TM Mode Analysis of a Periodic Thick Mushroom Structure

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Abstract

We analyzed a periodic thick mushroom structure for use as an artificial magnetic conductor using mode-matching method. The fields in each region were represented by either Floquet modes or waveguide modes. By applying tangential electric and magnetic field continuity conditions and using matrix equations, unknown coefficients and dispersion diagram were calculated. The proposed model can account for the effects of oblique incidence. Simulation time using the method was much faster than the commercial tools. We found that the current method produces accurate results of reflection phase and dispersion diagram.

Key words: Artificial magnetic conductor (AMC), Mushroom, Mode matching

1. INTRODUCTION

Periodic mushroom structures are widely used in the analysis and design of antennas and waveguides, because of their unique features as Artificial Magnetic Conductor (AMC) or Electromagnetic Bandgap (EBG) [1-3]. In conventional mushroom structures, conductors are assumed to be very thin, and there are few research about thick conductor. In this paper, plane wave scattering by the periodic mushroom structure with a conductor of finite thickness is analyzed utilizing a rigorous and accurate technique based on [4]. As an example, TM polarized plane wave incidence is studied. However, our analysis could be easily generalized to various configurations of periodic mushroom-like structure with different type and location of excitation sources.

2. FORMULATION

Firstly, we divide the mushroom structure (Fig.1) into three regions (a, above the mushroom cap; b, the cap; and c, between the base of the cap and the substrate) and represent the electromagnetic fields in each region. The incident magnetic and electric waves are represented by

\[ H_i^e(x, y) = e^{-j(k_{x,1} x - k_{y,1} y)} + \sum_{m = 0}^{\infty} A_{m,1} e^{-j(k_{x,m+1} x + k_{y,m+1} y)} \]  
\[ E_i^e(x, y) = \frac{k_{x,1}}{\omega \varepsilon_0} e^{-j(k_{x,1} x - k_{y,1} y)} - \frac{A_{m,1}}{\omega \varepsilon_0} e^{-j(k_{x,m+1} x + k_{y,m+1} y)} \]

where

\[ k_{x,m} = k_{x,1} + \frac{2\pi m}{p} \]  
\[ k_{y,m} = \begin{cases} \sqrt{k_{x,m}^2 - k_{y,m}^2} \quad |k_{y,m}| \geq |k_{y,1}| \\ -j \sqrt{k_{x,m}^2 - k_{y,m}^2} \quad |k_{y,m}| < |k_{y,1}| \end{cases} \]
where \( m \) is the Floquet mode number, and the \(-j\) term in (2b) means that fields decay with distance from the periodic structure (radiation condition). When the EM frequency is less than the grating frequency, only the zeroth mode \((m = 0)\) propagates from the periodic mushroom structure; the other modes are evanescent.

The scattered waves in region \( b \) are represented using an infinite number of the forward and the backward waveguide modes by

\[
H^b(x, y) = \sum_{i=0}^{\infty} \cos \xi_b \left( x + \frac{w_i}{2} \right) (C^+_e e^{-j\beta_b(x+h_i)} + C^-_e e^{j\beta_b(x+h_i)})
\]  
\[E^b(x, y) = \sum_{i=0}^{\infty} \frac{\beta_i \cos \xi_b \left( x + \frac{w_i}{2} \right)}{\omega_c} (-C^+_e e^{-j\beta_b(x+h_i)} + C^-_e e^{j\beta_b(x+h_i)})
\]

where

\[
\xi_b = \frac{\nu \pi}{w_i}
\]
\[
\beta_i = \left[ \sqrt{k^2 - \xi_b^2}, \left| k \right| \geq \left| \xi_b \right| \right] - j\sqrt{\xi_b^2 - k^2}, \left| k \right| < \left| \xi_b \right|
\]

Here, \( u \) is the waveguide mode number in region \( b \). When the frequency is below first cut-off frequency, only zeroth mode \((u = 0)\) propagates and the other modes are evanescent throughout that region.

In the same manner, the scattered waves in region \( c \) can be represented by

\[
H^c(x, y) = \sum_{i=0}^{\infty} D_i \cos \xi_c \left( x + \frac{w_i}{2} \right) \cos \beta_i \left( y + h_i + h_c \right)
\]  
\[E^c(x, y) = -\frac{1}{j\omega_c} \sum_{i=m}^{\infty} D_i \cos \xi_c \left( x + \frac{w_i}{2} \right) \sin \beta_i \left( y + h_i + h_c \right)
\]

where

\[
\xi_c = \frac{\nu \pi}{w_i}
\]
\[
\beta_i = \left[ \sqrt{k^2 - \xi_c^2}, \left| k \right| \geq \left| \xi_c \right| \right] - j\sqrt{\xi_c^2 - k^2}, \left| k \right| < \left| \xi_c \right|
\]

In (5), \( \nu \) is the waveguide mode number in region \( c \). The boundary condition \((E_c = 0 \text{ at } y = -h_i - h_c)\) was applied in each waveguide modes. Since we assumed the infinite periodic structure, the analysis of a unit cell is sufficient. The required boundary conditions are

\[
E_{x,y=0} = E_{x,y=\infty}, |x| \leq p / 2
\]

\[
H_{x,y=0} = H_{x,y=\infty}, |x| \leq p / 2
\]

\[
E_{x,y=\pm \infty} = E_{x,y=\infty}, |x| \leq p / 2
\]

\[
H_{x,y=\pm \infty} = H_{x,y=\infty}, |x| \leq p / 2
\]

Table I. Field notation

<table>
<thead>
<tr>
<th>Region</th>
<th>Field function</th>
<th>Orthogonal function</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>m</td>
<td>n</td>
</tr>
<tr>
<td>b</td>
<td>u</td>
<td>r</td>
</tr>
<tr>
<td>c</td>
<td>v</td>
<td>s</td>
</tr>
</tbody>
</table>

To obtain the unknown coefficients \((A_u, C^+_u, C^-_u, D_u)\), we should multiply some orthogonal functions defined for each region (Table 1). After applying the tangential continuity, the unknown coefficients can be calculated using the following equations.

\[
pk_u \delta_{u0} - pk_u A_u = \frac{w_i \nu}{2} \sum_{m=0}^{\infty} \left( -C^+_e e^{-j\beta_b(x+h_i)} + C^-_e e^{j\beta_b(x+h_i)} \right) G^m_{1m}
\]
\[
\sum_{m=0}^{\infty} \beta_i (C^+_e - C^-_e e^{-j\beta_b(x+h_i)}) \cdot G^m_{2m} = \frac{1}{2} \sum_{m=0}^{\infty} e^{j\beta_b h_i} \sin \beta_i \beta_i D_i \sin \beta_i h_c
\]
\[
G^m_{1m} + \sum_{m=0}^{\infty} A_u G^m_{1m} = (1 + \delta_{u0}) (C^+_e e^{-j\beta_b(x+h_i)} + C^-_e e^{j\beta_b(x+h_i)})
\]
\[
\frac{1}{2} (1 + \delta_{u0}) |x| (C^+_e + C^-_e e^{-j\beta_b(x+h_i)}) \cdot \sum_{m=0}^{\infty} G^m_{2m} D_i \cos \beta_i h_c
\]

To solve the equation, \( C^+_u \) and \( C^-_u \) must be transformed in terms of \( A_u \) and \( D_u \). From (8c) and (8d), we can obtain

\[
C^+_u = \frac{1}{-j2(1 + \delta_{u0}) \sin \beta_i h_i} \left( G^m_{1m} + \sum_{m=0}^{\infty} A_u G^m_{1m} e^{j\beta_b(x+h_i)} \right)
\]
\[
C^-_u = \frac{1}{j2(1 + \delta_{u0}) \sin \beta_i h_i} \left( G^m_{1m} + \sum_{m=0}^{\infty} A_u G^m_{1m} e^{-j\beta_b(x+h_i)} \right)
\]

Applying (9a) and (9b) to (8a) and (8b) yields

\[
A_u + \sum_{m=0}^{\infty} A_u \Gamma_{1m} + \left[ \sum_{m=0}^{\infty} D_i \Gamma_{2m} \right] = [\delta_{u0} - \Gamma_{0u}]
\]
\[
- \sum_{m=0}^{\infty} A_u \Gamma_{1m} + \left[ -\beta_i D_i + \sum_{m=0}^{\infty} D_i \Gamma_{4m} \right] = [-\Gamma_{0u}]
\]

where
\[
\Gamma_{1w} = \frac{-j w_1 \varepsilon_r \sum_{n=0}^{\infty} \beta_n \csc \beta_n h \cdot G_{1w} \cdot G_{1w}}{2 p k_0 \varepsilon_r (1 + \delta_{1w})} \quad (10a)
\]
\[
\Gamma_{2w} = \frac{j \cos \beta_n h \varepsilon_r \sum_{n=0}^{\infty} \beta_n \csc \beta_n h \cdot G_{2w} \cdot G_{2w}^*}{p k_0 \varepsilon_r (1 + \delta_{2w})} \quad (10b)
\]
\[
\Gamma_{3w} = \frac{-2 w_1 \sin \beta_n h \varepsilon_r \sum_{n=0}^{\infty} \beta_n \csc \beta_n h \cdot G_{3w} \cdot G_{3w}}{1 + \delta_{3w}} \quad (10c)
\]
\[
\Gamma_{4w} = \frac{4 \cos \beta_n h \varepsilon_r \sum_{n=0}^{\infty} \beta_n \csc \beta_n h \cdot G_{4w} \cdot G_{4w}^*}{1 + \delta_{4w}} \quad (10d)
\]

Truncation is required to solve the two matrix equations (9). If the maximum Floquet mode number is \( M \) and the maximum waveguide number is \( V \), the matrix size of (9) is \((2M + V + 2) \times (2M + V + 2)\). In general, the reflection phase \( \angle A_0 \) of the dominant mode and dispersion diagram is of prime interest in AMC. Dispersion diagram can be obtained using the determinant of (9). If \( \det(\text{matrix}) = 0 \), then \( R \to \infty \) and only surface waves exist.

3. RESULTS

We plotted some reflection phase and dispersion diagrams. Simulation using the formulated matrix equation was done within a few seconds, which is much faster than commercial simulators. The reflection amplitude of zeroth and first mode, and the reflection phase of zeroth mode are converged when the truncated mode number \( (M) \geq 12 \) (Fig. 2). In dispersion diagram, cutoff frequencies of the TM mode are 12 GHz, 16.5 GHz, and 8.7 GHz, respectively. We found that the results of the reflection phase (Fig. 3) and dispersion diagram (Fig. 4) agreed well with the output of commercial simulators (Ansoft HFSS and CST MWS).

![Reflection phase and dispersion diagrams](image)

Fig. 2. Reflection (a) amplitude and (b) phase with the truncation number \( (M) \). \( p = 5 \) mm, \( h_1 = 0.5 \) mm, \( h_2 = 1 \) mm, \( w_2 = 0.5 \) mm, \( \varepsilon_{r_a} = \varepsilon_{r_b} = \varepsilon_{r_c} = 1, \mu_{r_a} = \mu_{r_b} = \mu_{r_c} = 1 \).

Fig. 3. Reflection phase of 0th mode. \( p = 5 \) mm, \( h_1 = 0.5 \) mm, \( h_2 = 1 \) mm, \( w_2 = 0.5 \) mm, \( \varepsilon_{r_a} = \varepsilon_{r_b} = \varepsilon_{r_c} = 1, \mu_{r_a} = \mu_{r_b} = \mu_{r_c} = 1 \), (a) \( w_1 = 0.3 \) mm, \( \theta_i = 0^\circ \), (b) \( w_1 = 0.95 \) mm, \( \theta_i = 60^\circ \).

Fig. 4. Dispersion diagram. \( h_1 = 0.5 \) mm, \( h_2 = 1 \) mm, \( w_2 = 4.7 \) mm, (a) \( w_2 = 0.3 \) mm, \( \varepsilon_{r_c} = 1.0 \), (b) \( w_2 = 0.3 \) mm, \( \varepsilon_{r_c} = 4.0 \), (c) \( w_2 = 2.5 \) mm, \( \varepsilon_{r_c} = 4.0 \).

4. CONCLUSION

We used the mode-matching technique to analyze a periodic thick mushroom AMC. The fields in each region were represented by Floquet or waveguide modes. By applying tangential electric and magnetic field continuity
conditions and using matrix equations, unknown coefficients and dispersion diagrams were calculated. The proposed method was much faster than the conventional simulation tool (CST, HFSS) and had good accuracy.

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References


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