Incremental Best Relay Selection System with Outdated CSI in Rayleigh Fading Channels

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Abstract Recently, for spectral efficiency and power saving, an incremental best relay selection of a cooperative diversity scheme is proposed. However during the best relay selection process, there may exist a delay between channel estimation and actual data transmission. Consequently, this delay causes outdated channel state information (CSI). We analytically derive the effect of the outdated CSI to an incremental best relay selection diversity scheme. It is noted that the system performance deteriorates with decreasing the value of the correlation coefficient sensitively. When the correlation coefficient reduces from 1 to 0.9, the most performance degradation is denoted. However, the performance degradation is diminished with decreasing the correlation coefficient.

Key Words: Cooperative diversity, Relay, Best relay selection, CSI, Performance.
spatially independent relays which have usually single antenna instead of multiple antennas\(^{[3][4]}\). Especially in a parallel relay cooperation, the relays (usually the multiple relays) which satisfy the transmit condition forward the information to a destination for spectral efficiency. It is well known that the multiple active relays improve the diversity gain, however, decrease the spectral efficiency\(^{[5]-[7]}\).

Very recently, to overcome this inefficiency an incremental best relay selection\(^{[4][8]-[10]}\) which selects only one indirect (source-relay-destination) path is proposed. In \([9]\), the lower bound of the end-to-end outage probability is considered under the assumption that the selected best relay is not in outage. More accurate closed form of the end-to-end outage probability expression is investigated in \([10]\).

However, the incremental best relay selection for proactive scheme has to know all the exact instantaneous channels (i.e., between a source and a destination (S-D), a source and relays (S-R), and relays and a destination (R-D) channels) to select the best relay. And after the best relay selection process, the selected relay transmits the received information to a destination. Unfortunately, there may exist a delay between channel estimation and actual data transmission at the selected best relay. In other words, the instantaneous channel state information (CSI) used in relay selection can substantially differ from that of the current CSI at the data transmission, hence it causes the outdated CSI problems. It is well known the outdated CSI deteriorates the system performance\(^{[11][12]}\).

The effect of outdated CSI on a relay selection between R-D paths has been studied in \([12],[13]\). However, those opportunistic relay schemes, which select the best relay between R-D paths, only include the effect of outdated CSI to the R-D paths. To the best of our knowledge, the effect of the delayed CSI to the whole S-R-D paths of an incremental best relay selection scheme has not been considered in the literature.

Therefore, in this paper we derive the closed-form expression of the end-to-end outage probability of an incremental best relay selection scheme with outdated CSI in fading channel. Also we derive the probability density function (pdf) of each hop with outdated CSI which can be applied to the other diversity combining schemes. We assumed each channel has an independent and identically distributed Rayleigh fading channel and each relay has decode-and-forward (DF) protocol.

The remainder of this paper is organized as follows. In section II we introduce the system model of an incremental cooperative diversity scheme. The outage probability of the scheme with outdated CSI is derived analytically in section III. Finally, the numerical examples are depicted in section IV, and concluding remarks are given in section V.

### II. System model

Fig.1 shows an incremental best relay selection cooperative diversity system with a source (S), a destination(D) and \(K\) relays. \(R_k (k=1,2,...,K)\) denotes the \(k-th\) relay. And the slotted line denotes the selected best relay path. We assume independent and identically distributed Rayleigh fading across all links. \(h_{sr}, h_{sd}\) and \(h_{rd}\) denote the channel gain between S and \(R_k\), S and D, and \(R_k\) and D, respectively.

![Fig 1. System model](image-url)
Hence, $\gamma_{sr} = |h_{sr}|E_s/N_0$, $\gamma_{sd} = |h_{sd}|E_s/N_0$, and $\gamma_{rd} = |h_{rd}|E_r/N_0$ denote the instantaneous SNR between S and $R_k$, S and D, and $R_k$ and D, respectively. Where $E_s$ represents the transmit energy per symbol and $N_0$ is the noise power spectral density. Therefore, the average received SNR is given by $\tilde{\gamma}_{sr} = E[\gamma_{sr}]$, $\tilde{\gamma}_{sd} = E[\gamma_{sd}]$, and $\tilde{\gamma}_{rd} = E[\gamma_{rd}]$, where $E[\bullet]$ denotes the expectation operator.

During the first hop, S broadcasts its information to D as well as $R_k$ relays. If the received SNR exceeds a predetermined threshold at D, the transmission is completed. Otherwise, D sends a binary feedback to the source and the best relay selection process is started. The selection process of the best relay is based on the SNRs across the two hops of each relay. In the second hop, the best relay retransmits the received information to the destination. The best relay selection is performed jointly as follows [9]:

(a) Firstly, for each relay link, select minimum SNR link,

$$\Lambda_k = \min\{\gamma_{sr}, \gamma_{sd}\} \text{ for } k = 1, 2, \ldots, K$$

(b) Secondly, select the best relay $\gamma_{k*}$ across the relays,

$$\gamma_{k*} = \max_{k=1,2,\ldots,K} \{\Lambda_k\}$$

The received information both from the source and the selected best relay are combined using Maximal-ratio-combining (MRC) receiver at D.

### III. Performance Analysis

According to the Jake’s scattering model, the channel gain between the source and the $k$-th relay $h_k$ and its estimated(delayed) value $\hat{h}_k$ follow a joint complex Gaussian distribution with correlation coefficient $\rho_k = J_0(2\pi f_d T_k)$, where $J_0(\bullet)$ denotes the zero-order Bessel function of the first kind, $f_d$ is the Doppler frequency, $T_k$ stands for the time delay of the $k$-th relay. The received SNR of the $k$-th relay $\gamma_k$ conditioned on its delayed SNR $\hat{\gamma}_k$ follows a non-central chi-square distribution which is given by [11].

$$f_{\gamma_k}(\gamma_k | \hat{\gamma}_k) = \frac{K}{\gamma_k(1-\rho^2)} \left(\frac{\gamma_k}{\rho^2 \gamma_k}\right)^{\frac{K-1}{2}} e^{-\frac{\rho^2}{2(1-\rho^2)}} I_{K-1}\left(\frac{2K\sqrt{\rho^2 \gamma_k}}{\gamma_k(1-\rho^2)}\right)$$

where we assume each relay has same correlation coefficient $\rho_k = \rho$ and where $I_\alpha(x)$ denotes the $\alpha$-th order modified Bessel–function of the first kind.

$$I_\alpha(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\alpha+k}}{k! \Gamma(\alpha + k + 1)}, \quad x \geq 0$$

Therefore the probability density function (pdf) of the SNR between the best relay and the destination (R-D path) can be written by

$$f_{\gamma_k}(\gamma_k | \hat{\gamma}_k) = \int_{-\infty}^{\infty} f_{\gamma_k}(\gamma_k | \hat{\gamma}_k) f_{\hat{\gamma}_k}(\gamma_k | \hat{\gamma}_k) d\gamma_k$$

where the inner integral is the pdf of the delayed SNR $\hat{\gamma}_k$, which is given by [10]

$$f_{\hat{\gamma}_k}(\gamma_k | \hat{\gamma}_k) = \int_{-\infty}^{\infty} f_{\hat{\gamma}_k}(\gamma_k | \hat{\gamma}_k) f_{\gamma_k}(\gamma_k | \hat{\gamma}_k) d\gamma_k$$

By replacing Eq. (6) into Eq. (5), and by using the
identities of Eq. 6.614.3, Eq. 9.220.2 and Eq. 9.215.1 in [14], the pdf can be obtained by

\[
\begin{align*}
 f_{\gamma_{rd}}(\gamma_{rd}) & = \sum_{i=1}^{K} \left( \frac{K}{\gamma_{dr}} \right)^{i} \frac{1}{\gamma_{rd} - \gamma_{dr}} \left( e^{-\gamma_{rd}/\gamma_{dr}} - \frac{1}{\gamma_{dr}} \left( 1 - e^{-\gamma_{rd}/\gamma_{dr}} \right) \right) \\
 & \quad + \sum_{i=1}^{K} \left( \frac{K}{\gamma_{dr}} \right)^{i} \frac{1}{\gamma_{rd} + \gamma_{dr}} \left( 1 - e^{-\gamma_{rd}/\gamma_{dr}} \right)
\end{align*}
\]

(7)

where \( \widetilde{\gamma} = \gamma_{sr} / (\gamma_{sr} + \gamma_{rd}) \). As a special case of \( \rho = 1 \), the delayed SNR \( \gamma_{rd}' \) equals \( \gamma_{rd} \) (i.e., \( \gamma_{rd}' = \gamma_{rd} \)), hence Eq. (7) corresponds to Eq. (6). The pdf of the SNR between the source and the best relay (S-R path), \( f_{\gamma_{sr}}(\gamma_{sr}) \), can be easily obtained by replacing \( \gamma_{rd} \) and \( \gamma_{sr} \) with \( \gamma_{sr} \) and \( \gamma_{rd} \), respectively. Cumulative density function (CDF) of \( \gamma_{rd}' \) can be easily derived from Eq. (7), which is written by

\[
\begin{align*}
 F_{\gamma_{rd}}(\gamma_{rd}) & = \sum_{i=1}^{K} \left( \frac{K}{\gamma_{dr}} \right)^{i} \frac{1}{\gamma_{rd} - \gamma_{dr}} \left( 1 - e^{-\gamma_{rd}/\gamma_{dr}} \right) \\
 & \quad + \sum_{i=1}^{K} \left( \frac{K}{\gamma_{dr}} \right)^{i} \frac{1}{\gamma_{rd} + \gamma_{dr}} \left( 1 - e^{-\gamma_{rd}/\gamma_{dr}} \right)
\end{align*}
\]

(8)

Similarly the CDF of \( \gamma_{sr}' \) can be obtained from Eq. (8) by replacing \( \gamma_{rd} \) and \( \gamma_{sr} \) with \( \gamma_{sr} \) and \( \gamma_{rd} \), respectively.

Assume, each channel has equal average channel gain where \( \gamma_{sd} = \gamma_{sr} = \gamma_{rd} = \gamma_{e} \) and \( \gamma = \gamma_{e}/2 \). With this assumption, the CDF \( F_{\gamma_{rd}}(\gamma_{rd}) \) is identical to \( F_{\gamma_{rd}}(\gamma_{rd}) = F_{\gamma_{e}}(\gamma_{rd}) = F_{\gamma_{e}}(\gamma_{rd}) \), and is written by

\[
\begin{align*}
 F_{\gamma_{rd}}(\gamma_{rd}) & = \frac{K}{\gamma_{rd}} \left( 1 - e^{-\gamma_{rd}/\gamma_{e}} \right) \\
 & \quad + \frac{K}{\gamma_{rd}} \left( 1 - e^{-\gamma_{rd}/\gamma_{e}} \right)
\end{align*}
\]

The outage probability of the incremental relay system can be written by

\[
\begin{align*}
 P_{\text{out}} & = \Pr(\gamma_{ad} < \Gamma_{1}) \Pr(\gamma_{sr} < \Gamma) \\
 & \quad + \Pr(\gamma_{sr} > \Gamma, \gamma_{ad} < \Gamma, \gamma_{ad} + \gamma_{rd} < \Gamma)
\end{align*}
\]

(10)

where \( \Gamma_{1} = 2^{R} - 1 \), \( \Gamma = 2^{R} - 1 \), and \( R \) is the transmission rate in bit per second per Hz (bps/Hz). And where

\[
\begin{align*}
 \Pr(\gamma_{ad} < \Gamma_{1}) & = F_{\gamma_{ad}}(\Gamma_{1}) = 1 - e^{-\gamma_{ad}/\gamma_{e}} \\
 \Pr(\gamma_{sr} < \Gamma) & = F_{\gamma_{sr}}(\Gamma) = F_{\gamma_{e}}(\Gamma)
\end{align*}
\]

(11)

(12)

In Eq. (10), the probability term of \( \Pr(\gamma_{sr} > \Gamma, \gamma_{rd} < \Gamma - x) \) for \( 0 \leq x < \Gamma \) can be approximated by [15]

\[
\begin{align*}
 \Pr(\gamma_{sr} > \Gamma, \gamma_{rd} < \Gamma - x) & = \left[ 1 - F_{\gamma_{sr}}(\Gamma - x) \right] F_{\gamma_{rd}}(\Gamma - x)
\end{align*}
\]

(13)

With this approximation, the outage probability of Eq. (10) can be derived by

\[
\begin{align*}
 \Pr(\gamma_{sr} > \Gamma, \gamma_{rd} < \Gamma, \gamma_{ad} + \gamma_{rd} < \Gamma) & = \frac{1}{\gamma_{e}} \int_{\Gamma}^{\Gamma} \Pr(\gamma_{sr} > \Gamma, \gamma_{rd} < \Gamma - x) e^{-x/\gamma_{e}} dx \\
 & \approx \int_{0}^{\Gamma} \Pr(\gamma_{sr} > \Gamma, \gamma_{rd} < \Gamma - x) e^{-x/\gamma_{e}} dx
\end{align*}
\]

(14)

Replacing Eq. (8) into Eq. (14), and rearranging, we can obtain
By substituting Eq. (11), Eq. (12), and Eq. (15) into Eq. (10), we can have a closed-form expression for the end-to-end outage probability of the incremental-best-relay DF cooperative diversity with outdated CSI over Rayleigh fading channels. As a special case of correlation coefficient \( \rho = 1 \) that means there is no CSI delay, Eq. (15) is identical to Eq. (13) of [10] which does not include the effect of the CSI delay.

\[
Pr(\gamma > \Gamma, \gamma < \Gamma) = \sum_{k=2}^{K} \left[ \frac{1}{2} \left( 1 - \frac{2(1-\rho^2) + \rho^2}{(1-2\rho^2)} \right) \left( 1 - e^{-\frac{(0-\rho^2)}{(1-2\rho^2)}(1-\rho^2)} \right) \right] \]

\[
\sum_{k=2}^{K} \left[ \frac{1}{2} \left( 1 - \frac{2(1-\rho^2) + \rho^2}{(1-2\rho^2)} \right) \left( 1 - e^{-\frac{(0-\rho^2)}{(1-2\rho^2)}(1-\rho^2)} \right) \right] \]

\[
(15)
\]

IV. Numerical examples

Fig. 2 shows the outage probability for different values of correlation coefficient \( R = 3 \text{bps/Hz}, K = 10 \)

Fig. 2. Outage probability for different values of correlation coefficient \( R = 3 \text{bps/Hz}, K = 10 \)

Fig. 3 shows the outage probability for different number of relays \( R = 3 \text{bps/Hz} \)

Fig. 3. Outage probability for different number of \( K \) \( R = 3 \text{bps/Hz} \)

With this observation, we can conclude that the delay of CSI affects to the performance sensitively. To improve the performance of an incremental relay system, it is necessary that the delay time has to be minimized for high correlation between the real and the delayed channels.
V. Conclusions

Cooperative diversity system which is applied to the relaying system has diversity gain in fading channel, hence it is highly focused for power saving and network reliability in ad-hoc networks.

However, performance of an incremental best relay selection cooperative diversity system is sensitive for a delay which is mainly caused by the selection process.

We analytically derive the closed-form expression for the end-to-end outage probability of an incremental-best-relay DF cooperative diversity with outdated CSI over Rayleigh fading channels.

We noticed that the system performance deteriorates with decreasing the value of the correlation coefficient. When the correlation coefficient reduces from 1 to 0.9, the most performance degradation is denoted. However, the performance degradation is less sensitive with decreasing the correlation coefficient. Also as we expected, performance improves with the number of relays.

From the results of this paper, we can conclude the delayed CSI causes performance degradation sensitively. Therefore it is recommended that the time delay must be reduced to improve the performance of an incremental-best-relay DF cooperative diversity system.

References


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