Abstract: The inherent feature of the highly efficient spectrum usage has made Orthogonal Frequency Division Multiplexing (OFDM) preferable for Communication Standards. This study evaluated the performance of a Least Square (LS) estimator for a comb-type pilot insertion scheme over a fast fading Rayleigh channel. A High Peak-to-Average Power Ratio (PAPR) is one of the major downsides of the OFDM. The effects of an increase in the number of subcarriers on PAPR and the performance of the LS Estimator were studied. Increasing the number of subcarriers while keeping the pilots overhead constant resulted in improved performance of the LS estimator but the PAPR increased with increasing number of subcarriers. Therefore some trade-off between the number of subcarriers and the performance of the OFDM system is needed. The Mean Square Error (MSE) expression was also derived for the LS estimator in the case of a comb-type pilot arrangement. The MSE expression clearly explains the effects of the number of subcarriers on the performance of the LS estimator.

Keywords: OFDM, LS estimation, Peak-to-average power ratio, Comb-type pilot arrangement

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) transforms the frequency selective fading channel into flat fading channels and provides robustness against frequency selective fading channel impairments. Modern communication standards i.e. Long Term Evolution (LTE), Wi-Fi, Wi-Max, Digital Video Broadcasting (DVB) and Digital Audio Broadcasting (DAB) use OFDM because of its spectral efficiency and immunity to narrow band interference [1-3]. The spectral efficiency of OFDM is due its subcarriers overlapping nature in the frequency domain. The guard interval is used in OFDM to mitigate the Inter-Symbol Interference (ISI). This is another advantage of OFDM over single carrier communications systems, which use complex equalization schemes for ISI mitigation [4].

A High Peak-to-Average Power Ratio (PAPR) is one of the major downsides of the OFDM system [5, 6]. The performance of the OFDM system is degraded significantly by the impairments introduced by the frequency selective fading channel. Channel estimation is employed in OFDM to cancel out the impairments introduced by the multipath fading channel. The arrangement of the pilot tones in the OFDM symbol is based on the channel impulse response variations. If the channel impulse response varies rapidly, then the pilot tones are inserted into each OFDM symbol at the dedicated subcarriers [7]. On the other hand, the pilot tones are inserted into the OFDM symbols less frequently if the channel impulse response varies slowly. In the case of slow fading channels, the channel frequency response is estimated first and the estimated channel frequency response is then used to equalize the subsequent symbols [8]. In [9], the performance of the OFDM system was evaluated over the Rayleigh fading channel. The LS estimator has been used to estimate the channel frequency response at the pilot tones for the comb-type pilot tones insertion scheme. One dimensional interpolation techniques used to estimate the channel frequency response at the data tones are linear, low pass, time domain and second order interpolation techniques. Comparative analysis of interpolation techniques showed that a low pass interpolation outperforms the other one dimensional interpolation techniques.
In [10], the minimum mean square error (MMSE) estimator was realized using the DFT based estimator. Although the MMSE estimator was the optimum in terms of the mean square error (MSE), knowledge of the channel statistics is not easily available for practical channels. In [10], the channel auto correlation matrix and noise variance were calculated using the time domain channel impulse response, in which the noise after the first set of significant taps is suppressed. The proposed MMSE estimator shows better performance than the LS and is degraded more than the ideal MMSE estimator. Mahmoud et al in [11] compared the performance of the LS estimator and Kalman estimator for the comb-type pilot insertion technique. Low pass, spline and linear interpolation techniques were used to estimate the channel frequency response at the data tones. Simulations have shown that low pass interpolation has better performance than other interpolation techniques. The Kalman estimator outperforms the LS estimator with a low pass interpolation.

In this paper, the performance of the LS estimator was studied for a Rayleigh fading channel by varying the number of subcarriers per OFDM symbol. The effects of the number of subcarriers on PAPR were also studied. An increase in the number of subcarriers causes an increase in PAPR. The effect of an increase in the number of subcarriers for a particular pilot overhead results in performance improvement of the OFDM system with a LS estimator. Therefore, some trade-off must be made between the performance and the number of subcarriers for the OFDM system.

The remainder of the paper is organized as follows: Section 2 explains the OFDM system, and section 3 introduces PAPR. The channel estimation algorithm is described in section 4. MSE expression for the LS estimator based on comb-type pilot insertion scheme is derived in section 5. The simulation results are described in section 6 and finally, the paper is concluded in section 7.

Notations: ( )$^H$ is used to represent the Hermitian Transposition. Vectors and Metrics are denoted by the bold face italic lower case and upper case letters, respectively. $E\{\}$ denotes the expectation.

2. OFDM System Overview

Fig. 1 shows the OFDM system used to evaluate the performance of the LS estimator. Initially, the input bit sequence is mapped using the digital modulation schemes to yield the frequency domain sequence of symbols. The frequency domain sequence of the symbols is then divided into sets of size $N$. The $y^{th}$ set of symbols is $(x^{y}(0), x^{y}(1), ..., x^{y}(N_{p} - 1))$, where $N_{p}$ is the number of subcarriers reserved for the data per OFDM symbol. The $y^{th}$ set of symbols is then parallelized. The $N_{p}$ pilot symbols are then inserted into the OFDM symbols at the dedicated subcarriers according to the comb-type pilot arrangement. The parallelized frequency domain sequence of the symbols is passed through the IFFT block, which transforms the frequency domain sequence into a time domain sequence. The output of the IFFT block is given by:

$$z^{(y)}(n) = \sum_{k=0}^{N-1} x^{y}(k)e^{-j\frac{2\pi km}{N}} \quad n = 0, 1, ..., N - 1$$  (1)

The guard interval of the type of cyclic prefix with lengths greater than the highest delay spread of the multipath fading channel was added to mitigate the ISI. The addition of a cyclic prefix was carried out by the insertion of the last $L$ symbols at the start of each OFDM symbol:

$$d_{CP}(s) = \begin{cases} z^{(y)}(N - L + s), & s = 0, 1, 2, ..., L - 1 \\ z^{(y)}(L - s), & L - s = 0, \text{...,}, L + N - 1 \end{cases}$$  (2)

Finally, the signal at the transmitter side was serialized and transmitted through the multipath fading channel in the existence of Additive White Gaussian Noise (AWGN). The signal at the receiver side is given by the following:

$$c_{CP}(n) = \sum_{l=0}^{L-1} h(n, l)d_{CP}(n - l) + v(n)$$  (3)

where $h(n, l)$ is the channel impulse response at a particular instant, $n$, and $v(n)$ is the AWGN. The cyclic prefix was removed to cancel out ISI. The signal was passed through the FFT block to convert the time domain signal into frequency domain. The output of the FFT block is given by:
The channel estimation was performed using the received and known transmitted pilots. Equalization was performed after channel estimation. Finally, after parallel to serial conversion, the signal was demapped to yield the output bits.

3. Peak-to-Average Power Ratio

The peak-to-average power ratio (PAPR) of the OFDM system was given by the ratio of maximum power to the average power of a time domain OFDM signal [12]:

\[ PAPR = \frac{\text{max}[\text{envelope}^2]}{\text{mean}[\text{envelope}^2]} \]  \hspace{1cm} (5)

Different techniques have been adopted in the literature to reduce the high PAPR in OFDM. The partial transmit sequence [13], selective mapping [14] and clipping [15] have been used for PAPR reduction. Wang et al in [16], used a non-linear companding transform to reduce PAPR in the OFDM system. The OFDM symbol was first transformed into desirable statistics form defined by a linear piecewise function through the operation of the companding transform. The proposed technique performs better than the other non-linear transform algorithms. A baseband OFDM system with \( N \) subcarriers has PAPR \( N \) [17]. Therefore, the PAPR increases with increasing number of sub carriers. Several design constraints must be taken while designing an OFDM system, one of these constraints is the battery power consumption while designing a portable OFDM system. High peaks require the use of a linear filter with a greater linear range, which are not power efficient.

4. Channel Estimation Algorithm

This paper uses the comb-type pilot insertion technique. The channel frequency response at the pilot tones was estimated using the LS estimator. The operation of the LS estimator is based on a reduction of the square of the difference between the detected and estimated signal [18]. The insertion of the pilot at the dedicated subcarriers for a comb-type channel estimation is performed according to the following equation:

\[
x(k) = x(nP + l)
\]

\[
= \begin{cases} 
  x_p(n) & l = 0 \\
  \text{information data} & l = 1,2,3,...,P-1 
\end{cases} \hspace{1cm} (6)
\]

where \( P = \frac{N}{\text{Num of subcarriers per OFDM symbol}} \), \( k \) is the sub carrier index for the OFDM symbol in frequency domain and \( x_p(n) \) is the \( n^{th} \) pilot tone value. The pilot tones are inserted into the OFDM symbol in an equally spaced fashion to achieve optimal performance [19]. The performance of the estimation can be enhanced by increasing the number of pilots per OFDM symbol. On the other hand, this would cause an increase in the pilot overhead. Therefore, a trade-off must be made between the overhead and performance. The LS estimate of the channel at the pilot frequencies is given by [20]:

\[
h_{LS} = c_p x_p^{-1} \hspace{1cm} (7)
\]

where \( c_p \) and \( x_p \) are the vectors representing the received and transmitted pilots, respectively. The channel frequency response at the data tones was estimated after estimating the channel frequency response at the pilot tones using an interpolation technique. In this paper, one-dimensional interpolation techniques i.e. low pass, linear and spline interpolation were used because of their low computational complexity compared to two dimensional interpolation techniques. The channel response estimated at a subcarrier \( k \), where, \( mP \leq k < (m+1)P \), using a linear interpolation is given by the following [9]:

\[
h_{est}(k) = h_{est}(mP + l) = \left(1 - \frac{l}{P}\right) h_{LS}(m) + \frac{l}{P} h_{LS}(m + 1)
\]

\[
= h_{LS}(m) + \frac{l}{P} \left(h_{LS}(m+1) - h_{LS}(m)\right), 0 \leq l < P \hspace{1cm} (8)
\]

A linear interpolation has better performance than a piece wise interpolation [21]. Low pass interpolation interpolates in such a way that the sequence after the insertion of zeros between the pilot symbols was passed through the low pass Finite Impulse Response (FIR) filter. Spline interpolation is based on drawing a smooth curve by using several data points [7].

5. Mean Square Error Analysis

In this section, a MSE expression is derived for a LS estimation based on the comb-type pilot arrangement. The mean square error (MSE) at a particular subcarrier \( k \) is given by the following:
where \( h[n] \) and \( h_{\text{LS}}[n] \) are the actual channel impulse response and estimated channel impulse response in the time domain. The LS estimator estimates the channel impulse response without considering the channel noise in the estimation process, as depicted clearly in Fig. 3 in its conceptual view. The LS estimator minimizes the square of \( E[n] \). Therefore, the channel impulse response estimated by the LS estimator consists of the channel impulse response and noise.

\[
h_{\text{LS}}[n] = h[n] + \bar{v}[n]
\]  

(10)

where \( \bar{v}[n] = \frac{v[n]}{c[n]} \) is the noise vector divided by the transmitted pilots vector. (9) after inputting the value of \( h_{\text{LS}}[n] \) becomes:

\[
MSE[k] = E[|\text{FFT}(h[n] - h_{\text{LS}}[n])|^2]
\]  

(9)

\[
MSE[k] = E[|\text{FFT}(\bar{v}[n])|^2]
\]  

(11)

\[
MSE[k] = E[|\sum_{w=0}^{L} \bar{v}[w] e^{-j \frac{2 \pi k w}{P}}|^2]
\]  

(12)

\[
MSE[k] = L + E[|\bar{v}[w] e^{j H[f]}|^2]
\]  

(13)

\[
MSE[k] = \sum_{w=0}^{L} \sum_{f=0}^{F} E[|\bar{v}[w] e^{j H[f]}|^2]
\]  

(14)

\[
MSE[k] = L + E[|\bar{v}[w] e^{j H[f]}|^2]
\]  

(15)

\[
MSE[k] = L + E[|\sum_{w=0}^{L} \bar{v}[w] e^{-j \frac{2 \pi k w}{P}}|^2]
\]  

(16)

\[
MSE[k] = L + \frac{1}{P} \sigma_p^2 E\left[\left|\frac{1}{x[k]}\right|^2\right]
\]  

(17)

where \( \sigma_p \) is the average power of the \( p^{\text{th}} \) channel tap. The legend, ‘C’ and ‘E’, ‘Low pass’, ‘Linear’ and ‘spline’ represent the constant, exponential, low pass, linear and spline cubic interpolation techniques, respectively. The channel used for the performance evaluation of the OFDM system consists of \( L \) independent taps of a Gaussian distribution with a zero mean. The performance of the OFDM system was evaluated for two different power delay profiles i.e. exponential and constant [3]. The variance of each tap for an exponential power delay profile is given by the following equation:

\[
\sigma_l^2 = e^{-\frac{l}{\tau}} \quad l = 0, 1, 2 \ldots L - 1
\]  

(19)

The legend, ‘C’ and ‘E’, ‘Low pass’, ‘Linear’ and ‘spline’ represent the constant, exponential, low pass, linear and spline cubic interpolation techniques, respectively. The channel order, \( L=18 \), is used for Figs. 4-7. Figs. 4-7 present the effects of the increase in number of subcarriers on the performance of the LS estimator for BPSK and QPSK modulation schemes, respectively. The performance of the LS estimator improves with increasing number of subcarriers per OFDM symbol for a constant pilot overhead for both constant and exponential power delay profiles, respectively (Figs. 4-7). The performance of the LS estimator for different interpolation techniques, i.e., low passes, linear and spline interpolation technique is in the following order from the worst to best: Linear, spline and low pass for both a constant and exponential power delay profiles. This order of the performance of the interpolation techniques matches with that of the interpolation techniques performance in [9, 11].

Fig. 8 shows the performance of the LS estimator for the different digital modulation schemes i.e. BPSK, QPSK, 16-QAM, 32-QAM and 64-QAM for subcarriers \( N=512 \). The channel order \( L=25 \) was used for Fig. 8. The interpolation technique used in Fig. 8 is a low pass interpolation because of its better performance compared to linear and spline interpolation techniques.

**6. Simulation Results & Analysis**

This section presents the MATLAB® simulation results for a performance evaluation of the uncoded OFDM system with channel estimation. Table 1 lists the parameters used in the simulation. The channel used for the performance evaluation of the OFDM system consists of \( L \) independent taps of a Gaussian distribution with a zero mean. The performance of the OFDM system was evaluated for two different power delay profiles i.e. exponential and constant [3]. The variance of each tap for an exponential power delay profile is given by the following equation:

\[
\sigma_l^2 = e^{-\frac{l}{\tau}} \quad l = 0, 1, 2 \ldots L - 1
\]  

(19)

The legend, ‘C’ and ‘E’, ‘Low pass’, ‘Linear’ and ‘spline’ represent the constant, exponential, low pass, linear and spline cubic interpolation techniques, respectively. The channel order, \( L=18 \), is used for Figs. 4-7. Figs. 4-7 present the effects of the increase in number of subcarriers on the performance of the LS estimator for BPSK and QPSK modulation schemes, respectively. The performance of the LS estimator improves with increasing number of subcarriers per OFDM symbol for a constant pilot overhead for both constant and exponential power delay profiles, respectively (Figs. 4-7). The performance of the LS estimator for different interpolation techniques, i.e., low passes, linear and spline interpolation technique is in the following order from the worst to best: Linear, spline and low pass for both a constant and exponential power delay profiles. This order of the performance of the interpolation techniques matches with that of the interpolation techniques performance in [9, 11].

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**Table 1. Simulation Parameters.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarriers, N</td>
<td>256, 512</td>
</tr>
<tr>
<td>Size of the FFT</td>
<td>256, 512</td>
</tr>
<tr>
<td>Digital Modulation Schemes</td>
<td>BPSK, QPSK, 16-QAM, 32-QAM, 64-QAM</td>
</tr>
<tr>
<td>Cyclic Prefix Length</td>
<td>( \sqrt{64} ) i.e. 64, 128</td>
</tr>
<tr>
<td>Channel Estimation</td>
<td>Comb-Type</td>
</tr>
<tr>
<td>Pilot Ratio</td>
<td>1/8</td>
</tr>
</tbody>
</table>
to the other one-dimensional interpolation techniques. Fig. 8 clearly reveals the performance degradation for higher digital modulation schemes. This performance degradation is because of the nearby positioning of the constellation points for higher order modulation schemes. By comparing the performance of the OFDM system with constant and exponential power profiles, it is clear from Fig. 8 that the performance degradation of the OFDM system for the constant power delay profile is more than the exponential power delay profile. This performance degradation was attributed to the higher channel taps being less significant than the earlier ones for the exponential power delay profile, whereas all the taps are equally significant for a constant power delay profile. Therefore, the performance degradation due to the higher channel taps in the case of the exponential power delay profile is less significant than for a constant power delay profile.

Table 2 lists the PAPR calculated for different modulation schemes and different number of subcarriers. Table 2 clearly reveals an increase in PAPR for large numbers of subcarriers. On the other hand, the performance of the OFDM system with the LS estimator improves with increasing number of subcarriers for the constant pilot overhead. Therefore, some trade-off between the number of subcarriers and performance is needed. Other ways to reduce the PAPR without reducing the
number of subcarriers is to use PAPR reduction techniques. The PAPR calculated in Table 2 for uncoded OFDM with the BPSK modulation scheme was low compared to the other higher modulation schemes.

<table>
<thead>
<tr>
<th>Subcarriers</th>
<th>BPSK</th>
<th>QPSK</th>
<th>16-QAM</th>
<th>32-QAM</th>
<th>64-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=64</td>
<td>4.315</td>
<td>4.804</td>
<td>4.779</td>
<td>4.771</td>
<td>4.790</td>
</tr>
<tr>
<td>N=128</td>
<td>4.912</td>
<td>5.478</td>
<td>5.465</td>
<td>5.467</td>
<td>5.456</td>
</tr>
<tr>
<td>N=256</td>
<td>5.524</td>
<td>6.171</td>
<td>6.151</td>
<td>6.154</td>
<td>6.144</td>
</tr>
</tbody>
</table>

*All the values in this table are in dBs

7. Conclusions

This study examined the effects of an increase in the number of subcarriers on the performance of the LS estimator and PAPR. The expression for the MSE based on comb-type pilot insertion was also derived. The results showed that the derived MSE expression clearly explains the performance of the LS estimator in OFDM for a comb-type pilot arrangement over a fast fading Rayleigh channel. The performance of the LS estimator improves with increasing number of subcarriers for a constant pilot overhead but an increase in the number of subcarriers also causes an increase in PAPR. Therefore, some trade-off between the number of subcarriers and the performance of the OFDM system in terms of PAPR and BER is needed.

References


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