IMPLICATIVE VAGUE IDEALS IN BCK-ALGEBRAS

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Abstract. The notion of implicative vague ideals of BCK-algebras is defined, and several properties of it are investigated. Relations between a vague ideal and an implicative vague ideal is discussed. Characterizations of an implicative vague ideal are considered.

1. Introduction

Several authors from time to time have made a number of generalizations of Zadeh's fuzzy set theory [9]. Of these, the notion of vague set theory introduced by Gau and Buehrer [3] is of interest to us. Using the vague set in the sense of Gau and Buehrer, Biswas [2] studied vague groups. Jun and Park [4,8] studied vague ideals and vague deductive systems in subtraction algebras. In [6], the concept of vague BCK/BCI-algebras is discussed. S. S. Ahn, Y. U. Cho and C. H. Park [1] studied vague quick ideals of $BCK/BCI$-algebras. Y. B. Jun and K. J. Lee ([5]) introduced the notion of positive implicative vague ideals in BCK-algebras. They established relations between a vague ideal and a positive implicative ideals.

In this paper, we also use the notion of vague set in the sense of Gau and Buehrer to discuss the vague theory in BCK-algebras. We define the notion of implicative vague ideal of BCK-algebras and investigate several properties of it. We study a relation between a vague ideal and an implicative vague ideal. We establish characterizations of an implicative vague ideal.

2. Preliminaries

We review some definitions and properties that will be useful in our results.

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By a \textit{BCI-algebra} we mean an algebra \((X, \star, 0)\) of type \((2,0)\) satisfying the following conditions:

(a1) \((\forall x, y, z \in X) ((x \star y) \star (x \star z) \star (z \star y) = 0)\),
(a2) \((\forall x, y \in X) ((x \star (x \star y)) \star y = 0)\),
(a3) \((\forall x \in X) (x \star x = 0)\),
(a4) \((\forall x, y \in X) (x \star y = 0, y \star x = 0 \Rightarrow x = y)\).

A BCI-algebra \(X\) satisfying the additional condition:

(a5) \((\forall x \in X) (0 \star x = 0)\)

is called a \textit{BCK-algebra}. In any BCK/BCI-algebra \(X\) one can define a partial order “\(\leq\)” by putting \(x \leq y\) if and only if \(x \star y = 0\).

A BCK/BCI-algebra \(X\) has the following properties:

(b1) \((\forall x \in X) (x \star 0 = x)\).
(b2) \((\forall x, y, z \in X) ((x \star y) \star z = (x \star z) \star y)\).
(b3) \((\forall x, y, z \in X) (x \leq y \Rightarrow x \star z \leq y \star z, z \star y \leq z \star x)\).
(b4) \((\forall x, y, z \in X) ((x \star z) \star (y \star z) \leq x \star y)\).
(b5) \((\forall x, y \in X) (x \star (x \star (x \star y))) = x \star y)\).

A non-empty subset \(S\) of a BCK/BCI-algebra \(X\) is called a \textit{subalgebra} of \(X\) if \(x \star y \in S\) whenever \(x, y \in S\). A subset \(A\) of a BCK/BCI-algebra \(X\) is called an \textit{ideal} of \(X\) if it satisfies:

(c1) \(0 \in A\),
(c2) \((\forall x \in A) (\forall y \in X) (y \star x \in A \Rightarrow y \in A)\).

Note that every ideal \(A\) of a BCK/BCI-algebra \(X\) satisfies:

\((\forall x \in A) (\forall y \in X) (y \leq x \Rightarrow y \in A)\).

A subset \(A\) of a BCK-algebra \(X\) is called a \textit{positive implicative ideal} of \(X\) if it satisfies (c1) and

(c3) \((\forall x, y, z \in A)((x \star y) \star z \in A, y \star z \in A \Rightarrow x \star z \in A)\).

Note that any positive implicative ideal is an ideal, but the converse is not true in general.

A subset \(A\) of a BCK-algebra \(X\) is called an \textit{implicative ideal} of \(X\) if it satisfies (c1) and

(c4) \((\forall x, y, z \in A)((x \star (y \star x)) \star z \in A, z \in A \Rightarrow x \in A)\).

Note that any implicative ideal is an ideal, but the converse is not true in general.

We refer the reader to the book [7] for further information regarding BCK-algebras.
**Definition 2.1.** ([2]) A vague set $A$ in the universe of discourse $U$ is characterized by two membership functions given by:

1. A true membership function
   
   \[ t_A : U \rightarrow [0, 1], \]

   and

2. A false membership function

   \[ f_A : U \rightarrow [0, 1], \]

where $t_A(u)$ is a lower bound on the grade of membership of $u$ derived from the “evidence for $u$”, $f_A(u)$ is a lower bound on the negation of $u$ derived from the “evidence against $u$”, and

\[ t_A(u) + f_A(u) \leq 1. \]

Thus the grade of membership of $u$ in the vague set $A$ is bounded by a subinterval $[t_A(u), 1 - f_A(u)]$ of $[0, 1]$. This indicates that if the actual grade of membership of $u$ is $\mu(u)$, then

\[ t_A(u) \leq \mu(u) \leq 1 - f_A(u). \]

The vague set $A$ is written as

\[ A = \{ \langle u, [t_A(u), f_A(u)] \rangle \mid u \in U \}, \]

where the interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of $u$ in $A$, denoted by $V_A(u)$.

For $\alpha, \beta \in [0, 1]$ we now define $(\alpha, \beta)$-cut and $\alpha$-cut of a vague set. Recall that if $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ are two subintervals of $[0, 1]$, we can define a relation “$\succeq$” by putting $I_1 \succeq I_2$ if and only if $a_1 \geq a_2$ and $b_1 \geq b_2$.

**Definition 2.2.** ([2]) Let $A$ be a vague set of a universe $X$ with the true-membership function $t_A$ and the false-membership function $f_A$. The $(\alpha, \beta)$-cut of the vague set $A$ is a crisp subset $A_{(\alpha, \beta)}$ of the set $X$ given by

\[ A_{(\alpha, \beta)} = \{ x \in X \mid V_A(x) \succeq [\alpha, \beta] \}. \]

Clearly $A_{(0,0)} = X$. The $(\alpha, \beta)$-cuts of the vague set $A$ are also called vague-cuts of $A$.

**Definition 2.3.** ([2]) The $\alpha$-cut of the vague set $A$ is a crisp subset $A_\alpha$ of the set $X$ given by $A_\alpha = A_{(\alpha, \alpha)}$.

Note that $A_0 = X$, and if $\alpha \geq \beta$ then $A_\alpha \subseteq A_\beta$ and $A_{(\alpha, \beta)} = A_\alpha$. 
Equivalently, we can define the $\alpha$-cut as
$$A_\alpha = \{ x \in X \mid t_A(x) \geq \alpha \}.$$

3. Implicative vague ideals

For our discussion, we shall use the following notations on interval arithmetic:

Let $I[0,1]$ denote the family of all closed subintervals of $[0,1]$. We define the term “imax” to mean the maximum of two intervals as
$$\text{imax}(I_1, I_2) := [\max(a_1, a_2), \max(b_1, b_2)],$$
where $I_1 = [a_1, b_1], I_2 = [a_2, b_2] \in I[0,1]$. Similarly we define “imin”. The concepts of “imax” and “imin” could be extended to define “isup” and “iinf” of infinite number of elements of $I[0,1]$.

It is obvious that $L = \{I[0,1], \text{isup}, \text{iinf}, \geq\}$ is a lattice with universal bounds $[0,0]$ and $[1,1]$ (see [2]).

In what follows let $X$ denote $BCK$-algebra unless specified otherwise.

**Definition 3.1.** ([6]) A vague set $A$ of a $BCK/BCI$-algebra $X$ is called a vague $BCK/BCI$-algebra of $X$ if the following condition is true:

$$(\forall x, y \in X)(V_A(x * y) \succeq \text{imin}\{V_A(x), V_A(y)\}),$$

that is,

$$t_A(x * y) \geq \min\{t_A(x), t_A(y)\},$$

$$1 - f_A(x * y) \geq \min\{1 - f_A(x), 1 - f_A(y)\}$$

for all $x, y \in X$.

**Definition 3.2.** ([6]) A vague set $A$ of a $BCK$-algebra $X$ is called a vague ideal of $X$ if the following conditions are true:

(d1) $$(\forall x \in X)(V_A(0) \succeq V_A(x)),$$

(d2) $$(\forall x, y \in X)(V_A(x) \succeq \text{imin}\{V_A(x * y), V_A(y)\}).$$

that is,

$$t_A(0) \geq t_A(x), 1 - f_A(0) \geq 1 - f_A(x),$$

and

$$t_A(x) \geq \min\{t_A(x * y), t_A(y)\}$$

$$1 - f_A(x) \geq \min\{1 - f_A(x * y), 1 - f_A(y)\}$$

for all $x, y \in X$.

**Proposition 3.3.** ([6]) Every vague ideal $A$ of a $BCK$-algebra $X$ satisfies the following properties:

(i) $$(\forall x, y \in X)(x \leq y \Rightarrow V_A(x) \succeq V_A(y)),$$
(ii) $(\forall x, y, z \in X)(VA(x * z) \succeq \text{imin}\{VA((x * y) * z), VA(y * z)\})$.

**Definition 3.4.** ([5]) A vague set $A$ of a BCK-algebra $X$ is called a positive implicative vague ideal of $X$ if it satisfies (d1) and

(d3) $(\forall x, y, z \in X)(VA(x * z) \succeq \text{imin}\{VA((x * y) * z), VA(y * z)\})$

that is,

$$t_A(x * z) \geq \min\{t_A((x * y) * z), t_A(y * z)\},$$

$$1 - f_A(x * z) \geq \min\{1 - f_A((x * y) * z), 1 - f_A(y * z)\}$$

for all $x, y, z \in X$.

**Definition 3.5.** A vague set $A$ of a BCK-algebra $X$ is called an implicative vague ideal of a BCK-algebra $X$ if it satisfies (d1) and

(d4) $(\forall x, y, z \in X)(VA(x) \succeq \text{imin}\{VA((x * (y * x)) * z), VA(z)\})$

that is,

$$t_A(x) \geq \min\{t_A((x * (y * x)) * z), t_A(z)\},$$

$$1 - f_A(x) \geq \min\{1 - f_A((x * (y * x)) * z), 1 - f_A(z)\}$$

for all $x, y, z \in X$.

**Theorem 3.6.** Every implicative vague ideal of a BCK-algebra $X$ is a vague ideal of $X$.

**Proof.** Let $A$ be an implicative vague ideal of a BCK-algebra $X$. If we take $y := x$ in (d4) and use (b1), then we obtain (d2). Hence $A$ is a vague ideal of $X$. \hfill $\square$

**Example 3.7.** Let $X := \{0, 1, 2, 3, 4\}$ be a BCK-algebra([7]) in which the $*$-operation is given by the following table:

$$
\begin{array}{c|cccc}
* & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
2 & 2 & 1 & 0 & 1 \\
3 & 3 & 3 & 3 & 0 \\
4 & 4 & 4 & 4 & 0 \\
\end{array}
$$

Let $A$ be the vague set in $X$ defined as follows:

$$A = \{\langle0, [0.7, 0.2]\rangle, \langle1, [0.7, 0.2]\rangle, \langle2, [0.7, 0.2]\rangle, \langle3, [0.7, 0.2]\rangle, \langle4, [0.5, 0.3]\rangle\}$$

It is routine to verify that $A$ is an implicative vague ideal of $X$.

**Theorem 3.8.** Every implicative vague ideal of a BCK-algebra $X$ is a positive implicative ideal of $X$. 
Proof. Let \( A \) be an implicative vague ideal of a BCK-algebra \( X \). Since \( ((x \ast z) \ast z) \ast (y \ast z) \leq (x \ast z) \ast y = (x \ast y) \ast z \), using Proposition 3.3(i) we obtain
\[
V_A(((x \ast z) \ast z) \ast (y \ast z)) \succeq V_A((x \ast y) \ast z).
\]
(3.1)
Note that
\[
(x \ast z) \ast (x \ast (x \ast z)) = (x \ast (x \ast (x \ast z))) \ast z = (x \ast z) \ast z.
\]
(3.2)
It follows from (d4), (3.1) and (3.2) that
\[
V_A((x \ast z)) \succeq \min \{V_A(((x \ast z) \ast (x \ast (x \ast z))) \ast (y \ast z)), V_A((y \ast z)) \}
= \min \{V_A(((x \ast z) \ast (y \ast z)), V_A((y \ast z)) \}
\geq \min \{V_A((x \ast y) \ast z), V_A((y \ast z)) \}.
\]
Hence \( A \) is a positive implicative ideal of \( X \).

Example 3.9. Let \( X := \{0, 1, 2, 3, 4\} \) be a BCK-algebra([7]) in which the \( * \)-operation is given by the following table:

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<td>0</td>
</tr>
</tbody>
</table>

Let \( A \) be the vague set in \( X \) defined as follows:
\[
A = \{(0, [0, 0.7, 0.2]), (1, [0.5, 0.3]), (2, [0.7, 0.2]), (3, [0.5, 0.3]), (4, [0.5, 0.3])\}.
\]
It is routine to verify that \( A \) is a vague ideal of \( X \). But it is not an implicative vague ideal of \( X \), since
\[
V_A(1) \not\succeq \min \{V_A((1 \ast (3 \ast 1)) \ast 2), V_A(2) \}.
\]

Theorem 3.10.([5]) For a vague set \( A \) in a BCK-algebra \( X \), the following are equivalent:
\begin{enumerate}
\item \( A \) is a positive implicative ideal of \( X \).
\item \( (\forall x, y \in X)(V_A(x \ast y) \succeq V_A((x \ast y) \ast y)). \)
\item \( (\forall x, y, z \in X)(V_A((x \ast z) \ast (y \ast z)) \succeq V_A((x \ast y) \ast z)). \)
\end{enumerate}

Theorem 3.11. Let \( A \) be a positive implicative vague ideal of a BCK-algebra \( X \). Then \( A \) is an implicative vague ideal of \( X \) if and only if
\[
(\forall x, y, z \in X)(V_A(x \ast (x \ast y)) \succeq V_A(y \ast (y \ast x))).
\]
Using (3.4), (3.5) and Proposition 3.3(ii), we have

\[(x \ast (x \ast y)) \ast (y \ast (x \ast y)) \leq (x \ast (x \ast y)) \ast (y \ast x) = (x \ast (y \ast x)) \ast (x \ast y) \leq y \ast (y \ast x).\]

It follows from Proposition 3.3(i) that

\[V_A((x \ast (x \ast y)) \ast (y \ast (x \ast y))) \geq V_A(y \ast (y \ast x)). \tag{3.3}\]

Using (d4), (b1), (d1), and (3.3), we have

\[V_A(x \ast (x \ast y)) \geq \min \{V_A((x \ast (x \ast y)) \ast (y \ast (x \ast y))) \ast 0), V_A(0)\}
\[= V_A((x \ast (x \ast y)) \ast (y \ast (x \ast y))) \geq V_A(y \ast (y \ast x)).\]

Thus \(A\) satisfies (\ast). Conversely, let \(A\) be a positive implicative vague ideal of \(X\) which satisfies (\ast). Since \((y \ast (y \ast x)) \ast (y \ast x) \leq x \ast (y \ast x)\), it follows from Proposition 3.3(i), Theorem 3.10(2) and (\ast) that

\[V_A(x \ast (y \ast x)) \leq V_A((y \ast (y \ast x)) \ast (y \ast x)) \leq V_A(y \ast (y \ast x)) \leq V_A(x \ast (x \ast y)). \tag{3.4}\]

Since \((x \ast y) \ast z \leq x \ast y \leq x \ast (y \ast x)\) for any \(x, y, z \in X\), it follows from Proposition 3.3(i) that

\[V_A(x \ast (y \ast x)) \leq V_A((x \ast y) \ast z). \tag{3.5}\]

Using (3.4), (3.5) and Proposition 3.3(ii), we have

\[V_A(x) \geq \min \{V_A(x \ast (x \ast y)), V_A(x \ast y)\}
\[\geq \min \{V_A(x \ast (y \ast x)), V_A(x \ast y)\}
\[\geq \min \{V_A(x \ast (y \ast x)), V_A((x \ast y) \ast z), V_A(z)\}
\[\geq \min \{V_A(x \ast (y \ast x)), V_A(z)\}
\[= \min \{\min \{V_A((x \ast z) \ast (y \ast x)), V_A(z)\}, V_A(z)\}
\[= \min \{V_A((x \ast z) \ast (y \ast x)), V_A(z)\}.\]

Thus \(A\) is an implicative vague ideal of \(X\). \qed
**Theorem 3.12.** A vague ideal $A$ of a BCK-algebra $X$ is implicative if and only if

$$(* *) \quad (\forall x, y \in X)(V_A(x) \succeq V_A(x \ast (y \ast x))).$$

**Proof.** Suppose that a vague ideal $A$ of $X$ satisfies $(**)$.

It follows from (d2) and $(**)$ that

$$V_A(x) \succeq V_A(x \ast (y \ast x)) \succeq \text{imin}\{V_A((x \ast (y \ast x)) \ast z), V_A(z)\}.$$

Hence $A$ is an implicative vague ideal of $X$.

Conversely, assume that a vague ideal $A$ of $X$ is implicative. Putting $z := 0$ in (d4), we have

$$V_A(x) \succeq \text{imin}\{V_A((x \ast (y \ast x)) \ast 0), V_A(0)\} = \text{imin}\{V_A(x \ast (y \ast x)), V_A(0)\} \succeq V_A(x \ast (y \ast x)).$$

This completes the proof. \(\square\)

**Lemma 3.13.** ([6]) For a vague set $A$ in a BCK-algebra $X$, the following are equivalent:

1. $A$ is a vague ideal of $X$,
2. $A$ satisfies the following implication:

$$((\forall x, y, z \in X)((x \ast y) \ast z = 0 \Rightarrow V_A(x) \succeq \text{imin}\{V_A(y), V_A(z)\})).$$

**Theorem 3.14.** For a vague set $A$ in a BCK-algebra $X$, the following are equivalent:

1. $A$ is an implicative vague ideal of $X$,
2. $A$ satisfies the following implication:

$$((\forall x, a, b \in X)((x \ast (y \ast x)) \ast a) \ast b = 0 \Rightarrow V_A(x) \succeq \text{imin}\{V_A(a), V_A(b)\}).$$

**Proof.** Assume that $A$ is an implicative vague ideal of a BCK-algebra $X$. Then $A$ is a vague ideal of $X$ by Theorem 3.6. Let $x, y, a, b \in X$ be such that $((x \ast (y \ast x)) \ast a) \ast b = 0$. It follows from Theorem 3.12 and Lemma 3.13 that

$$V_A(x) \succeq V_A(x \ast (y \ast x)) \succeq \text{imin}\{V_A(a), V_A(b)\}.$$ 

Therefore $A$ satisfies (2).

Conversely, let $x, a, b \in X$ be such that $(x \ast a) \ast b = 0$. Then $((x \ast (0 \ast x)) \ast a) \ast b = (x \ast a) \ast b = 0$. By (2), we have $V_A(x) \succeq \text{imin}\{V_A(a), V_A(b)\}$. 
Thus $A$ is a vague ideal of $X$ by Lemma 3.13. Since $[(x * (y * x)) * (x * (y * x))] * 0 = 0$, it follow from (2) that
\[ V_A(x) \geq \text{imin}\{V_A(x * (y * x)), V_A(0)\} \]
\[ = V_A(x * (y * x)). \]
Thus $A$ is an implicative vague ideal of $X$ by Theorem 3.12.

**Corollary 3.15.** Let $A$ be a vague set of a BCK-algebra $X$. If $A$ satisfies the following inequality:
\[ V_A(x) \geq \text{imin}\{V_A(a_1), \ldots, V_A(a_n)\} \]
whenever $(\cdots((x * (y * x)) * a_1) \cdots) * a_n = 0$ for all $x, a_1, \ldots, a_n \in X$, then $A$ is an implicative vague ideal of $X$.

**Proof.** Straightforward.

**Theorem 3.16.** Let $A$ be an implicative vague ideal of a BCK-algebra $X$. Then for any $\alpha, \beta \in [0, 1]$, the vague-cut $A_{(\alpha, \beta)}$ of $A$ is a crisp implicative ideal of $X$.

**Proof.** Obviously, $0 \in A_{(\alpha, \beta)}$. Let $(x * (y * x)) * z \in A_{(\alpha, \beta)}$ and $z \in A_{(\alpha, \beta)}$. Then $V_A((x * (y * x)) * z) \geq [\alpha, \beta]$ and $V_A(z) \geq [\alpha, \beta]$, i.e., $t_A((x * (y * x)) * z) \geq \alpha$, $t_A(z) \geq \alpha$ and $1 - f_A((x * (y * x)) * z) \geq \beta$, $1 - f_A(z) \geq \beta$. It follows that
\[ t_A(x) \geq \text{imin}\{t_A((x * (y * x)) * z), t_A(z)\} \geq \alpha \]
and $1 - f_A(x) \geq \text{imin}\{1 - f((x * (y * x)) * z), 1 - f_A(z)\} \geq \beta$.
Hence $x \in A_{(\alpha, \beta)}$ and so $A_{(\alpha, \beta)}$ is an implicative ideal of $X$.

The ideals like $A_{(\alpha, \beta)}$ are also called vague cut implicative ideals of $X$.

**Theorem 3.17.** Any implicative ideal $I$ of a BCK-algebra $X$ is a vague cut-ideal of some implicative vague ideal of $X$.

**Proof.** Consider the vague set $A$ of $X$ given by
\[ V_A(x) = \begin{cases} [\alpha, \alpha] & \text{if } x \in I \\ [0, 0] & \text{if } x \notin I \end{cases} \]
where $\alpha \in (0, 1)$. Since $0 \in I$, we have $V_A(0) = [\alpha, \alpha] \succeq V_A(x)$ for all $x \in X$. Let $x, y, z \in X$ be such that $(x * (y * x)) * z \in I$ and $z \in I$. If $x \notin I$, then
\[ t_A(x) = 0 \leq \text{imin}\{t_A((x * (y * x)) * z), t_A(z)\} \]
and $1 - f_A(x) = 0 \leq \text{imin}\{1 - f_A((x * (y * x)) * z), 1 - f_A(z)\}$.
If $x \in I$, then
\[ t_A(x) = \alpha = \min\{t_A((x * (y * z)) * z), t_A(z)\} \]
and $1 - f_A(x) = \alpha = \min\{1 - f_A((x * (y * x)) * z), 1 - f_A(z)\}$.

Thus $A$ is an implicative vague ideal of $X$. Clearly, $I = A_{(\alpha, \alpha)}$.

**Theorem 3.18.** Let $A$ be an implicative vague ideal of a BCK-algebra $X$. Then the set
\[ I := \{x \in X|V_A(x) = V_A(0)\} \]
is a crisp implicative ideal of $X$.

**Proof.** Clearly, $0 \in I$. Let $x, y, z \in X$ be such that $(x * (y * x)) * z \in I$ and $z \in I$. Then $V_A((x * (y * x)) * z) = V_A(0)$ and $V_A(z) = V_A(0)$ and so
\[ V_A(x) \succeq \min\{V_A((x * (y * x)) * z), V_A(z)\} = V_A(0). \]
Since $V_A(0) \succeq V_A(x)$ for all $x \in X$, it follows that $V_A(x) = V_A(0)$. Hence $x \in I$. Therefore $I$ is a crisp implicative ideal of $X$. □

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