An adaptive modulation scheme is presented for multiuser orthogonal frequency-division multiplexing systems. The aim of the scheme is to minimize the total transmit power with a constraint on the transmission rate for users, assuming knowledge of the instantaneous channel gains for all users using a combined bit-loading and subcarrier allocation algorithm. The subcarrier allocation algorithm identifies the appropriate assignment of subcarriers to the users, while the bit-loading algorithm determines the number of bits given to each subcarrier. The proposed bit-loading algorithm is derived from the geometric progression of the additional transmission power required by the subcarriers and the arithmetic-geometric means inequality. This algorithm has a simple procedure and low computational complexity. A heuristic approach is also used for the subcarrier allocation algorithm, providing a trade-off between complexity and performance. Numerical results demonstrate that the proposed algorithms provide comparable performance with existing algorithms with low computational cost.

Keywords: Bit loading, subcarrier allocation, transmit power, adaptive modulation, OFDM.

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) is an attractive technique for high-speed data transmission over indoor and outdoor wireless communication systems due to its ability to combat intersymbol interference (ISI). Moreover, it has already been widely adopted by the European digital audio/video broadcasting (DAB, DVB-T) standards, high-speed modems over digital subscriber lines (DSL), broadband wireless local area networks, such as the IEEE 802.11a, as well as Hiperlan2, and the broadband wireless access standard IEEE 802.16a [1]. To further improve performance, OFDM systems also use adaptive modulation [2], which is composed of a bit-loading algorithm and a subcarrier allocation algorithm. The former allocates the optimum number of bits to subcarriers according to instantaneous channel transfer functions, so that each subcarrier can be modulated with a different modulation scheme, while the latter assigns the subcarriers to users based on instantaneous channel information in a multiuser frequency selective fading environment.

Previous research on adaptive modulation schemes has essentially been carried out in two different directions, namely: 1) margin adaptive (MA) and 2) rate adaptive (RA). The objective of MA is to minimize the overall transmit power under a data rate constraint [3]-[5], while the objective of RA is to maximize the data rate under a power constraint [6], [7]. Rhee and Cioffi [6] proposed a dynamic subcarrier allocation scheme, which assures that all users achieve a similar data rate by maximizing the worst user’s capacity. However, this scheme has high computational complexity. Jang and Lee [7] formulated the rate maximization problem and proved that the sum capacity is maximized when each subcarrier is assigned to the user with the best subchannel gain, and the power is then...
distributed using a water-filling algorithm. However, fairness is not considered in [7]. Thus, when the path loss differences among users are large, the users with higher average channel gains can be allocated most of the resources. In general wireless systems without prioritized services, all users should be offered equal data rates, as with the MA approach. In [3], a Lagrangian-based algorithm has been proposed to achieve power efficiency under a data rate constraint. However, although suboptimal, the prohibitively high computational complexity renders it almost impractical. Finally, an efficient bit-loading algorithm for DMT application [8] and a multiuser bit-loading algorithm for multicarrier systems [9] have been proposed, where the former adds or removes one bit at a time based on a two-dimensional look-up table, while the latter seeks a solution by iterations, which is optimal for certain channel conditions.

Accordingly, this paper proposes a new adaptive modulation scheme which combines a bit-loading algorithm and a subcarrier allocation algorithm based on the MA approach. The proposed bit-loading algorithm is derived from a simple mathematical derivation and does not require any iterations to adjust the power level, in contrast to conventional water-filling approaches [7], [10], [11]. In addition, a heuristic approach is used for the subcarrier allocation algorithm, creating a trade-off between speed and performance. Since the proposed adaptive modulation scheme has a comparable performance and low computational complexity, it would appear to be practical.

The organization of this paper is as follows. Section II describes the system model and formulates the minimum overall transmit power problem. In section III, the bit-loading algorithm is derived for a single-user system from a geometric progression and the arithmetic-geometric means inequality. The subcarrier allocation algorithm is presented in section IV, and section V compares the performance of the proposed method with other approaches. Finally, some conclusions are given in section VI.

II. Problem Description

In this paper, it is assumed that the system has K users and the k-th user has a data rate equal to Bk bits per OFDM symbol. Using the channel information, the transmitter applies the combined bit-loading and subcarrier allocation algorithm to assign each subcarrier to different users and determine the number of bits/OFDM symbols to be transmitted by each subcarrier. Depending on the number of bits assigned to a subcarrier, the adaptive modulator uses a corresponding modulation scheme to adjust the transmit power level.

When considering the problem of minimizing the total transmitted power for a fixed number of transmitted bits per OFDM symbol Bk for the k-th user, for one moment, the transmission power allocated to the n-th subcarrier for bn,k bits is equal to

\[ P_{n,b_{n,k}} = f(b_{n,k}) \alpha_{n,k}^{-\Delta}, \]  

where \( b_{n,k} \) is the number of bits allocated to the n-th subcarrier by the k-th user, \( \alpha_{n,k} \) is the magnitude of the channel gain of the n-th subcarrier as seen by the k-th user, and \( f(b_{n,k}) \) is the received power required per symbol by the subcarrier for the reliable reception of \( b_{n,k} \) information bits per symbol. With respect to the total transmitted power \( P_T \), the problem can be formulated as

\[ P_T^* = \min_{b_{n,k} \in D} \sum_{n=1}^{K} \sum_{k=1}^{N} P_{n,b_{n,k}}. \]  

Moreover, the minimization is under the constraint that \( B_k = \sum_{n=1}^{N} b_{n,k} \), where \( D = \{0, 1, 2, \cdots, J\} \) is the set of all possible values of \( b_{n,k} \), and more than one user sharing a subcarrier is not allowed.

III. Bit-Loading Algorithm for Single-User System

A new bit-loading algorithm is presented which is faster than conventional methods and completes the computation within a definite time. The proposed algorithm is based on the geometric progression of the additional transmission power required by the subcarriers and the arithmetic-geometric means inequality. In this section, the bit-loading algorithm is derived for a single-user environment.

The optimization problem in (2) is rewritten for the case of a single user as

\[ P_T^* = \min_{b_{n,k} \in D} \sum_{n=1}^{K} P_{n,b_{n,k}}. \]  

and the minimization is under the constraint \( B = \sum_{n=1}^{N} b_{n,k} \). The transmission power allocated to the n-th subcarrier, \( P_{n,b_{n,k}} \) can be expressed as the sum of the additional transmission power required to successively allocate up to \( b_{n,k} \) bits, one bit at a time, to the n-th subcarrier;

\[ P_{n,b_{n,k}} = \Delta P_{n,1} + \Delta P_{n,2} + \cdots + \Delta P_{n,b_{n,k}}, \]  

where \( \Delta P_{n,l} = f(l) - f(l-1)/\alpha_{n,k}^2 \). \( \Delta P_{n,l} \) denotes the additional power needed to transmit one additional bit through subcarrier \( n \) when the number of bits loaded on the subcarrier is \( l-1 \).

The power required to satisfy the desired performance depends on the number of allocated bits. As such, the
transmission power required by the \( n \)-th subcarrier for \( b_n \) bits to be transmitted, in the case of rectangular M-ary quadrature amplitude modulation (MQAM) signal constellations (\( b_n \) is even), can be determined as in [12] as

\[
f(b_n) \geq \frac{N_0}{3} \left[ Q \left( \frac{P_z}{4} \right) \right]^2 (2^{b_n} - 1),
\]

where \( N_0/2 \) is the variance of additive white Gaussian noise (AWGN), \( P_z \) is the power of a symbol error, and \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \). For non-rectangular QAM signal constellations (\( b_n \in \{3, 5, 7, \ldots\} \)), the required transmission power can also be expressed as (5) without the equal sign [12].

Meanwhile, in the case of binary phase shift keying, \( b_n = 1 \), the required transmission power \( f(1) \) can be approximated as \( \{N_0/2|Q^{-1}(P_z)|^2\}/2 \), which is known to be slightly larger than the right-hand side of (5).

However, for simplicity and practicality, when deriving the bit allocation algorithm to minimize the total transmitted power, \( f(b_n) \) from (5) with the equality sign is used as the transmission power required by the \( n \)-th subcarrier for \( b_n \in \{1, 2, \ldots\} \) bits to be transmitted.

For the problem of minimizing the bit allocation, (3) can be rewritten as

\[
P_T^* = \min_{b_n} \min_{1 \leq n \leq N} \left[ \Delta P_{1,1} + \cdots + \Delta P_{1,b_n} \right] + \left[ \Delta P_{2,1} + \cdots + \Delta P_{2,b_n} \right] + \cdots + \left[ \Delta P_{N,1} + \cdots + \Delta P_{N,b_n} \right].
\]

It can be easily shown that, if the sum of the last additional transmission power required by \( N \) subcarriers, \( \Delta P_{1,b_n} + \Delta P_{2,b_n} + \cdots + \Delta P_{N,b_n} \), is the minimum, then \( P_T \) is the minimum value. Also, from (4) and (5), it is understood that the sequence \( \Delta P_{1,b_n}, \Delta P_{2,b_n}, \cdots, \Delta P_{N,b_n} \) is the geometric progression in which the initial term is \( f(1)/\alpha_n^* \), and the common ratio is 2. To find the minimum value of \( \Delta P_{1,b_n} + \Delta P_{2,b_n} + \cdots + \Delta P_{N,b_n} \), advantage is taken of the arithmetic-geometric means inequality:

\[
\Delta P_{1,b_n} + \Delta P_{2,b_n} + \cdots + \Delta P_{N,b_n} \geq N \frac{f(1)}{\sqrt[2]{\alpha_1 \alpha_2 \cdots \alpha_n}} 2^{b_n - 1}. \tag{7}
\]

When each term on the left side of (7) is equal, the sum is the minimum. Therefore, if each term on the left side has the same value as

\[
f(1) \frac{2^{b_n - 1}}{\alpha_n} = \frac{f(1)}{\alpha_1} \frac{2^{b_1 - 1}}{\alpha_1} + \frac{f(1)}{\alpha_2} \frac{2^{b_2 - 1}}{\alpha_2} + \cdots + \frac{f(1)}{\alpha_N} \frac{2^{b_N - 1}}{\alpha_N}, \tag{8}
\]

then the total transmitted power \( P_T \) is the minimum value. From (8), the optimal number of bits for the \( n \)-th subcarrier is obtained as

\[
b_n = \log_2 \frac{B}{N} - \log_2 \left( \frac{2}{N} \right), \quad \text{for } n = 1, 2, \cdots, N, \tag{9}
\]

Thereafter, (9) is used to calculate the bits allocated to the subcarriers, where \( b_n, n = 1, 2, \cdots, N \), must be integers and the sum of these bits \( \sum b_n \) must be equal to \( B \). Thus, for actual modulation/demodulation, \( b_n \) is rounded-off as an integer, such that \( \hat{b}_n = \text{round}(b_n) \), for \( n = 1, 2, \cdots, N \). When the sum of \( \hat{b}_n \), coincides with \( B \), the bit allocation procedure is finished. Otherwise, as many bits as the number of \( B - \sum \hat{b}_n \) should be added to or subtracted from the appropriate subcarriers. Let’s denote \( B - \sum \hat{b}_n \) by \( R \). If \( R \) is positive, \( R \) subcarriers are selected in a descending order of \( b_n - \hat{b}_n \), and one more bit is allocated to each of these subcarriers. In contrast, if \( R \) is negative, one bit is subtracted from each subcarrier among \( |R| \) subcarriers in the ascending order of \( b_n - \hat{b}_n \). The total number of added or subtracted bits is no more than \( N/2 \).

The following is a summary of the proposed bit-loading algorithm:

**Step 1.** Compute the number of bits for each subcarrier using (9).

**Step 2.** Calculate \( \hat{b}_n = \text{round}(b_n) \) and \( R = B - \sum \hat{b}_n \).

**Step 3.** Adjust \( \hat{b}_n \) to satisfy the constraint \( B = \sum \hat{b}_n \):

- If \( R = 0 \), then the procedure is finished.
- If \( R > 0 \), then select \( R \) subcarriers based on the descending order of the value of \( b_n - \hat{b}_n \) and add one bit to each of these subcarriers.
- If \( R < 0 \), then select \( |R| \) subcarriers based on the ascending order of the value of \( b_n - \hat{b}_n \) and subtract one bit from each of these subcarriers.

Figure 1 shows an example of the allocated bits and power for a test channel with 128 subcarriers, 512 bits to be transmitted, and a 10^{-3} target BER. The proposed algorithm does not require any iterations, in contrast to a water-filling approach, regardless of the channel environment, and its computational complexity is \( O(N + \log_2 N) \). The computation complexity of Lai’s algorithm [13] used in [3] is \( O(J-1) \).
can undergo independent fading, as the users may not all be in the same locations. Thus, subcarriers that appear in deep fade to one user may not be in deep fade for other users. Hence, each user should be allocated suitable subcarriers based on consideration of the channel gains for all users and fairness among all users, which is the essential purpose of the subcarrier allocation algorithm. Then, once all the subcarriers are allocated to the users using the subcarrier allocation algorithm, the single-user bit-loading algorithm given in section III is applied to each user of the allocated subcarriers. As such, this section presents a heuristic method which provides a trade-off between swiftness and performance.

The proposed subcarrier allocation algorithm specifically focuses on utilizing channels with high gains. Thus, in the case of two or more users with higher channel gains than threshold $\alpha_\text{thr}$, selected by trial and error in a certain subcarrier, the selection of that subcarrier is waived. After the rest of the subcarriers are allocated to the users with the highest gain, the waived subcarriers are then allocated. Thus, users with a lower channel gain have a fair chance when bidding against more powerful users. In the second iteration, if there are still two or more unallocated users with a high channel gain in a waived subcarrier, the user throughput, $C_k$, is employed as a criterion of selection:

$$C_k = \sum_{n^*} \frac{b_{\text{avg}}}{N} \log_2 \left( 1 + \frac{f(b_{\text{avg}}) \alpha_n^2}{N_0 b_{\text{avg}}} \right),$$

where $n^*$ denotes the waived subcarriers, $b_{\text{avg}}$ is the total bandwidth, and $f(b_{\text{avg}})$ is the calculated power, assuming that the average number of bits is allocated to each subcarrier. In a $K$-user OFDM system with $N$-subcarriers, each user is assumed to be allocated $s_k$ subcarriers, where $S_k = N/K$.

The following is the proposed subcarrier allocation algorithm:

**Step 1.** Every user’s average channel gain is normalized to one.

**Step 2.** First iteration

for subcarrier $n = 1:N$

if the number of subcarriers with $\alpha_n^2 > \alpha_{\text{thr}}$ is 2 or more

waive allocation

else

$k^* \leftarrow \arg \max \{\alpha_{n,k}^2\}$

allocate the $n$-th subcarrier to user $k^*$

if the number of selected subcarriers is $s_k$

stop allocation for user $k^*$

end if

end if

end for

Table 1. Complexity of bit-loading algorithms.

<table>
<thead>
<tr>
<th>Bit-loading algorithm</th>
<th>Order of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hughes-Hartogs’ algorithm [10]</td>
<td>$O(B \times N \log_2 N)$</td>
</tr>
<tr>
<td>Chow’s algorithm [11]</td>
<td>$O(k \times N + N \log_2 N)$</td>
</tr>
<tr>
<td>Proposed algorithm</td>
<td>$O(N + N \log_2 N)$</td>
</tr>
</tbody>
</table>

$B$: total number of loaded bits, $N$: number of subcarriers, $k$: required number of iterations, $J$: maximum number of information bits per symbol.

$B + N \log_2(N)$, where $J$ is the maximum number of information bits per symbol. The order of operations for the proposed and other algorithms is tabulated in Table 1. Clearly, the proposed algorithm requires far fewer operations than the other algorithms.

IV. Subcarrier Allocation Algorithm

In multiuser OFDM systems, any of the multiple user signals
Step 3. Second iteration for waived subcarrier
if the number of subcarriers with \( \alpha_{i,x} > \alpha_{i,y} \) is 2 or more
\[ k^* \leftarrow \arg \min \left( C_i \right) \]
allocate the \( n \)-th subcarrier to user \( k^* \)
if the number of selected subcarriers is \( s_k \)
stop allocation for user \( k^* \)
end if
else
\[ k^* \leftarrow \arg \max \left( \alpha_{i,x} \right) \]
allocate the \( n \)-th subcarrier to user \( k^* \)
if the number of selected subcarriers is \( s_k \)
stop allocation for user \( k^* \)
end if
end if
end for

The order of operations for the proposed algorithm and other algorithms is tabulated in Table 2. As the number of subcarriers increases, the computational burden of the proposed algorithm decreases compared to that of other algorithms.

### Table 2. Complexity of subcarrier allocation algorithms.

<table>
<thead>
<tr>
<th>Subcarrier allocation algorithm</th>
<th>Order of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kivanc’s RCG algorithm [5]</td>
<td>( O(KN + N \log_2 N) )</td>
</tr>
<tr>
<td>Rhee’s algorithm [6]</td>
<td>( O(2KN \log_2 N) )</td>
</tr>
<tr>
<td>Kivanc’s ACG algorithm [5]</td>
<td>( O(KN) )</td>
</tr>
<tr>
<td>Wong’s algorithm [3]</td>
<td>( O(N^2) )</td>
</tr>
<tr>
<td>Proposed algorithm</td>
<td>( O(2KN) )</td>
</tr>
</tbody>
</table>

Note: \( K \) number of users, \( N \) number of subcarriers

V. Simulation Results

To demonstrate the performance of the proposed algorithm for both single-user and multiuser OFDM, computer simulations were carried out in a frequency selective Rayleigh fading channel with additive Gaussian noise. It was assumed that the single-sided power spectral density level \( N_0 \) was equal to one, and no channel coding scheme was used. The bit error rate (BER) and the average transmit power were adopted as the performance measures of the bit allocation algorithms.

Figure 2 shows the BER versus the bit SNR in a single-user OFDM system with 128 subcarriers. The BER performance of the proposed subcarrier allocation algorithm lay between that of Wong’s algorithm and Kivanc’s ACG algorithm.

The performance of a multiuser OFDM system using the proposed bit-loading and subcarrier allocation algorithm was also compared with that using Wong’s algorithm. After the same number of subcarriers was selected for each user using the subcarrier allocation algorithms, the subcarriers allocated for each user were loaded with the appropriate number of bits level, \( N_0 \). The BER for the proposed algorithm was slightly higher than that for Hughes-Hartogs’ algorithm [10], as expected, yet it was lower than that for Chow’s algorithm [11]. Table 3 shows the average transmit power allocated to all subcarriers in the single-user OFDM system with 128 subcarriers and 512 bits to transmit for various target BERs. The average transmit power for the proposed algorithm was within 0.7 dB of that of Hughes-Hartogs’ algorithm. The proposed algorithm outperformed Chow’s by about 5.1 dB.

To compare the performance of the subcarrier allocation algorithms, computer simulations were performed in which an equal number of bits were allocated to all subcarriers in the single-user OFDM system with 128 subcarriers and 512 bits to transmit for various target BERs. The average transmit power for the proposed algorithm was within 0.7 dB of that of Hughes-Hartogs’ algorithm. The proposed algorithm outperformed Chow’s by about 5.1 dB.

![Fig. 2. BER performance comparison of bit-loading algorithms in a single-user OFDM system with 128 subcarriers.](image)

### Table 3. Average transmit power for various target BERs (\( p_e \)) (dB)

<table>
<thead>
<tr>
<th>Bit-loading algorithm</th>
<th>( p_e = 10^{-3} )</th>
<th>( p_e = 10^{-4} )</th>
<th>( p_e = 10^{-5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed algorithm</td>
<td>23.3273</td>
<td>31.6729</td>
<td>40.1322</td>
</tr>
</tbody>
</table>
VI. Conclusion

This paper presented a new adaptive modulation scheme, consisting of a bit-loading algorithm and a subcarrier allocation algorithm, for multiuser OFDM systems. The proposed bit-loading algorithm is derived from the fact that the sequence of the additional transmission power required by each subcarrier is a geometric progression and the minimum total transmitted power is obtained when the additional transmission power required by each subcarrier is the same according to the arithmetic-geometric means inequality. As such, the proposed algorithm does not require any iteration; rather, it involves calculating the number of bits for each subcarrier based on a formula, followed by an adding or subtracting procedure for the remaining bits, which are no more than \( N/2 \). The proposed subcarrier allocation algorithm is a heuristic method which balances the trade-off between speed and performance.

Simulation results in single-user and multiuser OFDM systems show that the proposed algorithms can achieve a similar BER performance to existing algorithms with low computational cost. As a result, the proposed adaptive modulation scheme is efficient and feasible for practical implementation.

References


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