \gamma_0 \) approaches 0, the data rate approaches infinity and thus, the delay \( D_{PC} \) approaches 0. However, decreasing \( \gamma_0 \) to 0 causes an unacceptable packet error probability and then \( D_{PC} \) in (14) has no meaning. In Fig. 1, \( D_{PC} \) given in (14) is plotted as a function of \( \gamma_0 \) for various system parameters. We note that there is the optimum \( \gamma_0^* \) that achieves minimum delay. The optimum required SIR \( \gamma_0^* \) that minimizes \( D_{PC} \) is calculated as

\[
\gamma_0^* = -\frac{1}{b} \left( \text{lambertw} \left( -1, \frac{1}{aL} \right) + \frac{1}{L} \right).
\]

(15)

where lambertw\((-1, x)\) is the –1st branch of the solutions to \( \exp(w) = x \). The derivation of (15) is given in Appendix A. With a substitution of (15) into (14), the minimized mean packet delay \( D_{PC}^* \) is given as

\[
D_{PC}^* = D_{PC}(\gamma_0 = \gamma_0^*) = \frac{LN_t(1/g)\gamma_0^*}{P_0(1 - ae^{-bg_0})^L}.
\]

(16)

2. Rate Control with a Fixed Power

For rate control with a fixed power, the transmission power is fixed at \( P_0 \) and the data rate \( R \) is controlled to maintain \( E_s/N_t = \gamma_0 \) as follows:

\[
E_s/N_t = \frac{gP_0}{RN_t} = \gamma_0 \Rightarrow R = \frac{gP_0}{\gamma_0N_t}.
\]

(17)

Then, the packet transmission delay from (10) is given as

\[
D_{RC}(g) = \frac{LN_t\gamma_0}{P_0[1 - ae^{-bg_0}]},
\]

(18)

\[
\overline{D}_{RC} = E_g \left[ D_{RC}(g) \right] = \frac{LN_t^2(1/g)\gamma_0}{P_0[1 - ae^{-bg_0}]^L}.
\]

(19)

1) In CDMA networks, the data rate can be changed by multi-code allocation or a variable spreading gain scheme. Although the data rate has discrete values according to the number of codes or spreading gain used and its range is limited in a practical system, we assume an unlimited and real-valued data rate for tractable derivation.
From (14) and (19), we note that the power control scheme with the fixed rate and the rate control scheme with the fixed power have an identical mean packet transmission delay. Consequently, the optimum $\gamma_0$ and the minimum mean packet delay for the rate control scheme with a fixed power are also identical to (15) and (16) as follows.

$$\gamma_0^* = \frac{-1}{b} \left\{ \text{lambertw} \left(-1,-\frac{e^{-1/L}}{aL} \right) + \frac{1}{L} \right\},$$  \hspace{1cm} (20)

$$D_{RC}^* = D_{RC}(\gamma_0 = \gamma_{0,RC}^*) = \frac{LN_i \left(\frac{1}{g} \gamma_0^* \right)}{P_0 \left(1-ae^{-b_0 \gamma_0^*} \right)^L}.$$  \hspace{1cm} (21)

3. Optimum Power/Rate Control

For optimum power/rate control, power and rate are jointly optimized to minimize the mean transmission delay given in (10). First, we optimize $R$ by solving the following equation:

$$R^*(g) = \arg \min_R D(g).$$  \hspace{1cm} (22)

From (9) and (22), we get

$$R^*(g) = \arg \min_R \frac{L}{R \left[1-ae^{-bgP/RN_i} \right]^L} = \frac{-bgP}{N_i \left\{ \text{lambertw} \left(-1,-\frac{\exp(-1/L)}{aL} \right) + \frac{1}{L} \right\}},$$  \hspace{1cm} (23)

from which we note that the optimum data rate is proportional to the received power and thus, the optimized received SIR $\gamma_0^*$ should be maintained at a constant irrespective of channel gain as follows:

$$\gamma_0^* = \frac{gP}{R^*(g)N_i} = -\frac{\text{lambertw} \left(-1,-\frac{\exp(-1/L)}{aL} \right) + \frac{1}{L}}{b}.$$  \hspace{1cm} (24)

From (15), (20), and (24), we note that the SIR is maintained at an identical constant in order to achieve a minimum mean transmission delay irrespective of the power/rate control scheme.

By substituting (23) into (10), the equation for the optimum power for a given mean transmission power $P_0$ can be written as

$$P^* = \arg \min_P D(R = R^*(g)) \text{ subject to } \bar{P} = P_0$$

$$= \arg \min_P E_g \left[ \frac{LN_i \left\{ \text{lambertw} \left(-1,-\frac{e^{-1/L}}{aL} \right) + \frac{1}{L} \right\}}{bgP \left(1-ae^{-b_0 \gamma_0^*} \right)^L} \right]$$

subject to $\bar{P} = P_0$  \hspace{1cm} (25)

$$= \arg \min_P \int \frac{\Pr(g)}{gP} dg \text{ subject to } \int \Pr(g)Pdg = P_0,$$

where $\Pr(g)$ is the probability density function of the channel gain $g$. We can solve the above optimization problem by using the LaGrange multiplier as follows:

$$\Lambda = \int \frac{\Pr(g)}{gP} dg + \lambda \int \Pr(g)Pdg$$

$$= \int \Pr(g) \left( \frac{1}{gP} + \lambda P \right) dg.$$  \hspace{1cm} (26)

The partial derivatives of (26) with respect to $P$ is given as

$$\frac{\partial \Lambda}{\partial P} = \int \Pr(g) \left( -\frac{1}{gP^2} + \lambda \right) dg,$$  \hspace{1cm} (27)

and then $P$, which makes (27) equal to 0, is given by

$$P = \frac{1}{\sqrt{g} \lambda^2},$$  \hspace{1cm} (28)

where $\lambda$ is determined by the mean transmission power condition $\int \Pr(g)Pdg = P_0$ as follows:

$$\lambda = \frac{1}{P_0^2} \left( \frac{1}{\sqrt{g}} \right)^2.$$  \hspace{1cm} (29)

By substituting (29) into (28), we obtain the optimum power control rule as follows:

$$P^* = \frac{1}{\sqrt{g} \frac{1}{\sqrt{g}}}.$$  \hspace{1cm} (30)

By substituting (30) into (23) again, the optimum data rate is determined as follows:

$$R^* = \frac{\sqrt{g}P_0}{N_i \left( \frac{1}{\sqrt{g}} \right)^L}.$$  \hspace{1cm} (31)
From (30) and (31), we note that it is optimal to control the transmission power to be inverse-linear to the square root of the channel gain and to the control data rate to be proportional to the square root of the channel gain. Therefore, the mean transmission delay with the optimum power/rate control \( D^{*}_{PRC} \) is given as

\[
D^{*}_{PRC} = E_g \left[ D(g) \right] = \frac{LN \left( \frac{\sqrt{g}}{g} \right) P_0^*}{P_0 \left( 1 - ae^{-b \sigma^2} \right)^L}. \tag{32}
\]

From (16), (21), and (32), we can calculate a reduction factor \( \mu \), which is defined by the ratio of \( D^{*}_{PRC} \) to \( D^{*}_{RC}(= D^{*}_{RC}) \), as follows:

\[
\mu = \frac{D^{*}_{PRC}}{D^{*}_{RC}(= D^{*}_{RC})} = \frac{\left( \frac{\sqrt{g}}{g} \right)}{1/g}. \tag{33}
\]

If we take \( x = \frac{\sqrt{g}}{g} \) and use a special form of Schwartz’s inequality \((x^a)^2 \leq x^2\) [17], we can show the following inequality:

\[ 0 \leq \mu \leq 1, \tag{34} \]

which implies that the proposed scheme always achieves a smaller average mean transmission delay than constant power or constant rate control schemes.

IV. Performance Evaluation under Lognormal Shadowing Environments

To compare the transmission delays for different control schemes in a mobile communication environment, we consider the case when a user experiences lognormal shadowing. Then, the channel gain \( g \) is given as

\[ g = 10^{\zeta/10}, \tag{35} \]

where \( \zeta \) is the attenuation in decibel due to shadowing and follows a Gaussian distribution with a zero mean and standard deviation \( \sigma \) as follows:

\[ p_\zeta(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left( -\frac{x^2}{2\sigma^2} \right). \tag{36} \]

Since we are focusing on the case when power/rate control is performed to adapt to slow channel variation, the short-term variation due to fast fading is averaged out. Furthermore, we can assume that the variation due to fast fading can also be reduced by multi-path diversity combining, and thus it is not included in (35).

First, to obtain \( \mu \), we calculate \( \frac{1}{g} \) and \( \frac{1}{\sqrt{g}} \) as follows:

\[
\frac{1}{g} = \int_{-\infty}^{\infty} p_\zeta(x) 10^{-x/20} \, dx = \exp \left( \frac{\sigma^2 (\ln 10)^2}{800} \right), \tag{37}
\]

\[
\frac{1}{\sqrt{g}} = \int_{-\infty}^{\infty} p_\zeta(x) 10^{-x/10} \, dx = \exp \left( \frac{\sigma^2 (\ln 10)^2}{200} \right). \tag{38}
\]

From (33), (37), and (38), \( \mu \) is written as follows:

\[
\mu = \exp \left( - \frac{\sigma^2 (\ln 10)^2}{400} \right). \tag{39}
\]

where we note that \( \mu \) exponentially decreases as \( \sigma \) increases, which means that the proposed scheme reduces transmission delay more significantly under severe channel variation.

In Fig. 2, average mean transmission delays for a nonadaptive scheme and three different control schemes are plotted as the function of \( \sigma \) for the case when binary phase shift keying (BPSK) is employed \((a=0.5, b=1)\) and \( P_0/N_0=50 \) dB\(^2\) for the packet length \( L=1000 \) and 5000, respectively. We simulated packet transmissions with independently generated channel gains \( g \) of \( 10^6 \) samples and averaged the mean transmission delays for three different control schemes. Figure 2 also shows the analytical results using (16), (21), and (32) and shows that they nicely fit the simulation results.

For a nonadaptive scheme where neither the data rate nor the transmission power is controlled, we searched for the optimum fixed data rate by simulation and plotted the corresponding average mean transmission delay for \( L=1000 \) in Fig. 2. This demonstrates that a resource control scheme against channel variation is very important to reduce the delay. There is a common observation irrespective of control schemes that average mean transmission delays show an abrupt increase as \( \sigma \) increases. This is because the channel compensation factors \( \frac{1}{g} \) and \( \frac{1}{\sqrt{g}} \) in (16), (21), and (32) exponentially increase as \( \sigma^2 \) increases.

The optimum power/rate control achieves a significant reduction in the mean transmission time compared to the one-dimensional control schemes. As \( \sigma \) increases, the range of the channel variation increases and thus, the amount of reduction in the average mean transmission delay by optimum

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2) We used \( P_0/N_0 \) of 50 dB in our analysis by considering a typical data service with \( R_0 \) of 19.2 kbps. We need \( E_b/N_0 \) of 10.5 dB at BER range of \( 10^{-6} \) in AWGN channel with BPSK modulation, and this results in \( P_0/N_0 \) of approximately 53.3 dB by the relation that \( P_0/N_0 = R_0 \cdot E_b/N_0 \).
power/rate control increases. For example, the optimum power/rate scheme reduces the average mean transmission delay to 1/6 of those of the other two one-dimensional control schemes for $\sigma = 12$.

![Fig. 2. Average mean transmission delay for a nonadaptive scheme and three different control schemes with $L=1000$ and 5000, $P_0/N_0=50$ dB, BPSK.](image)

For instantaneous delay-sensitive data, such as realtime streaming data, there may be a tolerable threshold in the delay to satisfy a given service quality. In this case, it is more important to control the instantaneous delay $t_d$ given in (6) to not exceed the threshold than to minimize the mean transmission delay. In order to assess the network performance in this point of view, we simulated the probabilities of delay exceeding the tolerable threshold and plotted them as the function of the threshold for three different control algorithms (Fig. 3) when $\sigma = 12$ and $L=1000$ and 5000, respectively. We note that the optimum power/rate control also minimizes the probability of intolerable delay among three kinds of control schemes over almost all the range of the tolerable data rate threshold. While the power-only control and the rate-only control achieves an identical performance as the average mean transmission delay, there are differences in the instantaneous delay performance between them. This is because they have a different distribution of the instantaneous data rate even though they have an identical transmission delay in the mean sense.

In addition, there are two points to be addressed in Fig. 3. First, the power-only control scheme produces an almost piecewise linear curve. This can be explained using (6). Because $R$ is fixed and thus $t_d$ in (6) has a discrete distribution over the multiples of a constant $L/R$. Secondly, there is an approximate scaling factor of 5 for the delay threshold between the plots in (a) and (b). The explanation starts from an evaluation of the optimum SNR $\gamma_0^*$ in (15), and this reveals that $\gamma_0^*$ is nearly the same for $L=1000$ and $L=5000$. Therefore, $R_0$ is also nearly the same for $L=1000$ and $L=5000$ from (13). Consequently, there is an approximate scaling factor of 5 for the discontinuous points between the curves for $L=1000$ and 5000 in Fig. 3, because the discontinuous points are made at multiples of $L/R_0$.

![Fig. 3. Probabilities of instantaneous delay, $t_d$ exceeding a tolerable threshold for three different control schemes with $L=1000$ and 5000, $P_0/N_0=50$ dB, $\sigma=12$, BPSK.](image)

V. Conclusions

In this paper, we proposed an optimum power/rate control scheme for packet data transmission in CDMA networks where the power and the data rate are jointly controlled in order to achieve the minimum mean transmission delay. We demonstrated that controlling transmission power to be inverse-linear to the square root of the channel gains and controlling the data rate to be proportional to the square root of
the channel gain is optimum. We compared the performance of the proposed scheme to the rate-only and the power-only control schemes. As the variation of channel gains increased, the amount of reduction in transmission delay by the proposed scheme against the rate-only or power-only control schemes increased. With our numerical results, we demonstrated that the proposed control achieves a several times reduction in the mean transmission delay. Moreover, the proposed control achieves the best performance among three different control schemes in terms of the probability of instantaneous delay exceeding a given tolerable threshold.

In this investigation, for the sake of a compact analysis, we did not investigate automatic request control schemes with combining or incremental redundancy. Instead, we left this for a further study because the incremental redundancy scheme will be widely used in the 4th generation mobile communication systems [18], [19].

Appendix A

The optimum required SIR \( \gamma_0^* \) that minimizes \( D_{\text{PC}} \) is calculated by setting the derivative of (14) to equal to 0. Then, the optimum required SIR \( \gamma_0^* \) must satisfy the following equation with a variable \( x \):

\[
a(Lbx + 1) - e^{bx} = 0. \tag{A.1}
\]

In Fig. A.1, \( y=a(Lbx+1)-e^{bx} \) is plotted as a function of \( x \) for several system parameter sets. We note that there are two solutions to \( y=0 \) at each curve; however, the smaller one of these solutions corresponds to the local maximum point. The other solution corresponds to the minimum point and is expressed in a function form of \( a, b \) and \( L \) as follows:

\[
\gamma_0^* = -\frac{1}{b} \left\{ \text{lambertw} \left( -1, -\frac{e^{-1/L}}{aL} \right) + \frac{1}{L} \right\} \tag{A.2}
\]

where \( \text{lambertw}(\cdot, \cdot) \) is the \( -1 \)-st branch of the solutions to \( \text{exp}(w) = x \) [16]. Figure A.2 shows \( \gamma_0^* \) as a function of \( L \) for several system parameters.

References


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