Asynchronous Multilevel Search Strategy for Fast Acquisition of AltBOC Signals

Binhee Kim, Seung-Hyun Kong
CCS Graduate School for Green Transportation, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea

ABSTRACT

Alternative binary offset carrier (AltBOC) signals can be approximated by four synchronized direct sequence spread spectrum (DSSS) signals, each pair of which is a quadrature phase shift keyed (QPSK) signal at a different frequency. Therefore, depending on the strength of an incoming AltBOC signal, an acquisition technique can reduce the mean acquisition time (MAT) by searching the four DSSS signals asynchronously; the search for each of the four DSSS signals can start at one of the evenly separated hypotheses on the two-dimensional hypothesis space. And detection sensitivity can be improved by multiple levels when different numbers of search results for the same hypothesis are combined. In this paper, we propose a fast AltBOC acquisition technique that has an asynchronous search strategy and efficiently utilizes the output of the four search results to increase the sensitivity level when sensitivity improvement is needed. We provide a complete theoretical analysis and demonstrate with numerous Monte Carlo simulations that the MAT of the proposed technique is much smaller than conventional AltBOC acquisition techniques.

Keywords: AltBOC, acquisition, asynchronous search, search strategy

1. INTRODUCTION

The next-generation GNSS is being deployed (Parkinson et al. 1996, Kaplan & Hegarty 2005, Misra & Enge 2006), as planned. The U.S. government built a Global Positioning System (GPS) modernization plan that adds new civil and military signals to the legacy GPS (Kaplan & Hegarty 2005). The European Union is also deploying Galileo for civilian and public use (Kaplan & Hegarty 2005), Russia is planning to transmit a new civil signal from Global Navigation Satellite System, and the Chinese Compass system is expected to achieve a full operational capability by 2020 (Montenbruck et al. 2012).

Among the modulation schemes employed in the next-generation GNSS, alternating binary offset carrier (AltBOC) modulation has been adopted to the multiple new signals including Galileo E5 and Compass B2 signals. The AltBOC modulation belongs to the family of binary offset carrier (BOC) modulation. However, unlike the BOC modulation, AltBOC signals carry different signal components on each side of their split spectrum. For example, Galileo E5 has a double-sideband spectrum, where each sideband can be approximated by the spectrum of a quadrature phase shift keying (QPSK) modulated signal whose in-phase and quadrature-phase components are two different direct sequence spread spectrum (DSSS) signals. As a result, Galileo E5 signal offers unprecedented performance with a theoretical code tracking error less than 5 cm at signal strength of 35 dB-Hz (Sleewaegen et al. 2004), which is superior to other existing or planned GNSS signals. In addition, due to the multiple synchronized DSSS signals conveyed in an AltBOC signal, the detection sensitivity of the AltBOC signal can be higher than existing GNSS signals.

However, the enhanced performance of the AltBOC signals comes with a cost such as larger mean acquisition computation, longer mean acquisition time (MAT), or more receiver hardware complexity than a legacy GPS signal.
To reduce the cost for the fast AltBOC acquisition, a few techniques have been suggested to acquire the primary codes (the direct spreading sequence) of an AltBOC signal in the presence of the secondary code and unknown navigation data bits. For example, there are single side-band acquisition (SSB), double side-band acquisition (DSB), full-band independent code (FIC) acquisition, and Direct-AltBOC acquisition (Margaria et al. 2008, Shivaramaiah 2011) techniques. In the SSB technique (Margaria et al. 2008), a receiver uses a single bandpass filter (BPF) with 20.46 MHz bandwidth (BW) to acquire one or two primary codes in one of the two QPSK signals of the Galileo E5 signal, i.e., the technique may try to acquire one of the four primary codes in E5aQ, E5bQ, E5al, and E5bl, or to acquire two primary codes in E5al and E5aQ or in E5bl and E5bQ. In the DSB technique (Margaria et al. 2008), a receiver has two BPFs with 20.46 MHz BW to process two QPSK signals simultaneously and can non-coherently combine the search result of the two primary codes in [E5aQ, E5bQ] or the four primary codes in [E5al, E5aQ, E5bl, and E5bQ]. However, in the FIC technique (Shivaramaiah 2011), a receiver has a BPF with 51.15 MHz BW and can non-coherently combine any set of the four primary code signals. Since a Galileo E5 signal is equivalent to an 8-PSK signal, in the Direct-AltBOC technique (Shivaramaiah 2011), a receiver is equipped with a BPF with 51.15 MHz BW and correlates the incoming signal with a receiver replica of the 8-PSK signal generated with a look-up-table. In general, the DSB, FIC, and Direct-AltBOC techniques are preferred acquisition techniques to the SSB technique for higher detection sensitivity. However, due to the secondary code and unknown navigation data bits, the Direct-AltBOC technique has higher algorithmic and computational complexity than other techniques, and the FIC technique suffers from a larger noise power than the DSB technique for wider BPF bandwidth.

Despite of the multiplicity of the primary codes, all of the Galileo E5 signal acquisition techniques introduced in the literature employ a synchronized search strategy, where the code phase and Doppler frequency hypotheses of the primary codes being tested are always the same during the search process, and a single detection level strategy, where the number of auto-correlation function (ACF) outputs constructing a detection variable is fixed. In this paper, we propose a time-efficient asynchronous search strategy to reduce the MAT for AltBOC signal acquisition. The proposed strategy achieves about 4 times smaller MAT than the DSB and SSB techniques for a strong signal (C/N0 > 43 dB) and has about similar MAT to the DSB and SSB techniques for moderate and weak signals (C/N0 < 36 dB).

The rest of this paper is organized as follows. Section 2 introduces the Galileo E5 signal, and Section 3 introduces the conventional AltBOC signal acquisition techniques. The proposed asynchronous multilevel search strategy is introduced in Section 4. The statistical performance analysis of the proposed technique is provided in Section 5, and performance comparison to the conventional techniques using numerous Monte Carlo simulation results is provided in Section 6. The conclusion of the paper is in Section 7.

2. GALILEO E5 SIGNAL

Among a number of AltBOC signals in the next generation GNSS, we consider Galileo E5 signal as an example in this paper. The Galileo E5 signal consists of four sets of primary codes and secondary codes, and it can be modeled as the sum of two QPSK signals (i.e., E5a(=E5aLÆE5aQ) and E5b(=E5bLÆE5bQ)). In this regard, ALB represents the phase difference between A and B, and () and () represent the in-phase signal component and the quadrature-phase signal component, respectively. E5a(=E5aLÆE5aQ) and E5b(=E5bLÆE5bQ) signals are transmitted at a carrier frequency of 1176.45 MHz and 1207.14 MHz, respectively. For the Galileo E5 signal, the in-phase signal component is transmitted by spreading the navigation data bit signal as a primary code and a secondary code, and the quadrature-phase signal component is transmitted by spreading the pilot signal as another primary code and secondary code. The four signal components, E5a data channel (E5a), E5a pilot channel (E5a), E5b data channel (E5b), and E5b pilot channel (E5b), are spread by four synchronized primary codes of the same chip rate 10.23 MHz (=fr, where fr is the reference frequency) so that the main lobes of E5a and E5b spectra span 2fr. The transmitted signal from Galileo E5 satellite is expressed (Galileo Project 2013)

\[
s(t) = \frac{1}{2\sqrt{2}} \left[ (E^I_L(t) + jE^Q_L(t))(sc_a(t) - jsc_b(t - \frac{T}{4})) \\
+ (E^I_a(t) + jE^Q_a(t))(sc_a(t) + jsc_b(t - \frac{T}{4})) \\
+ (E^I_b(t) + jE^Q_b(t))(sc_a(t) - jsc_b(t - \frac{T}{4})) \\
+ (E^I_b(t) + jE^Q_b(t))(sc_a(t) + jsc_b(t - \frac{T}{4})) \right],
\]

where

\[
\begin{align*}
E^I_L(t) &= E^I_a(t)E^I_b(t)E^I_a(t) \\
E^Q_L(t) &= E^Q_a(t)E^Q_b(t)E^Q_a(t) \\
E^I_a(t) &= E^I_a(t)E^I_b(t)E^I_b(t) \\
E^Q_a(t) &= E^Q_a(t)E^Q_b(t)E^Q_b(t) \\
E^I_b(t) &= E^I_b(t)E^I_a(t)E^I_a(t) \\
E^Q_b(t) &= E^Q_b(t)E^Q_a(t)E^Q_a(t)
\end{align*}
\]
are the product components
\[ sc_x(t) = \sum_{k=1}^{n} a_k \cdot \text{rect}_{T_s}(t - kT_s / 8) \]  \hspace{1cm} (6)
\[ sc_y(t) = \sum_{k=1}^{n} b_k \cdot \text{rect}_{T_s}(t - kT_s / 8) \]  \hspace{1cm} (7)

are the subcarrier functions of the signal and product components, respectively. \( T_s \) denotes the subcarrier period, \( T_s(=1.57T_n) \) denotes the chip width of the Galileo E5 signal, and the coefficients \( a_k \) and \( b_k \) are summarized in Table 1.

Since an AltBOC signal can be modeled as the sum of two QPSK signals separated in frequency by \( 3f_a \), the incoming signal to a GNSS receiver can be expressed as
\[
r(t) = \alpha \left[ E_a(t - \tau) + jE_a^\theta(t - \tau) \right] e^{j(2\pi f_{sc} + \phi_a)} + \beta \left[ E_b(t - \tau) + jE_b^\theta(t - \tau) \right] e^{j(2\pi f_{sc} + \phi_b)} + \beta r_p(t) + \alpha(t), \]

where \( r_p(t) \) represents the sum of the product components as
\[
r_p(t) = \alpha \left[ E_a^p(t - \tau) + jE_a^\theta(t - \tau) \right] e^{j(2\pi f_{sp} + \phi_a)} + \beta \left[ E_b^p(t - \tau) + jE_b^\theta(t - \tau) \right] e^{j(2\pi f_{sp} + \phi_b)} \]

and \( \tau, f_{lo}, f_{la}, f_{lb}, f_{na}, \) and \( f_{nl} \) are the code phase, IF frequency, Doppler frequency of the E5a channel, Doppler frequency of E5b channel, subcarrier frequency 1.5\( f_a \) of the signal components, subcarrier frequency of the product components, respectively, \( \phi_a, \phi_b, \phi_p, \phi_{\nu} \) and \( \phi_{\nu} \) are unknown phase offsets, \( \alpha \) and \( \beta \) represent magnitudes, and \( w(t) \) is a complex additive white Gaussian noise with two-sided power spectral density \( N_0 / 2 \). The received power ratio between the signal components \( \alpha^2 \) and the product components \( \beta^2 \) is about 6 (Lestarquit et al. 2008). Note that the maximum difference \( f_{sp} \) between \( f_{sc} \) is 65Hz due to the difference of the carrier frequencies of E5a and E5b signals (Wallner et al. 2005). When \( T = 1 \) ms is the integration interval and 1/(2\( T \)) = 500 Hz is the frequency search step size (assumed in this paper), the 65 Hz difference is negligible so that we can further assume \( f_{sp} = f_{sc} \) without loss of generality.

### Table 1. AltBOC subcarrier coefficients.

<table>
<thead>
<tr>
<th>k</th>
<th>( a_k )</th>
<th>( b_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \sqrt{2} + 1 )</td>
<td>( -\sqrt{2} + 1 )</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>( -\sqrt{2} - 1 )</td>
<td>( \sqrt{2} - 1 )</td>
</tr>
<tr>
<td>5</td>
<td>( -\sqrt{2} + 1 )</td>
<td>( \sqrt{2} - 1 )</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>( \sqrt{2} + 1 )</td>
<td>( -\sqrt{2} + 1 )</td>
</tr>
</tbody>
</table>

### 3. CONVENTIONAL ALTBOC SIGNAL ACQUISITION

The front end of a GNSS receiver uses a number of analogue filters whose precise characteristics are difficult to model with theoretical expressions. However, for an algebraic simplicity in this paper, we assume that a filter \( H(f) \) in the front end is an ideal linear bandlimited filter such that the amplitude response and the derivative of the phase response with respect to the frequency \( f \) are constant as
\[
[|H(f)|] = A, \]  \hspace{1cm} (10)
\[
-\frac{1}{2\pi} \frac{d\phi_f(f)}{df} = \tau, \]  \hspace{1cm} (11)

respectively, for all frequency \( f \) within the passband of bandwidth \( B_e \), where \( \phi_f(f) = 2\pi f \) and \( \nu \in [1, 2, 3] \) is the filter index so that the filter transfer function can be expressed as
\[
H_n(f) = A e^{j(2\pi \nu f + \theta_n)} = (A e^{j\theta_n}) e^{-j2\pi \nu f}, \]  \hspace{1cm} (12)

where \( \theta_n \) is a phase offset. In addition, \( \tau << T_c \) and \( A_1 = 1/\sqrt{B_e} \) are assumed for an ideal filter, and, in the following, \( h(t) \) denotes the time-domain expression of \( H(f) \).

In the following, among a few conventional AltBOC acquisition techniques introduced in the literature, Direct-AltBOC, SSB, and DSB techniques, shown in Fig. 1, respectively, are briefly introduced for a comparison with the proposed technique in Section 6. The detection variable of the Direct-AltBOC technique is found in (Shivaramaiah 2011) as
\[
j_1 = \sum_{n=-N_1}^{N_1} \left[ \left( E_a[n - \hat{n}] + jE_a^\theta[n - \hat{n}] \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) \right] e^{j2\pi f_{sc} n / T} + \left( E_a[n - \hat{n}] + jE_a^\theta[n - \hat{n}] \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) + \frac{1}{2} \left( E_a[n - \hat{n}] + jE_a^\theta[n - \hat{n}] \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) + \frac{1}{2} \left( E_a[n - \hat{n}] + jE_a^\theta[n - \hat{n}] \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) + \left( E_{a[n - \hat{n}] + jE_{a[n - \hat{n}]}} \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) + \left( E_{a[n - \hat{n}] + jE_{a[n - \hat{n}]}} \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) + \left( E_{a[n - \hat{n}] + jE_{a[n - \hat{n}]}} \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) + \left( E_{a[n - \hat{n}] + jE_{a[n - \hat{n}]}} \right) \left( sc[n - \hat{n}] + jsc[n - \hat{n} - \frac{N_1}{2}] \right) \]  \hspace{1cm} (13)

where
\[
r_p(t) = (h(t) * r(t)) e^{-j2\pi f_{sp} t / T}, \]  \hspace{1cm} (14)

\( N_1 = f_s T, f_s \) and \( T \) denote the sampling frequency and the integration interval, respectively, \( \hat{n} \) and \( n_a \) denote the discrete-time representation of the estimated code delay.

http://www.gnss.or.kr
filter, and, in the following, in the literature, Direct-AltBOC, SSB, and DSB techniques, shown in Figs. 1a-c, respectively.

\[
\begin{align*}
\hat{f}_d & = \sum_{n} r_n \left[ E^o_n[n-n_c] - j E^o_n[n-n_c] \right] \\
\hat{f}_d & = \sum_{n} r_n \left[ E^o_n[n-n_c] + j E^o_n[n-n_c] \right] \\
\hat{f}_d & = \sum_{n} r_n \left[ E^o_n[n-n_c] - j E^o_n[n-n_c] \right]
\end{align*}
\]

where

\[
\begin{align*}
\hat{r}(t) & = (b(t) \ast r(t)) e^{-j2\pi f_c t} \\
\hat{r}(t) & = (b(t) \ast r(t)) e^{-j2\pi f_c t}
\end{align*}
\]

and then the four outputs are non-coherently combined to build a detection variable (Shivaramaiah 2011)

\[
y_1 = |y_1|^2 + |y_2|^2 + |y_3|^2 + |y_4|^2
\]

that has a non-central or central \( \chi^2 \) distribution with 4 degrees of freedom (DOF) depending on the accuracy of \( \hat{r} \) and \( \hat{f}_d \). Using Eqs. (15-18), the detection variable of the SSB technique can be expressed as (Shivaramaiah 2011)

\[
y_1 = |y_1|^2 + |y_2|^2
\]

that has a non-central or central \( \chi^2 \) distribution with DOF 2 depending on the accuracy of \( \hat{r} \) and \( \hat{f}_d \).

4. PROPOSED TECHNIQUE FOR A FAST ALTBOC ACQUISITION

In this section, we describe the details of the proposed AltBOC acquisition technique, shown in Figs. 2 and 3. Fig. 2 illustrates the asynchronous search strategy employed in the three stages of the proposed technique, where the overall Doppler frequency search range is divided into four equal size small range segments that are searched in...
parallel. The size of a range segment and the direction of the Doppler frequency search step increase are represented by arrows along the Doppler frequency axis for three stages. At the 1st stage, the four DSSS signals of Galileo E5 are searched in parallel and asynchronously such that the Doppler frequency hypotheses for the four DSSS signals are within the four respective range segments. The four arrows illustrate the initial Doppler frequencies and the final Doppler frequencies for each of the four DSSS signal searches in three stages. In the 2nd stage, the four arrows for the four DSSS signals continues in the same direction to search the four DSSS signals in the next Doppler frequency range segments. In the 3rd stage, the four arrows for the four DSSS signals are doubled in length so that the search of the four DSSS signals can be completed for all Doppler frequencies. Note that during the 1st (2nd) stage the AltBOC signal is completely searched over the entire two-dimensional hypothesis plane by testing only one (two) of the four DSSS signals at each hypothesis.

And during the first (second) half of the 3rd stage, the proposed technique can search the AltBOC signal using the three (four) of the four DSSS signals for all hypotheses. For algebraic simplicity in this paper, we assume that the search result of the first half of the 3rd stage is only stored in the memory block shown in Fig. 3 and the comparison of the detection variable to the detection threshold is only performed in the second half.

Fig. 3 shows the schematic diagram of the proposed technique. The two QPSK signals in \( r(t) \) are filtered, sampled with sampling frequency \( f_s = 1/T_o \) and carrier removed. The switches associated with numerically controlled oscillators (NCO) are on \( D_q \) nodes while the signal search is in process but on \( D_v \) in the verification mode. Notice that the conjugate of the NCO output is to generate a carrier with a negative frequency of the given input frequency, which is shown in Fig. 4. There

are four arms for the four DSSS signals in the diagram, and each arm represents a path of a complex signal. The complex correlations are integrated for \( N_f \) samples and yield

\[
\begin{align*}
X_1^1(\hat{n}) &= \sum_{n=0}^{N_f-1} (h_n(t) * r(t)) e^{j2\pi f_s T_o (n + \hat{n})} \times (E_1[n - \hat{n}] - jE_2[n - \hat{n}]) \\
X_1^2(\hat{n}) &= \sum_{n=0}^{N_f-1} (h_n(t) * r(t)) e^{j2\pi f_s T_o (n + \hat{n})} \times (E_1[n - \hat{n}] + jE_2[n - \hat{n}]) \\
X_2^1(\hat{n}) &= \sum_{n=0}^{N_f-1} (h_n(t) * r(t)) e^{j2\pi f_s T_o (n + \hat{n})} \times (E_2[n - \hat{n}] - jE_1[n - \hat{n}]) \\
X_2^2(\hat{n}) &= \sum_{n=0}^{N_f-1} (h_n(t) * r(t)) e^{j2\pi f_s T_o (n + \hat{n})} \times (E_2[n - \hat{n}] + jE_1[n - \hat{n}])
\end{align*}
\]

where \( \hat{n} \) is the code phase hypothesis for all four DSSS signals, and \( \hat{f}_{d,m} (m=1,2,3,4) \) is the Doppler frequency hypothesis for the \( m \)-th DSSS signal. Since \( \hat{f}_{d,m} \neq \hat{f}_{d,k} \) for any \( m \neq k \) \( k \in \{1,2,3,4\} \), the 1st stage has four detection variables, \( y_{p,m} (m=1,2,3,4) \), that are compared to a common threshold \( y_t \) of the 1st stage.

\[
\begin{align*}
y_{p1,1} &= |X_1^1(\hat{f}_{d,1})|^2 \\
y_{p2,1} &= |X_1^2(\hat{f}_{d,1})|^2 \\
y_{p1,2} &= |X_2^1(\hat{f}_{d,2})|^2 \\
y_{p2,2} &= |X_2^2(\hat{f}_{d,2})|^2
\end{align*}
\]

http://www.gnss.or.kr
where \( \hat{f}_{d,4} = -f_{d,4} \) and \( \hat{f}_{d,2} = -f_{d,2} \). In addition, \( y_{p,m}(m=1,2,3,4) \) for all \( \hat{n} \) and \( \hat{y}_{m,\hat{n}} \) is stored in the memory block to use in later stages. When \( y_1 \) is sufficiently high, and the code phase hypothesis and \( m \)-th Doppler frequency hypothesis \( f_{d,m} \) are correct, there can be only one decision block declaring a signal detection. When the signal detection at the \( m \)-th decision block is declared, we set \( \hat{f}_{d,1} \) and \( \hat{f}_{d,2} \) to \( f_{d,m} \) (case \( m' = 1 \), \( 2 \), \( 3 \), \( 4 \), \( 25 \)), respectively, and the noise variance can be found as

\[
\sigma^2(t) = \frac{W_n}{4} T
\]

where the second and third fourth terms of the equations in Eqs. (36-39) correspond to the search results obtained during the first half of the 3rd stage that takes \( N_s T \) seconds, the 1st stage (Eqs. (27-30)), and the 2nd stage (Eqs. (27-30)), respectively, that are stored in the memory block. Notice that by the end of the 3rd stage every code phase and Doppler frequency hypotheses is tested using a detection variable made from the search result(s) of one, two, and four of the four DSSS signals in the 1st, 2nd, and 3rd stages, respectively, which enables the multilevel detection sensitivity of the proposed technique.

5. PERFORMANCE ANALYSIS

In this section, we derive algebraic expressions for the performance analysis of the SSB, DSB, AltBOC, and proposed techniques.

5.1 Conventional DSB and SSB Techniques

The magnitude of an ACF output for the correct code phase and Doppler frequency hypotheses (cell) in the DSB and SSB techniques can be expressed using \( y_2 \) (Eqs. (32-35)) and \( y_3 \) (Eqs. (36-39)) as

\[
S = \begin{cases} \sqrt{y_2}, & \text{for DSB} \\ \sqrt{y_1}, & \text{for SSB} \end{cases}
\]

respectively, and the noise variance can be found as

\[
S = \begin{cases} 4f_T N_s, & \text{for DSB} \\ 2f_T N_s, & \text{for SSB} \end{cases}
\]

In the case of the DSB technique, the distribution of \( y_2 \) is a noncentral \( \chi^2 \) distribution with DOF 4 for the \( H_1 \) cell as

\[
P_{\chi^2}(y_2) = \frac{1}{V} \exp \left( -\frac{(y_2 + S^2)}{V} \right) \sqrt{\frac{y_2}{S^2}} I_4 \left( 2S \sqrt{\frac{y_2}{V}} \right).
\]

where \( I_4(\cdot) \) is the first-order modified Bessel function of the first kind. And the distribution of \( y_2 \) for an incorrect hypothesis (\( H_0 \) cell) has a central \( \chi^2 \) distribution with DOF 4 as

\[
P_{\chi^2}(y_2) = \frac{1}{V} \exp \left( -\frac{y_2}{V} \right).
\]

In the case of the SSB technique, the distribution of \( y_3 \) for the \( H_1 \) cell is a noncentral \( \chi^2 \) distribution with DOF 2 as

\[
P_{\chi^2}(y_3) = \frac{1}{V} \exp \left( -\frac{(y_3 + S^2)}{V} \right) I_2 \left( 2S \sqrt{\frac{y_3}{V}} \right).
\]

where \( I_2(\cdot) \) is the zeroth-order modified Bessel function of...
the first kind. On the other hand, for an cell has a central distribution with DOF 2

\[ P_1(y_i) = \frac{1}{V} \exp \left( -\frac{y_i}{V} \right). \]  

(45)

Using \( P_1 \) (Eqs. (42) and (45)) and \( P_0 \) (Eqs. (43) and (45)), the false alarm, detection, and misdetection probabilities are (Abu-Rgheff 2007)

\[
P_1(y_i) = \left[ 1 + \frac{y_i^2}{V} \right] \exp \left( -\frac{y_i}{V} \right) \]  

(46)

\[
P_2(y_i) = Q \left( \frac{\sqrt{2}}{V} \sqrt{\frac{2y_i}{V}} \right) \]  

(47)

\[
P_3(y_i) = 1 - P_3(y_i). \]  

(48)

respectively, where \( g(a, b) \) is the lower incomplete Gamma function, \( c = 0 \) for SSB, \( c = 2 \) for DSB, and \( Q_{ab}(a, b) \) is the Marcum’s Q-function (Marcum 1950) of order \( M \) as

\[
Q_{ab}(a, b) = \int_a^\infty u^{M-1} \exp \left( -\frac{u^2 + a^2}{2} \right) J_{M-i}(au) du. \]  

(49)

5.2 First and second Stages of the Proposed Technique

In the following subsections, \( y_{pi} \) denotes the detection variable at the \( i \)-th stage (\( i = 1, 2, 3 \)) of the proposed technique. The ACF peak of the -th stage for the \( H_1 \) cell is

\[
S_i = \sqrt{V_{pi}}, \]  

(50)

and the noise variance of \( y_{pi} \) is

\[
V_i = icTN_y. \]  

(51)

Since detection variables of the 1st and 2nd stages have non-central \( \chi^2 \) distribution with DOF 2 for the \( H_1 \) cell and central \( \chi^2 \) distribution with DOF 2 for an \( H_0 \) cell, the distribution of \( y_{pi} \) and \( y_{p2} \) for the \( H_1 \) cell can be found as

\[
P_{pi}(y_{pi}) = \frac{1}{V_i} \exp \left[ -\frac{(y_{pi} + S_i^2)}{2V_i} \right] I_0 \left( \frac{2S_i \sqrt{V_i}}{V_i} \right), \]  

(52)

where \( i = 1, 2 \), \( I_0(\cdot) \), is the zeroth-order modified Bessel function of the first kind, \( y_{pi} \) and \( y_{p2} \) the distribution of and for an \( H_0 \) cell is

\[
P_{pi}(y_{pi}) = \frac{1}{V_i} \exp \left( -\frac{y_{pi}}{V_i} \right). \]  

(53)

where \( i = 1, 2 \). Using \( P_{ai} \) (Eq. (52)) and \( P_{ao} \) (Eq. (53)), we can find the false alarm, detection, and misdetection probabilities when four asynchronous correlators are used for a serial search scheme as shown in Fig. 3. The false alarm probability when the four hypotheses being tested are cells, detection probability, false alarm probability when one of the four hypotheses being tested are the \( H_1 \) cell, and misdetection probability are derived as (Kim & Kong 2014)

\[
P_{f1}(y_i) = P_{ab}(y_{pi}^{N0} > y_i) = 1 - \left[ 1 - \exp \left( -\frac{y_i}{V_i} \right) \right]^N \]  

(54)

\[
P_{o1}(y_i) = P_{ai}(y_{pi} > y_i) = Q \left( \frac{S_i \sqrt{2V_i}}{V_i} \right) \]  

(55)

\[
P_{f1}(y_i) = P_{ab}(y_{pi}^{N0} > y_i) = \left[ 1 - \left( 1 - \exp \left( -\frac{y_i}{V_i} \right) \right)^{N-i} \right] \times \left[ 1 - Q \left( \frac{S_i \sqrt{2V_i}}{V_i} \right) \right] \]  

(56)

\[
P_{m1}(y_i) = 1 - P_{o1}(y_i) - P_{f1}(y_i), \]  

(57)

where \( N (= 4) \) is the number of hypotheses being tested asynchronously, and \( X^{N0} \) denotes the N-th minimum of X.

5.3 Third Stage of the Proposed Technique

In the 3rd stage of the proposed technique, the detection variable has the same distribution to that of the DSB technique, so that the ACF peak for the \( H_1 \) cell is

\[
S_i = \sqrt{V_{pi}}, \]  

(58)

and the noise variance is

\[
V_i = 4icTN_y. \]  

(59)

For the \( H_1 \) cell, the distribution of \( y_{pi} \) is a noncentral \( \chi^2 \) distribution with DOF 4 as

\[
P_{ai}(y_{pi}) = \frac{1}{V_i} \exp \left[ -\frac{(y_{pi} + S_i^2)}{2V_i} \right] I_2 \left( \frac{2S_i \sqrt{V_i}}{V_i} \right), \]  

(60)

and the distribution of \( y_{p3} \) for an \( H_0 \) cell has a central \( \chi^2 \) distribution with DOF 4 as

\[
P_{ao}(y_{p3}) = \frac{y_{p3}}{V_3} \exp \left( -\frac{y_{p3}}{V_3} \right). \]  

(61)

Using \( P_{ai} \) (Eq. (60)) and \( P_{ao} \) (Eq. (61)), the false alarm probability when the four hypotheses being tested are \( H_0 \) cells, detection probability, false alarm probability when
one of the four hypotheses being tested are the $H_i$ cell, and misdetection probability are derived as (Kim & Kong 2014)

$$P_{p,3}(y_i) = 1 - \left[ P_{b0}(y_{p,3} < y_i) \right]^3$$

$$= 1 - \left[ 1 + \frac{y_i}{V_j} \right] \exp \left( -\frac{y_i}{V_j} \right)$$

(62)

$$P_{d,3}(y_i) = P_b(y_{p,3} > y_i) = Q_i \left[ \frac{2}{V_j} \sqrt{\frac{2y_i}{V_j}} \right]$$

(63)

$$P_{d,3}(y_i) = P_{b0}(y_{p,3} < y_i) P_b(y_{p,3} > y_i)$$

$$= \left[ 1 - Q_i \left( \frac{2}{V_j} \sqrt{\frac{2y_i}{V_j}} \right) \right] \left[ 1 - \left( 1 + \frac{y_i}{V_j} \right) \exp \left( -\frac{y_i}{V_j} \right) \right]$$

(64)

$$P_{f,3}(y_i) = 1 - P_{d,3}(y_i) - P_{d,3}(y_i)$$

(65)

respectively.

### 5.4 Mean Acquisition Time

The search state diagram of the proposed technique is illustrated in Fig. 5. Each node represents four hypotheses that are simultaneously tested by the four asynchronous correlators. It shows that verification mode that takes $pT$ seconds is initiated when a signal detection is declared at any stage, and that the technique moves to the $(i+1)$-th stage when signal is not detected by the end of the $i$-th stage ($i=1,2$). Notice that the search state diagram shows a non-circular search scheme, where the search operation does not return to the 1st stage to re-initialize the search when signal is not detected by the end of the 3rd stage. Since when a GNSS receiver fails to detect a satellite signal, the receiver begins searching for another satellite signal rather than searching the satellite signal again.

Using Eqs. (54-57) and Eqs. (62-65), the mean acquisition time (MAT) of the proposed technique for a circular search scheme can be found as

$$H(T) = \frac{1}{F_c} \left[ \left( 1 - H_{d,1}(T) H_{d,2}(T) H_{d,3}(T) N_T \right) \times \left( H_{d,1}(T) \sum_{k=0}^{T-1} H_{d,0}(T) \right) + H_{d,2}(T) H_{d,3}(T) N_T \sum_{k=0}^{T-1} H_{d,0}(T) \right] \times N_T \sum_{k=0}^{T-1} H_{d,0}(T),$$

(66)

where $F_c$ is the number of hypotheses in a stage, and $N_T = F_c$ indicates the time needed for the first half of the 3rd stage. The branch transfer functions for the correct hypothesis, the correct hypothesis missed, and an incorrect hypothesis are

$$H_{d,1}(T) = P_{d,1} T^{p+1}$$

(67)

$$H_{d,2}(T) = P_{d,2} T + P_{p,2} T^{p+1}$$

(68)

$$H_{d,3}(T) = (1 - P_{p,3}) T + P_{p,3} T^{p+1},$$

(69)

where $i=1,2,3$. On the other hand, for the non-circular search scheme as illustrated in Fig. 5, a practical search transfer function can be derived as

$$H_i(T) = \frac{1}{F_c} \left( H_{d,1}(T) \sum_{k=0}^{T-1} H_{d,0}(T) \right) + H_{d,2}(T) H_{d,3}(T) N_T \sum_{k=0}^{T-1} H_{d,0}(T)$$

(70)

Utilizing the algebraic method to obtain MAT introduced in (Viterbi 1995), the MAT of the proposed technique can be derived as

http://dx.doi.org/10.11003/JPNT.2015.4.4.161
can be expressed as

\[ \mu = \frac{dH(T)}{dT} \]

\[ \approx (p + 1)P_{0,1} + \left(pP_{3,1} + 1\right)P_{0,1} F_{c} - \frac{1}{2} \]

\[ + \left(P_{0,1} + P_{3,1}\right) \left(P_{0,1} + \left(pP_{3,1} + 1\right)P_{0,1}\right) \]

\[ + \left(P_{0,1} + P_{3,1}\right) \left(P_{0,1} + \left(pP_{3,1} + 1\right)P_{0,1}\right) \]

\[ + P_{0,1} \left(pP_{3,1} + 1\right)F_{c} + \left(pP_{3,1} + 1\right) \left(F_{c} - \frac{1}{2}\right) \]

\[ + P_{0,1} (p + 1) + N_{a} + F_{c} P_{0,3} (pP_{3,1} + 1) \]

\[ + \frac{F_{c} - 1}{2} \]

\[ P_{0,1} (pP_{3,1} + 1) + \left(P_{3,1} + 3\right) F_{c} + 1 \]

\[ + pF_{c} (P_{3,1} + P_{3,2} + P_{3,3}) \] (71)

For a comparison, the overall transfer function of the conventional SSB and DSB techniques can be expressed as

\[ H_{i}(T) = \frac{1}{F_{c} \sum_{k=0}^{3} H_{i}^{k}(T)} \]

\[ + H_{d}(T)H_{d}^{c-1}, \]

where \( F_{c}(=4F_{c}) \) is the number of total hypotheses to test, and

\[ H_{d}(T) = P_{0,1} T^{p+1} \]

\[ H_{d}(T) = P_{0,1} T \]

\[ H_{d}(T) = (1 - P_{1})T + P_{1} T^{p+1}, \]

where \( i=1,2,3 \). Using Eqs. (72-75), the MAT of the conventional SSB and DSB techniques can be expressed as

\[ \mu_{c} = \frac{dH(T)}{dT} \]

\[ = \left[p + 1\right] P_{0,1} F_{c} + \left(F_{c} - \frac{1}{2}\right) \]

\[ + F_{c} (1 + \left(F_{c} - 1\right)(1 + pP_{3,1})). \] (76)

6. NUMERICAL RESULTS

In this section, the performance of the proposed technique is demonstrated using 10^5 Monte Carlo simulations for a Galileo E5 receiver with two BPFs having 2f, BW for E5a and E5b signals. Both E5a and E5b signals are sampled at the same sampling frequency \( f_{s} = 2f \), and integrated with an interval \( T_{s}=1 \) ms to produce a search result. The total number of Doppler frequency hypotheses to test is \( F_{c} = 20 \) by choosing the Doppler frequency search step size of 525 Hz, which is a reasonable choice, since the Doppler frequency search step size can be from 1/(2T_{s}) to 2/(3T_{s}) (Kaplan & Hegarty 2005). And \( N_{o}=5\times10^{6} \) is the number of all hypotheses to search in the first half of the 3rd stage.

Fig. 6 shows numerically evaluated detection threshold (for \( i=1,2,3 \)) to the 1st stage signal power ratio, i.e., \( \gamma_{i} / |S_{i}|^{2} \), to achieve a constant false alarm rate (CFAR) \( P_{f} = 10^{-3} \) in Eqs. (54, 56, 62, 65) for a wide range of the \( C/N_{0} \) of the incoming Galileo E5a signal. Note that to achieve \( P_{f} = 10^{-3} \), it should be \( \gamma_{i} \geq \gamma \) for \( i=1,2 \). Note also that \( \gamma_{2} \) and \( \gamma_{3} \) are larger than \( \gamma \) for the SSB technique and that for the DSB technique, respectively, which is because the proposed technique has a slightly higher false alarm probability, \( P_{f} \), than the \( P_{f} \) (Eq. (46)) of the conventional SSB and DSB techniques for the same \( C/N_{0} \) of the incoming signal.

Fig. 7 shows the detection probabilities \( P_{D} \) of the proposed technique and the conventional techniques for \( CFAR=10^{-3} \). In general, the detection variable of the DSB technique is 3dB higher than that of the SSB technique. However, the detection threshold for the DSB technique is slightly higher than that for the SSB technique, which results in about 2 dB gain in the \( P_{D} \). In addition, as expected from the comparison

http://www.gnss.or.kr
of the detection thresholds of the proposed technique to the conventional techniques in Fig. 6, the detection probabilities of the 2nd and 3rd stages of the proposed technique are slightly smaller than those of the SSB and DSB techniques, respectively. Note that the detection probability of the proposed technique increases as the stage increases, so that the proposed technique has multilevel detection sensitivity.

Fig. 8 shows the MAT from the $10^4$ Monte Carlo simulations and from the numerical evaluation of the algebraic expressions in Eqs. (71, 76) for the non-circular search scheme. The theoretical MAT derived in subsection 5.4 matches the MAT’s obtained from Monte Carlo simulations for the SSB, DSB, and proposed techniques. As expected, the proposed technique can achieve about 4 times faster acquisition for strong signals, and the MAT of the proposed technique is about the average of the SSB and DSB techniques for moderate and weak signals. Note that the MAT at low $C/N_0$ ($\leq 36$) dB shows a saturation, since the signal is not detectable and the MAT is mostly for testing all of the hypotheses in the two-dimensional hypothesis space once. Notice that the proposed technique shows a similar performance improvement to the multiple dwell search technique. However, the proposed technique is different from the multiple dwell search technique in that the proposed technique performs an asynchronous search and combines search results for different DSSS signals to improve sensitivity level. In addition, the multiple dwell search technique can be integrated into the proposed technique, which can further reduce the MAT, especially, for strong signals at high $C/N_0$ ($\leq 42$) dB. Therefore, the proposed technique can be very useful to quickly search and acquire the line-of-sight signals in open sky environments.

6. CONCLUSION

A fast AltBOC signal acquisition technique using an asynchronous search strategy has been proposed. The proposed technique can have multilevel detection sensitivity, which allowing a fast detection of the target signal when the signal strength is high enough. A complete algebraic analysis to evaluate the performance of the proposed technique has been provided, and it has been demonstrated with numerous Monte Carlo simulations that the proposed technique can achieve a multiple times smaller MAT than the conventional Galileo E5 primary code acquisition techniques for strong incoming AltBOC signals. Therefore, it can be found that the proposed technique is very useful for receivers to acquire the primary code of the Galileo E5 signals quickly in open sky environments.

ACKNOWLEDGMENTS

This research (2013R1A2A2A01067863) was supported by Mid-career Researcher Program through NRF grant funded by the Korean government (MEST).

REFERENCES

Lestarquit, L., Artaud, G., & Issler, J. L. 2008, AltBOC for dummies or everything you always wanted to know about AltBOC, in ION ITM, Savannah, GA, Sep 2008
Marcum, J. I. 1950, A Table of Q-functions, Rand Corp.
Binhee Kim & Seung-Hyun Kong   Asynchronous Multilevel AltBOC Acquisition

Report, RM-339, Jan. 1950
Shivaramaiah, N. C. 2011, Enhanced Receiver Techniques for Galileo E5 AltBOC Signal Processing, PhD Dissertation, School of Surveying and Spatial Information Systems, University of New South Wales, Sydney, Australia

Binhee Kim received B.S. and M.S. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2008 and 2010, respectively. She is a researcher in the CCS Graduate School for Green Transportation at KAIST, where she received a Ph.D. degree in 2015. Her research interests include radar signal processing, GNSS signal processing, and detection and estimation for navigation systems.

Seung-Hyun Kong received a B.S.E.E. from Sogang University, Korea, in 1992, an M.S.E.E. from Polytechnic University, New York, in 1994, and a Ph.D. degree in aeronautics and astronautics from Stanford University, CA, in 2006. From 1997 to 2004, he was with Samsung Electronics Inc. and Nexpilot Inc., both in Korea, where his research focus was on 2G CDMA and 3G UMTS PHY and mobile positioning technologies. In 2006, he was involved with hybrid positioning technology development using wireless location signature and Assisted GNSS at Polaris Wireless, Inc. and from 2007 to 2009, he was a research staff at Qualcomm Research Center, San Diego, CA, where his R&D focused on indoor location technologies and advanced GNSS technologies. Since 2010, he has been an Assistant Professor at the Department of Aerospace Engineering in the Korea Advanced Institute of Science and Technology (KAIST). His research interests include super-resolution signal processing, detection and estimation for navigation systems, and assisted GNSS in wireless communication systems.

http://www.gnss.or.kr