Terminal Sliding Mode Control of Nonlinear Systems Using Self-Recurrent Wavelet Neural Network

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Abstract: In this paper, we design a terminal sliding mode controller based on self-recurrent wavelet neural network (SRWNN) for the second-order nonlinear systems with model uncertainties. The terminal sliding mode control (TSMC) method can drive the tracking errors to zero within finite time in comparison with the classical sliding mode control (CSMC) method. In addition, the TSMC method has advantages such as the improved performance, robustness, reliability and precision. We employ the SRWNN to approximate model uncertainties. The weights of SRWNN are trained by adaptation laws induced from Lyapunov stability theorem. Finally, we carry out simulations for Duffing system and the wing rock phenomena to illustrate the effectiveness of the proposed control scheme.

Keywords: terminal sliding mode control, self-recurrent wavelet neural network, nonlinear systems, Lyapunov stability theorem

I. Introduction

Sliding mode control (SMC) is a well-known method to control linear or nonlinear systems. In addition, SMC systems have been applied to the control of real nonlinear systems with uncertainties such as the pendulum system, the biped robot and the spacecraft because of the robustness to parameter perturbations and external disturbances [1-3]. In order to design SMC systems, the linear sliding surface is widely used to describe the desired performance. Its representative characteristic is that the convergence of system states to the equilibrium point is usually asymptotic. However, it does not ensure the convergence in finite time.

Recently, a new control method called the terminal sliding mode control (TSMC) method was developed [4-9]. The TSMC method has a nonlinear sliding surface based on the concept of terminal attractor [7]. In addition, the TSMC method has following advantages compared with the classical sliding mode control (CSMC) method [5]. First, the tracking error of system using the TSMC is driven to zero within finite time, but the CSMC method guarantees the convergence to origin within infinite time [8]. Second, the TSMC method has the improved performance, which results from the elimination of chattering phenomenon in a control input. Third, the TSMC method has the improved robustness, which results from the dependence of terminal slider stability upon the rate of change of uncertainties. Forth, the TSMC method has the improved reliability, which results from the elimination of interpolation region. Finally, the TSMC method guarantees the improved tracking precision compared with the CSMC method in uncertain dynamic nonlinear systems. Though the TSMC method has those advantages, the approximation error must be assumed to be bounded by some known functions or constants. Therefore, we employ the neural networks (NNs) because we cannot know the bounded function or constant in real situation. Many NNs such as multi-layer perceptions (MLPs) [10], radial basis function networks (RBFNs) [11], recurrent neural networks (RNNs), wavelet neural networks (WNNs) [12] were utilized to control and identify nonlinear systems. Among these NNs, SRWNN which has the fast convergence ability of WNN and the dynamic mapping ability of RNN was proposed to control nonlinear systems in [13,14]. In this paper, we propose the SRWNN based TSMC method for the nonlinear system with model uncertainties. In the proposed control system, the SRWNN is used to approximate model uncertainties. Adaptation laws of weights of the SRWNN are induced from the Lyapunov stability theorem, which is used to guarantee the stability of the proposed control system. Finally, we carry out computer simulations for second-order nonlinear systems such as Duffing system and the wing rock phenomena in order to verify the effectiveness of the proposed control scheme.
II. Self Recurrent Wavelet Neural Network

The SRWNN structure is shown in Fig. 1. As shown in the figure, we consider the SRWNN structure with multi-input and single-output. That is, the SRWNN has $N_i$ inputs, one output, and $N_i \times N_o$ mother wavelets. The basic structure of SRWNN consists of four layers as follows [13,14]:

The layer 1 is an input layer. This layer accepts the input variables and transmits the accepted inputs to the next layer and output layer, directly.

The layer 2 is a mother wavelet layer. Each node of this layer has a mother wavelet and a self-feedback loop. In this paper, we select the first derivative of a Gaussian function, $\phi_{\beta}(x) = -x \exp \left( -\frac{1}{2} x^2 \right)$, of a mother wavelet function. A wavelet $\phi_{\beta}(x)$ of each node is derived from its mother wavelet function $\phi(x)$ as follows:

$$\phi_{\beta}(x) = \phi \left( \frac{u_{\beta} - m_{\beta}}{d_{\beta}} \right) \text{ with } z_{\beta} = \frac{u_{\beta} - m_{\beta}}{d_{\beta}},$$  \hspace{1cm} (1)

where $m_{\beta}$ and $d_{\beta}$ are the translation factor and the dilation factor of the wavelets, respectively.

The subscript $\beta$ indicates the $k$-th input term of the $j$-th wavelet. In addition, inputs of this layer for discrete time $n$ can be expressed as follows:

$$u_{\beta}(n) = x_i(n) + \phi_{\beta}(n-1) \alpha_{\beta},$$ \hspace{1cm} (2)

where $\alpha_{\beta}$ denotes the weight of the self-feedback loop.

The input of this layer contains the memory term $\phi_{\beta}(n-1)$, which can store the past information of network. That is, the information of current dynamics of the system is maintained for the next step. Thus, even if the SRWNN has less mother wavelet than the WNN, it can attract the system with complex dynamics well. Also, we overcome the disadvantage of WNN which cannot confront the unexpected change of system because it does not have memories. Here, $\alpha_{\beta}$ is a factor to represent the ratio of information storage. These aspects are the apparent dissimilar point between the WNN and the SRWNN.

From (2), we can know that the SRWNN is the generalization form of WNN because the structure of SRWNN is the same as that of WNN when $\alpha_{\beta}$ is equal to zero.

The layer 3 is a product layer. The nodes in this layer are given by the product of mother wavelets. That is, outputs of layer 2 is the input of each nodes in layer 2. The former contents are shown as follows:

$$\phi_j(x) = \prod_{k=1}^{N_i} \phi(\tau_{j,\beta})$$

The layer 4 is an output layer. The node of output is a linear combination of consequences obtained from the output of layer 3. In addition, the node of output layer accepts directly input values from the input layer. Therefore, the SRWNN output is composed by self-recurrent wavelets and parameters as follows:

$$y = \hat{\mathbf{w}}^T Y + \hat{A}^T X,$$ \hspace{1cm} (4)

where $\hat{\mathbf{w}}$ is estimated uncertainties, $\hat{\mathbf{w}} = [w_1, \cdots, w_{N_i}]^T$ and $\hat{\mathbf{A}} = [\alpha_1, \cdots, \alpha_{N_i}]^T$ are weighting vectors trained by tuning law. In addition, $Y = [y_1, \cdots, y_{N_i}]^T$ is the output vector of wavelet function and $X = [x_1, \cdots, x_{N_i}]^T$ is the input vector.

III. Terminal Sliding Mode Control of Nonlinear Systems

In this paper, we consider a second order nonlinear dynamic system with uncertainties as following form:

$$\ddot{x} = f(x) + u + d,$$ \hspace{1cm} (5)

where $x$ is the output of interest, $f(x)$ is the uncertain nonlinear function, $u$ is a control input and $d$ is a disturbance. It is assumed that the actual nonlinear function is expressed as follows:

$$f(x) = f_{\lambda}(x) + \Delta f(x),$$ \hspace{1cm} (6)

where $f_{\lambda}(x)$ is the nominal nonlinear function and $\Delta f(x)$ denotes the uncertain term. Then, the (5) can be written in the following form:

$$\ddot{x} = f_{\lambda}(x) + u + \Xi,$$ \hspace{1cm} (7)
where $\Xi = \Delta f(x) + d$. From the universal approximation theorem [15], there exists the optimal SRWNN in the form of (5). Therefore, it can uniformly approximate unknown uncertainties as follows:

$$\Xi = \Xi' + \epsilon = W^T Y + A^T X + e,$$  \hspace{1cm} (8)

where $W'$ and $A'$ are the optimal weighting vectors of $W$ and $A$, respectively. That vectors achieve the minimum reconstruction error. In addition, $\epsilon$ is the element of reconstruction error vector which is bounded by $|\epsilon| \leq E$ and $E$ is positive constant.

The TSMC method has a nonlinear sliding surface in order to obtain the finite time convergence of the system tracking error. Thus, we define the terminal sliding surface as follows:

$$s = \dot{e} + \alpha e^\gamma,$$  \hspace{1cm} (9)

where $\alpha > 0$, $0 < \gamma < 1$ and both denominator and numerator of $\gamma$ are odd integers.

At terminal sliding surface, the concept of terminal attractor is used [16]. The fundamental departure of dynamical systems which shows terminal attracting characteristics is the ability to converge to their natural equilibrium point within finite time.

**Theorem 1** [17]: We can easily compute that (9) will reach to zero within the following finite time $t_f$:

$$t_f = \int_0^e \frac{de}{e(0)}} = \frac{\int_0^e}{\alpha(1-\gamma)},$$  \hspace{1cm} (10)

where $e(0)$ is the initial value of $e$ at $t=0$.

**Proof**: When $s$ is equal to zero, the terminal sliding surface can be described as follows:

$$\dot{e} = -\alpha e^\gamma.$$  \hspace{1cm} (11)

In the TSMC method, the system tracking error is determined by (11). In addition, the system is infinitely stable in the TSMC method because (11) defines the exponentially stability. Here, if we select the proper $\gamma$, we can obtain a final time. This fact shows that the system is infinitely stable. The convergence time for a solution of (11) is given by

$$\frac{de}{dt} = -\alpha e^\gamma \Rightarrow \dot{t}_f = \int_0^e \frac{de}{\alpha e^\gamma},$$  \hspace{1cm} (12)

where $e(0)$ is the initial value of $e$ at $t=0$. From (12), we verify that the system tracking error converges to zero within finite time.

**Theorem 2**: Suppose that the given system (7) is controlled by the following control input using the terminal sliding surface (9):

$$u = -f_N + \dot{x}_d - \alpha \gamma \dot{e}e^{\gamma-1} \dot{e} - W^T Y - A^T X - \dot{E} \text{sig}(s) - k \text{sig}(s),$$  \hspace{1cm} (13)

where $k$ is any positive value and $x_d$ is a desired trajectory. In addition, $\dot{W}, \dot{A}$ and $\dot{E}$ are trained by adaptation laws as follows:

$$\dot{W} = \lambda_w s Y,$$  \hspace{1cm} (14)

$$\dot{A} = \lambda_a s X,$$  \hspace{1cm} (15)

$$\dot{E} = \lambda_e |s|,$$  \hspace{1cm} (16)

where $\lambda_w, \lambda_a$ and $\lambda_e$ are positive tuning gain. Then, the given system (7) is stable within finite time.

**Proof**: We consider the following Lyapunov function:

$$V = \frac{1}{2} s^T + \frac{1}{2\lambda_w} W^T W + \frac{1}{2\lambda_a} A^T A + \frac{1}{2\lambda_e} E^T E,$$  \hspace{1cm} (17)

where $\dot{W} = W' - W$, $\dot{A} = A' - A$ and $\dot{E} = E' - E$.

Using (9), we define the differential form of sliding surface as follows:

$$\dot{s} = \dot{e} + \alpha \gamma \dot{e} e^{\gamma-1} \dot{e} = \dot{x} - \dot{x}_d - \alpha \gamma \dot{e} e^{\gamma-1} \dot{e}$$

$$= f_N + u - \dot{x}_d - \alpha \gamma \dot{e} e^{\gamma-1} \dot{e}$$

$$= f_N + u + W^T Y + A^T X + e - \dot{x}_d + \alpha \gamma \dot{e} e^{\gamma-1} \dot{e}.$$  \hspace{1cm} (18)

By applying (13), we obtain the simple form of $\dot{s}$ as follows:

$$\dot{s} = s \left[ f - \dot{x}_d - \alpha \gamma \dot{e} e^{\gamma-1} \dot{e} - W^T Y - A^T X - \dot{E} \text{sig}(s) - k \text{sig}(s) \right]$$

$$= s \left[ W^T Y + A^T X + e - \dot{E} \text{sig}(s) - k \text{sig}(s) \right]$$

By applying (14), (15), (16) and (19) into it, $V$ can be represented as follows:

$$\dot{V} = s \dot{s} + \frac{1}{\lambda_w} W^T \dot{W} + \frac{1}{\lambda_a} A^T \dot{A} - \frac{1}{\lambda_e} E \dot{E}$$

$$= s W^T Y + s A^T X + e - \dot{E} \text{sig}(s) - k \text{sig}(s)$$

$$- \frac{1}{\lambda_w} W^T \dot{W} - \frac{1}{\lambda_a} A^T \dot{A} - \frac{1}{\lambda_e} E \dot{E}$$

$$= W^T \left( s Y - \frac{1}{\lambda_w} \dot{W} \right) + A^T \left( s X - \frac{1}{\lambda_a} \dot{A} \right)$$

$$+ e - \dot{E} \text{sig}(s) - k \text{sig}(s) - \frac{1}{\lambda_e} E \dot{E}$$

$$\leq |s| \dot{s} - |\dot{E} \text{sig}(s)| - \frac{1}{\lambda_e} E \dot{E}.$$

Taking the time derivative of Lyapunov function (17), and substituting (14), (15), (16) and (19) into it, $\dot{V}$ can be represented as follows:
\[ \leq E - \dot{E} - k_s - \frac{1}{\lambda_c} \dot{\tilde{E}} \]
\[ = \dot{E} \left( s - \frac{1}{\lambda_c} E \right) - k_s \]
\[ = -k_s |s| \leq 0. \]  

(20)

Here, $\tilde{W}, \tilde{A}, \tilde{E}$ and $s$ are bounded because $\dot{V}$ is negative semi-definite. We define a function as follows:

\[ B(t) = -\dot{V}(s, \tilde{W}, \tilde{A}, \tilde{E}). \]  

(21)

Integrating $B(t)$ with respect to time, we obtain the following form:

\[ \int_0^t B(\tau) d\tau \leq V(s(0), \tilde{W}, \tilde{A}, \tilde{E}) - V(s, \tilde{W}, \tilde{A}, \tilde{E}). \]  

(22)

Since $V(s(0), \tilde{W}, \tilde{A}, \tilde{E})$ is bounded and $V(s, \tilde{W}, \tilde{A}, \tilde{E})$ is nonincreasing and bounded, we can conclude the following result:

\[ \lim_{t \to \infty} \int_0^t B(\tau) d\tau < \infty. \]  

(23)

In addition, the limit $B(t) = 0$ because $B(t)$ is bounded by Barbati's lemma. That is, $s \to 0$ as $t \to 0$. Consequently, according to the Lyapunov stability criterion, the stability of the system is guaranteed within finite time.

IV. Simulations and Results

In this section, we apply the proposed control algorithm to two representative nonlinear systems. First, Duffing system, which is the continuous-time chaotic system, is considered to demonstrate the tracking ability of proposed control system. Next, we consider the wing rock motion for regulation. In addition, we compare the performance of the TSMC method based on SRWNN with that of the CSMC method using the bounded approximation error. Here, we consider the magnitude of error.

1. Duffing System

In this subsection, we consider the control of Duffing system. The state equation of Duffing system is as follows:

\[ \ddot{x} - p_2 \dot{x} - p_3 x - x^3 + q \cos(\omega t) + u + \Xi, \]  

(24)

where $p_2 = 1.1$, $p_3 = 0.4$, $q = 2.1$ and $\omega = 1.8$. In addition, the disturbance of this system is bounded by a positive constant.

The tracking control objective of Duffing system is to follow the unstable periodic solution of Duffing system. Because of the variation of $q$, Duffing system can be chaotic or periodic. Thus, Duffing system may have either a chaotic or a periodic solution.

We simulate the CSMC method and the TSMC method for Duffing system with 100%, 70% and 50% parametric uncertainty of $p_1$, $p_2$ and $q$, respectively. In this simulation, we design the control input $u$ to follow the desired trajectory as $x_d = \sin(\pi t/2)$. In addition, it is assumed that the disturbance $d(t) = \sin(t)$. Besides, we choose the proper parameters selected as $\alpha = 50$ and $\epsilon(0) = 0.5$ in both the CSMC method and the TSMC method. In the CSMC method, the bounded value is chosen as 4.6. In addition, the simulation parameters of the TSMC method are chosen as $\lambda_\alpha = 0.00001$, $\lambda_\epsilon = 0.00001$, $\lambda_\gamma = 0.01$ and $\gamma = 1/3$.

Table 1. The comparison of MSE for the Duffing system.

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<th>TSMC</th>
<th>CSMC</th>
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<tr>
<td>MSE</td>
<td>0.1161</td>
<td>0.1858</td>
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![Graph 2. Tracking results for the Duffing system.](image)

![Graph 3. Tracking error for the Duffing system.](image)
The results of tracking performance for the TSMC method and the CSMC method with uncertainties are compared in Figs. 2 and 3. From these figures, we can verify that the TSMC method has faster convergence speed than the CSMC method. Table 1 shows simulation results of the CSMC method and the TSMC method, where the mean square error (MSE) of the TSMC method is lower than that of the CSMC method.

2. The Wing Rock Motion

In this subsection, we consider the wing rock phenomena. The TSMC method is applied to the wing rock motion control of aircraft to investigate the regulation ability of the proposed control method. The differential equation of wing rock control and the aerodynamic rolling moment are given by [18]. Therefore, we rewrite the equation of wing rock motion as follows:

$$\ddot{\phi} = b_0 + b_1 \dot{\phi} + b_2 \phi - b_3 \dot{\phi} + b_4 \dot{\phi} + b_5 \phi + u + \Xi,$$

(25)

where $\phi$ is a roll angle, $u$ is a control input and $b_i$ ($i = 0, 1, 2, 3, 4, 5$) are aerodynamic parameters of the angle of attack.

We determine the nominal aerodynamic parameters as $b_0 = 0$, $b_1 = -0.01859521$, $b_2 = 0.015162375$, $b_3 = -0.06245153$, $b_4 = 0.00954708$ and $b_5 = 0.02145291$ [18]. The derivation of the TSMC method and the CSMC method does not need the use of aerodynamic parameters on the structure of aerodynamic function. To investigate the effectiveness of proposed control method, we carry out simulations for two initial conditions: 1) the small initial condition is $\phi = 1^\circ$, $\dot{\phi} = 0.5^\circ$/s; 2) the large initial condition is $\phi = 30^\circ$, $\dot{\phi} = 10^\circ$/s. For the small initial condition, a limit cycle oscillation is obtained and for the large initial condition, the roll angle is divergent [19]. Therefore, we can see the unstability of systems for the large initial condition in the uncontrolled nonlinear wing rock motion systems. Here, the TSMC method and the CSMC method are used to solve the problem, such as the unstability of large initial condition.

We simulate the CSMC method and the TSMC method for Duffing system with 100%, 70%, 60%, 30% and 10% parametric uncertainty of $b_1$, $b_2$, $b_3$, $b_4$ and $b_5$, respectively. In this simulation, it is assumed that the disturbance $d(t) = 0.5$. In addition, we choose the proper parameter selected as $\alpha = 5$ in both the CSMC method and the TSMC method. In the CSMC, the bounded value is chosen as 0.15. Besides, the simulation parameters of the TSMC method are chosen as $\gamma = 1/5$, $\lambda_0 = 0.000001$, $\lambda_1 = 0.000001$ and $\lambda_2 = 0.001$.

The state responses of the TSMC method and the CSMC method with unknown uncertainties for small initial conditions are shown in Fig. 4. In addition, those of the TSMC method and the CSMC method for large initial conditions are shown in Fig. 5. From these figures, we can certainly see that the TSMC method has faster convergence speed than the CSMC method. Moreover, Table 2 compares simulation results of the CSMC method and the TSMC method for small and large initial condition, where the MSE of TSMC method is lower than that of the CSMC method.

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<th>TSMC</th>
<th>CSMC</th>
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<tbody>
<tr>
<td>MSE for small initial condition</td>
<td>9.8908×10⁻⁴</td>
<td>0.0019</td>
</tr>
<tr>
<td>MSE for large initial condition</td>
<td>0.1007</td>
<td>0.1260</td>
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Fig. 4. Regulation results of wing rock motion for small initial condition.

Fig. 5. Regulation results of wing rock motion for large initial condition.
V. Conclusion

In this paper, we have designed a SRWNN based terminal sliding mode controller for nonlinear system with uncertainties. The TSMC method has been utilized to improve the performance, the robustness, the reliability and the precision in contrast with the CSMC method. In addition, by using the SRWNN, the problem caused by uncertainties of systems have been solved easily. In our proposed control system, the SRWNN with the simple structure was used to approximate unknown uncertainties. The weights of SRWNN were trained by adaptation law based on the Lyapunov stability theorem. Finally, the proposed terminal sliding mode controller based on SRWNN has been applied to a second order dynamic nonlinear system, such as Duffing system and the wing rock phenomena. From the computer simulation results, we show that the convergence speed and the accuracy of the TSMC method are improved compared with those of the CSMC method.

Reference

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