SOME RESULTS ON THE SECOND BOUNDED
COHOMOLOGY OF A PERFECT GROUP

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Abstract. For a discrete group $G$, the kernel of a homomorphism from bounded cohomology $\hat{H}^*(G)$ of $G$ to the ordinary cohomology $H^*(G)$ of $G$ is called the singular part of $\hat{H}^*(G)$. We give some results on the space of the singular part of the second bounded cohomology of $G$. Also some results on the second bounded cohomology of a uniformly perfect group are given.

1. Introduction

Throughout this paper, every group is considered as a discrete group. We review the definition of bounded cohomology.

Let $G$ be a group. For a positive integer $n \geq 1$, let $C^n(G)$ be the space of all functions $G^n \rightarrow \mathbb{R}$, where $G^n = G \times G \times \cdots \times G$. The ordinary cohomology of $G$ with real coefficients is given by the cohomology of the complex

$$0 \to \mathbb{R} \xrightarrow{\partial_{-1}=0} C(G) \xrightarrow{\partial_1} C^2(G) \xrightarrow{\partial_2} C^3(G) \xrightarrow{\partial_3} \cdots,$$

where the boundary operator $\partial_n$ for $n \geq 1$ is defined by the formula

$$\partial_n(f)(g_1, \ldots, g_{n+1}) = f(g_2, \ldots, g_{n+1})$$

$$+ \sum_{i=1}^{n} (-1)^i f(g_1, \ldots, g_i g_{i+1}, \ldots, g_{n+1}) + (-1)^{n+1} f(g_1, \ldots, g_n).$$

We denote it by $H^*(G)$. Let $B^n(G)$ be the space of all bounded functions $G^n \rightarrow \mathbb{R}$, that is,

$$B^n(G) = \{ f: G^n \rightarrow \mathbb{R} \mid \|f\| < \infty \},$$

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where \( \| f \| = \sup \{ | f(g_1, \cdots, g_n) | \mid (g_1, \cdots, g_n) \in G^n \} \). It is easy to check that the sequence

\[
(1.2) \quad 0 \to \mathbb{R} \xrightarrow{d_0=0} B(G) \xrightarrow{d_1} B^2(G) \xrightarrow{d_2} B^3(G) \xrightarrow{d_3} \cdots
\]

is a complex, where the boundary operator \( d_n \) is defined by the same formula as in (1.1).

**Definition 1.1.** The \( n \)-th cohomology of the complex (1.2) is called the \( n \)-th bounded cohomology of \( G \) with trivial coefficients \( \mathbb{R} \) and is denoted by \( \hat{H}^n(G) \).

Notice that both the ordinary cohomology \( H^\ast(G) \) and bounded cohomology \( \hat{H}^\ast(G) \) of a group \( G \) are the vector spaces over \( \mathbb{R} \). From now on, we will understand the dimension of \( \hat{H}^n(G) \) as its dimension as a vector space over \( \mathbb{R} \).

Notice that for any group \( G \), from the complex in (1.2), we have

\[
\hat{H}^0(G) = \ker(d_0) = \mathbb{R}.
\]

Also

\[
\ker(d_1) = \{ f \in B(G) \mid d_1(f) = 0 \} = \{ f \in B(G) \mid f(g_2) - f(g_1g_2) + f(g_1) = 0 \quad \text{for} \quad g_1, g_2 \in G \}.
\]

Thus \( \ker(d_1) \) is the space of all bounded homomorphisms \( G \to \mathbb{R} \). Since there is no bounded homomorphisms \( G \to \mathbb{R} \), we have

\[
\hat{H}^1(G) = \ker(d_1) = 0.
\]

Thus, for any group \( G \) the second bounded cohomology of \( G \) with trivial coefficients \( \mathbb{R} \) should be investigated first. In [5] and [3] the second bounded cohomology of some interesting groups are calculated.

Amenable groups play a special role in the theory of bounded cohomology. Recall that a group \( G \) is called amenable if it admits an invariant mean, that is, if there is a linear map \( m: B(G) \to \mathbb{R} \) satisfying the following three conditions:

i. \( m(f) \geq 0 \) if \( f \geq 0 \)

ii. \( m(1_G) = 1 \), where \( 1_G \) is the constant function taking the value 1 everywhere on \( G \),

iii. \( m(g \cdot f) = m(f) \) for every \( g \in G \) and \( f \in B(G) \), where \( (g \cdot f)(x) = f(g^{-1}x) \).

For example, finite groups, abelian groups, solvable groups, subgroups and the homomorphic image of an amenable group are amenable. It is
known that no group which contains a free group on two generators can be amenable.

In [4], the following is proved.

**Theorem 1.1.** If a group $G$ is amenable, then $\hat{H}^n(G)$ is zero for every $n \geq 1$.

Thus, in particular, $\hat{H}^2(G)$ is zero for an amenable group $G$. On the other hand, we have the following.

**Theorem 1.2.** Let $F$ be a free group of rank greater than 1. Then $\hat{H}^2(F)$ is infinite dimensional as a vector space over $\mathbb{R}$.

In [3] Grigorchuk proved Theorem 1.2 by constructing explicitly the infinitely many linearly independent generators based on pseudocharacters (Definition 2.1).

From the inclusion $\iota_n : B^n(G) \hookrightarrow C^n(G)$, we consider the following commutative diagram of complexes

\[
\begin{array}{cccc}
B^{n-1}(G) & \xrightarrow{d_{n-1}} & B^n(G) & \xrightarrow{d_n} & B^{n+1}(G) \\
\downarrow{\iota_{n-1}} & & \downarrow{\iota_n} & & \downarrow{\iota_{n+1}} \\
C^{n-1}(G) & \xrightarrow{\partial_{n-1}} & C^n(G) & \xrightarrow{\partial_n} & C^{n+1}(G).
\end{array}
\]

It is easy to see that $\ker d_n \subset \ker \partial_n$. Notice that the inclusion $B^*(G) \hookrightarrow C^*(G)$ induces a homomorphism $\hat{H}^*(G) \to H^*(G)$ which is in general neither injective nor surjective.

**Definition 1.2.** We define a vector space $\hat{H}^n_s(G) \subset \hat{H}^n(G)$ by

\[
\hat{H}^n_s(G) = (\text{Im} \partial_{n-1} \cap \ker d_n)/\text{Im} d_{n-1}
\]

and we call $\hat{H}^n_s(G)$ the singular part of $\hat{H}^n(G)$.

Similarly, we define a vector space $\hat{H}^n_b(G) \subset H^n(G)$ by

\[
\hat{H}^n_b(G) = \ker d_n/(\text{Im} \partial_{n-1} \cap \ker d_n)
\]

and we call $\hat{H}^n_b(G)$ the bounded part of $H^*(G)$.

Notice that the spaces $\hat{H}^n_s(G) \subset \hat{H}^n(G)$ and $\hat{H}^n_b(G) \subset H^n(G)$ are the kernel and the image, respectively, of the induced homomorphism $\hat{H}^*(G) \to H^*(G)$.

**Theorem 1.3.** There is an isomorphism of vector spaces

\[
\hat{H}^*(G) \cong \hat{H}^n_s(G) \oplus \hat{H}^n_b(G).
\]
This is Corollary 1.15 in [3].

The singular part of the second bounded cohomology is more subtle and is closely related to the dimension of $\hat{H}^2(G)$ for many interesting groups, such as free groups, surface groups and knot groups [3]. In the following section, we find some properties of $\hat{H}^2_s(G)$ in terms of the pseudocharacters. We also investigate the dimension of the second bounded cohomology for a uniformly perfect group.

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2. The second bounded cohomology of a perfect group

In [3], Grigorchuck described the singular part $\hat{H}^2_s(G)$ in terms of pseudocharacters.

Definition 2.1. A function $f : G \to \mathbb{R}$ is called a *quasicharacter* if there a constant $C \geq 0$ such that for every $x, y \in G$

$$|f(x) - f(xy) + f(y)| \leq C.$$  

A quasicharacter $f : G \to \mathbb{R}$ is called a *pseudocharacter* if $f(g^n) = n \cdot f(g)$ for every integer $n$ and every $g \in G$.

We denote by $QX(G)$ the vector space of all quasicharacters of $G$ and by $X(G)$ the space of all additive characters (homomorphisms) $f : G \to \mathbb{R}$. We also denote by $PX(G)$ the space of all pseudocharacters of $G$.

Theorem 2.1. There is an isomorphism of vector space

$$\hat{H}^2_s(G) \cong PX(G)/X(G).$$

This is Theorem 3.5 in [3].

Proposition 2.2. If $\hat{H}^2_s(G) \neq 0$, then there exists at least one element $x \in G$ of infinite order.

Proof. Since $\hat{H}^2_s(G) \neq 0$, there is at least one pseudocharacter $\alpha \in PX(G) \setminus X(G)$. Then $\alpha(g^n) = n \cdot \alpha(g)$ for every integer $n$ and $g \in G$. This shows that $\alpha(g) = 0$ for every $g \in G$ of finite order. Since $\alpha \neq 0$, there must be at least one element $x \in G$ such that $\alpha(x) \neq 0$ and so the order of $x$ is infinite. $\square$
Corollary 2.3. Let $G$ be a torsion group. Then the dimension of $\hat{H}^2(G)$ is less than or equal to the dimension of $H^2(G)$.

Proof. Since $G$ is a torsion group, the order of every element of $G$ is finite. Hence by Proposition 2.2, every pseudocharacter of $G$ is zero and the singular part of the second bounded cohomology of $G$ is zero. Thus the induced homomorphism $\hat{H}^2(G) \to H^2(G)$ is injective. □

Corollary 2.4. Let $\hat{H}^2_s(G) \neq 0$. Then $G$ contains an infinite cyclic group.

Proof. From Proposition 2.2, $G$ contains an element $x$ of infinite order. The group generated by $x$ is an infinite cyclic subgroup of $G$. □

Remark 2.1. Let $G = \mathbb{Z} \ast \mathbb{Z}$ a free group of rank 2, generated by $x$ and $y$. First recall that the ordinary cohomology of a free group is zero for every degree greater than 1. Hence $H^2(G) = 0$. On the other hand, by Theorem 1.2, $H^2(G)$ is infinite dimensional. Hence $\hat{H}^2(G) = \hat{H}_s^2(G)$. In [3], it is proved that

$$\hat{H}^2_s(G) \cong PX_0(G),$$

where $PX_0(G)$ is a space of pseudochatacters of $G$ vanishing on the generators $x$ and $y$.

Recall that there is a five-term exact sequence in the ordinary cohomology [2]:

Theorem 2.5. Let $N \leq G$ be a normal subgroup of $G$. Then there is an exact sequence

$$0 \to H^1(G/N) \to H^1(G) \to H^1(N)^{G/N} \to H^2(G/N) \to H^2(G),$$

where $H^1(N)^{G/N}$ is a $G/N$-invariant subspace of $H^*(N)$.

Similarly, there is a five-term exact sequence in the bounded cohomology:

Theorem 2.6. Let $N \leq G$ be a normal subgroup of $G$. Then there is an exact sequence

$$0 \to \hat{H}^2(G/N) \to \hat{H}^2(G) \to \hat{H}^2(N)^{G/N} \to \hat{H}^3(G/N) \to \hat{H}^3(G),$$

where $\hat{H}^2(N)^{G/N}$ is a $G/N$-invariant subspace of $\hat{H}^2(N)$.

Proof. This is Theorem 12.4.2 in [6] □
Proposition 2.7. Let $G = \langle x \rangle * \langle y \rangle$ be a free group generated by $x$ and $y$. Let $N$ be the normal closure of $\langle y \rangle$. Then $N$ is a proper normal subgroup of $G$ and infinitely generated. Furthermore, $\widetilde{H}^2(N)$ is infinite dimensional.

Proof. By definition, it is clear that $N$ is a nontrivial normal subgroup of $G$. As it is well known, we have $G/N \cong \langle x \rangle$. Hence the index of $N$ in $G$ is infinite and $\langle y \rangle \subset N \neq G$. Also recall that a finitely generated normal subgroup of a nontrivial free group $F$ is of finite index in $F$. Hence $N$ must be infinitely generated. Since $G/N \cong \langle x \rangle$, the quotient $G/N$ is abelian and so amenable. So by Theorem 1.1, $\tilde{H}^n(G/N) = 0$ for every $n \geq 1$. Hence by Theorem 2.6 there is an isomorphism of vector spaces

$$\tilde{H}^2(G) \cong \tilde{H}^2(N)^{G/N}.$$ 

Since $\tilde{H}^2(G)$ is infinite dimensional by Theorem 1.2, $\tilde{H}^2(N)^{G/N}$ is infinite dimensional and so is $\tilde{H}^2(N)$. 

Notice that for a normal subgroup $N \leq G$, if the quotient $G/N$ is amenable, then there the inclusion homomorphism $N \to G$ induces an injective homomorphism $\tilde{H}^2(G) \to \tilde{H}^2(N)$. For example, for the commutator subgroup $[G, G]$ of $G$, the quotient $G/[G, G]$ is abelian and so amenable. Thus there is an injective homomorphism $\tilde{H}^2(G) \to \tilde{H}^2([G, G])$.

As a corollary to Theorem 2.6, we prove that Lyndon-Hochschild-Serre spectral sequence for $\tilde{H}^2(G)$.

**Corollary 2.8.** Let $N$ be a normal subgroup of $G$. Suppose $\tilde{H}^2(G/N) = 0$. Then there the spectral sequence $E_2^{pq} = \tilde{H}^p(G/N, \tilde{H}^q(N))$ converging to $\tilde{H}^2(G)$.

**Proof.** Suppose $\tilde{H}^2(G/N)$ is zero. Then by Theorem 2.6 the homomorphism $\tilde{H}^2(G) \to \tilde{H}^2(N)^{G/N}$ is injective. For $E_2^{pq} = \tilde{H}^p(G/N, \tilde{H}^q(N))$, we fix $(p, q) = (0, 2)$. Then we have

$$E_2^{0,2} = \tilde{H}^0(G/N, \tilde{H}^2(N)) = \tilde{H}^2(N)^{G/N}.$$ 

Consider

$$0 = E_2^{-2,3} \to E_2^{0,2} = \tilde{H}^2(N)^{G/N} \xrightarrow{d_2} E_2^{2,1} = \tilde{H}^2(G/N, \tilde{H}^1(N)) = 0.$$
where the last term is zero because the first bounded cohomology of any group is zero. This gives
\[ E_0^{0,2} = \ker d_2 = E_2^{0,2} = \hat{H}^2(N)^{G/N}. \]

For \( E_4^{0,2} \), we consider
\[ 0 = E_3^{-3,4} \to E_3^{0,2} = \hat{H}^2(N)^{G/N} \xrightarrow{d_3} E_3^{3,0} = \hat{H}^3(G/N, \hat{H}^0(N)) = \hat{H}^3(G/N), \]
where the last term follows from \( \hat{H}^0(N) = \mathbb{R} \). Then by the exactness of the five-term exact sequence above, we have
\[
E_4^{0,2} = \ker d_3 = \ker(\hat{H}^2(N)^{G/N} \to \hat{H}^3(G/N))
= \text{image of } (\hat{H}^2(G) \hookrightarrow \hat{H}^2(N)^{G/N})
\cong \hat{H}^2(G).
\]
Thus
\[ E_4^{0,2} \cong \hat{H}^2(G). \]

For \( r = 4 \), we consider
\[ 0 = E_4^{-4,5} \xrightarrow{d_4} E_4^{0,2} = \hat{H}^2(G) \xrightarrow{d_4} E_4^{-1,4} = 0. \]
So \( E_5^{0,2} = \ker d_4 = E_4^{0,2} \). Now it is easy to see that for every \( r \geq 5 \) we have
\[ E_r^{0,2} = E_4^{0,2} = \hat{H}^2(G). \]
Hence we have \( E_r^{0,2} \) converges to \( \hat{H}^2(G) \).

Corollary 2.8 is still true without the condition \( \hat{H}^2(G/N) = 0 \). Related to Theorem 2.6, the following more general version of Lyndon-Hochschild-Serre spectral sequence is proved in [7]:

**Lemma 2.9.** Let \( N \) be a normal subgroup \( N \leq G \). Then \( \hat{H}^*(G/N) \) admits a natural structure of bounded \( G/N \)-module and there exists a spectral sequence \( (E_r) \) converging to \( \hat{H}^*(G) \) in which \( E_2^{pq} = \hat{H}^p(G/N, \hat{H}^q(N)) \) for every \( p, q \geq 0 \).

Recall that a subgroup of a free group is also free. Proposition 2.7 gives another example for the fact that the rank of a subgroup of a free group of finite rank is not necessarily finite.

Recall that we say a group \( G \) is perfect if \( G = [G, G] \), where \( [G, G] \) is the commutator subgroup of \( G \). Recall that \( H_1(G, \mathbb{Z}) \cong G/[G, G] \). Hence \( H^1(G) = X(G) = 0 \) for a perfect group \( G \). Thus from Theorem 2.1, we have \( \hat{H}^2_s(G) \cong PX(G) \).
Definition 2.2. A group $G$ is said to be uniformly perfect if there is a positive integer $N$ such that every element of $G$ can be presented as a product of at most $N$ commutators.

For example, it is known that the alternating groups $A_n$ for $n \geq 5$ and $G = SL(2, \mathbb{R})$ are uniformly perfect [8].

In [5], the following theorem is proved:

Theorem 2.10. If $G$ is uniformly perfect, then homomorphism $\tilde{H}^2(G) \rightarrow H^2(G)$ is injective.

Definition 2.3. A group such that $H_1(G, \mathbb{Z}) = H_2(G, \mathbb{Z}) = 0$ is called superperfect. Furthermore, if $H_n(G, \mathbb{Z}) = 0$ for all $n \geq 1$, then $G$ is called acyclic.

Notice that acyclic groups are superperfect, and superperfect groups are perfect. The alternating group $A_5$ is (uniformly) perfect, but not superperfect. If $G$ is superperfect, then by Universal Coefficient Theorem $H^1(G) = H^2(G) = 0$. Also, since $H^1(X) = X(G)$, we have

$$\tilde{H}^2(G) = \tilde{H}_s^2(G) \cong PX(G)/X(G) = PX(G).$$

One of interesting examples of acyclic groups is binate groups [1].

Definition 2.4. A group $G$ is called binate if it is the direct limit of subgroups $G_{\lambda}$ where for each $\lambda$ there exist $\mu \geq \lambda$, $u_{\lambda} \in G_{\mu} - G_{\lambda}$ and $\phi_{\lambda}: G_{\lambda} \rightarrow G_{\mu}$ such that $g = [u_{\lambda}, \phi_{\lambda}g]$ for all $g \in G_{\lambda}$.

As shown in [1], a binate group $G$ is acyclic and so superperfect. By Universal Coefficient Theorem, $H^n(G) = 0$ for every $n \geq 1$. In particular, $H^1(G) = H^2(G) = 0$. Also it is shown in [1] that every element of a binate group $G$ is a commutator. Thus it is uniformly perfect. Hence $\tilde{H}^2(G) = 0$ by Theorem 2.10. Then it is clear that $PX(G) = 0$.

Recall that the quotient of a perfect group is also perfect.

Proposition 2.11. Let $Z$ be the center of a superperfect group $G$. If the quotient $G/Z$ is uniformly perfect and $\tilde{H}^2(G) \neq 0$, then the center $Z$ is nontrivial. In particular, $Z$ has at least one element of infinite order.

Proof. By Theorem 2.6, the exact sequence $0 \rightarrow Z \rightarrow G \rightarrow G/Z \rightarrow 1$ induces a five-term exact sequence

$$0 \rightarrow \tilde{H}^2(G/Z) \rightarrow \tilde{H}^2(G) \rightarrow \tilde{H}^2(Z) \rightarrow \tilde{H}^2(G/Z) \rightarrow \tilde{H}^3(G).$$

First notice that, since $Z$ is the center of $G$, the conjugation action of $G/Z$ on $\tilde{H}^2(Z)$ is trivial and so $\tilde{H}^2(Z)^{G/Z} \cong \tilde{H}^2(Z)$. Recall that the
The second bounded cohomology

center $Z$ is abelian and so it is amenable. So by Theorem 1.1 $\hat{H}^*(Z) = 0$. Hence there is an isomorphism

$$\hat{H}^2(G/Z) \cong \hat{H}^2(G)$$

and so $\hat{H}^2(G/Z) \neq 0$. Since $G/Z$ is uniformly perfect, there is an injective homomorphism $\hat{H}^2(G/Z) \hookrightarrow H^2(G/Z)$ by Theorem 2.10 and so $H^2(G/Z) \neq 0$. Recall that $G$ is perfect. So the quotient $G/Z$ is also perfect and $H^1(G/Z) = 0$. Also notice that $H^2(G) = 0$. Thus from Theorem 2.5, there is an isomorphism $H^1(Z) \cong H^2(G/Z)$. Hence $H^1(Z) \neq 0$ so that $Z$ is nontrivial. If every element of $Z$ is torsion, then $0 = X(Z) \cong H^1(Z)$. Hence there is at least one element $z \in Z$ such that the order of $z$ is infinite.

**Remark 2.2.** From the Proposition 2.11, notice that $\hat{H}^2(G/Z) \cong \hat{H}^2(G) = \hat{H}^2_s(G)$ and $\hat{H}^2_s(G/Z) = 0$. Thus, even the spaces of the second bounded cohomology of two groups are isomorphic, their singular parts are not necessarily isomorphic.

**Definition 2.5.** A group $G$ is of type $FP_n$ if $Z$ is type of $FP_n$ as a $\mathbb{Z}G$-module. More precisely, there is a partial projective resolution $P_n \rightarrow \cdots \rightarrow P_0 \rightarrow \mathbb{Z} \rightarrow 0$ of finite type, that is, such that each $P_k$ is finitely generated for $0 \leq k \leq n$.

As proved in [2], a group $G$ is of type $FP_n$ if and only if for every partial projective resolution $P_k \rightarrow \cdots P_0 \rightarrow \mathbb{Z} \rightarrow 0$ of finite type with $k < n$, $\ker\{P_k \rightarrow P_{k-1}\}$ is finitely generated.

**Proposition 2.12.** If a group $G$ is of type $FP_n$, then the dimension of $H^k(G)$ is finite for $k \leq n$.

**Proof.** Since $G$ is of type $FP_n$ and $\mathbb{Z}$ is finitely generated abelian group, every $H_k(G, \mathbb{Z})$ for $k \leq n$ is a finitely generated abelian group. Hence we can write

$$H_k(G, \mathbb{Z}) \cong \mathbb{Z}^{r_k} \oplus T_k,$$

where the rank $r_k \geq 0$ is finite and $T_k$ is the torsion subgroup of $H_k(G, \mathbb{Z})$. Then

$$H^k(G) \cong \text{Hom}_{\mathbb{Z}}(H_k(G, \mathbb{Z}), \mathbb{R}) = \mathbb{R}^{r_k},$$

where the first isomorphism follows from Universal Coefficient Theorem. Hence the dimension of $H^k(G)$ is finite. $\square$
Remark 2.3. Recall that the group $H_2(G, \mathbb{Z})$ is called the Schur multiplier of $G$. It is an important invariant of a group that has applications in many areas. Notice that a superperfect group is one whose abelianization and also Schur multiplier vanish. A group $G = F/K$ with $F$ free, it is known [2] that $H_2(G, \mathbb{Z}) \cong (K \cap [F, F])/[F, K]$.

We first see how the dimension of the second bounded cohomology of a uniformly perfect group $G$ which is of type $FP_2$ is affiliated with $H_2(G, \mathbb{Z})$.

Corollary 2.13. Let $G$ be uniformly perfect and of type $FP_2$. Then the dimension of $\tilde{H}^2(G)$ is finite. Especially, its maximum is achieved by the rank of $H_2(G, \mathbb{Z})$.

Proof. Since $G$ is of type $FP_2$, by Proposition 2.12 the dimension of $H^2(G)$ is equal to the rank of $H_2(G, \mathbb{Z})$, which is finite. Since $G$ is uniformly perfect, there is an injective homomorphism $\tilde{H}^2(G) \hookrightarrow H^2(G)$ and so the dimension of $\tilde{H}^2(G)$ is less than equal to the dimension of $H^2(G)$.

Theorem 2.14. Let $G$ be uniformly perfect. If $G$ is finitely presented with $m$ generators and $n$ relations, then the dimension of $\tilde{H}^2(G)$ is at most $n - m$.

Proof. Notice that a finitely generated group is of type $FP_2$, and so by Corollary 2.13 the dimension of $\tilde{H}^2(G)$ is finite. Let $r = rk_{\mathbb{Z}}(G_{ab})$ the rank of the abelianization of $G$. As it is well known, the group $H_2(G, \mathbb{Z})$ can be generated by $n - m + r$ elements. Since $G$ is uniformly perfect, it is perfect and so $r = 0$. Thus $H_2(G, \mathbb{Z})$ can be generated by $n - m$ elements. Since $G$ is uniformly perfect, similarly as shown in Corollary 2.13, the dimension of $\tilde{H}^2(G)$ is at most $n - m$.

References


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