Identification of the Jiles-Atherton Model Parameters Using Simulated Annealing Method

Baodong Bai * and Jiayin Wang *

Abstract –This paper presents a method and the experimental measurement system for the determination of Jiles–Atherton model parameters of the 30ZH120 electrical steel sheet. The paper utilizes Epstein Square devices to proceed with the experiment and measurement on a group of hysteresis loops of some certain transformers which use the 30ZH120 electrical steel sheet under two different lap ways. The approach relies on the simulated annealing optimization method in order to minimize the error between the measured and modeled hysteresis curves and yield the best five Jiles–Atherton model parameters. A convenient program, based on the Simulink platform, that can identify the J-A model parameters automatically from the experimental saturated hysteresis loop which is used to model the nonlinear characteristics of the electrical steel sheet, is developed. Research shows that the simulated annealing optimization method gets satisfactory results.

Keywords: Parameters, J-A model, Electrical steel sheet, Simulated annealing

1. Introduction

Transformers and reactors are essential components in the overall power system, and the accurate modeling of ferromagnetic materials in magnetic field numerical analysis is essential to predict and analyze the performance of electric power equipment. We usually use average magnetization curves instead of hysteresis loops, which can more truly reflect the characteristics of ferromagnetic materials. The description of magnetization processes in soft magnetic materials proposed by Jiles and Atherton still remains one of the most widely used models. The Jiles-Atherton (J-A) hysteresis model has several attractive advantages, including the facts that only five physically related material model parameters are required, and that parameter determination can be accomplished by using only one measured hysteresis loop that runs far enough into saturation. Thus, it is desirable and convenient to apply the J-A model in magnetic field numerical calculations.

The identification of the parameters for the J-A hysteresis model turns out to be a difficult process. Many scholars are studying methods of parameter identification. There are two kinds of methods usually used for determining the five parameters of the J-A hysteresis model.

Optimization methods, applied to fit the Jiles–Atherton hysteresis loop model into measured data, have been investigated by many researchers. The nonlinear least-squares method, which is used for parameter fitting of classical and extended J–A models, is presented in [1]. A satisfactory result from optimization by the genetic algorithm is obtained in [2]. Parameters of the Jiles–Atherton hysteresis model are identified using a real coded genetic algorithm in [3]. In [4] and [5], the five parameters are identified by the stochastic optimization method “simulated annealing”, which is simple for application and appropriate to our research work.

2. Method

2.1 The J-A hysteresis model

The J-A hysteresis model is widely used for the modeling of the nonlinear characteristics of magnetic materials, especially for soft magnetic materials such as electrical steel. The description of the magnetization processes in soft magnetic materials proposed by Jiles and Atherton still remains one of the most widely used models. It is given in the form of a set of non-linear and first-order ordinary differential equations. Therefore, it is relatively simple to use for modeling hysteresis loops of soft magnetic materials. Table I shows the five model parameters and shows that they have a clear physical interpretation.

In the original J-A hysteresis model, the total magnetization \( M \) (Eq. (1)) of the ferromagnetic material is decomposed into an irreversible component \( M_{irr} \) (Eq. (2)) and a reversible magnetization component \( M_{rev} \) (Eq. (3)), as follows:

\[
M(t) = M_{irr}(t) + M_{rev}(t)
\]
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Table 1. Physical properties of J–A model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Physical property</th>
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</thead>
<tbody>
<tr>
<td>$M_s$</td>
<td>Saturation magnetization</td>
</tr>
<tr>
<td>$a$</td>
<td>Domain interaction</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Shape parameter of $M_{an}$</td>
</tr>
<tr>
<td>$c$</td>
<td>Reversibility coefficient</td>
</tr>
<tr>
<td>$k$</td>
<td>Linked to hysteresis loss</td>
</tr>
</tbody>
</table>

In the original J-A hysteresis model, the total magnetization $M$ (Eq. (1)) of the ferromagnetic material is decomposed into an irreversible component $M_{irr}$ (Eq. (2)) and a reversible magnetization component $M_{rev}$ (Eq. (3)), as follows:

$$ M = M_{rev} + M_{irr} $$  \hspace{1cm} (1)

The irreversible magnetization component $M_{irr}$ represents the energy loss caused by the domain wall translation and wall pinning:

$$ \frac{dM_{irr}}{dH} = \frac{M_{an} - M_{an}}{k\delta - \alpha (M_{an} - M_{rev})} $$  \hspace{1cm} (2)

whereas the reversible magnetization component $M_{rev}$ takes the form:

$$ M_{rev} = c(M_{an} - M_{rev}) $$  \hspace{1cm} (3)

The anhysteretic magnetization $M_{an}$ (i.e., hysteresis curve if there are no losses) follows the Langevin function:

$$ M_{an} = M_0 \left[ \coth \left( \frac{H + \alpha M}{a} \right) - \frac{a}{H + \alpha M} \right] $$  \hspace{1cm} (4)

The combination of Eqs. (1)–(3) leads to the total differential magnetization susceptibility $dM/dH$ which, in its original form, has derivatives with respect to $H$. This equation was reformulated into a differential equation in time by multiplying the left and the right sides by $dH/dt$, thus resulting in:

$$ \frac{dM}{dt} = (1-c) \frac{dH}{dt} \frac{(M_{an} - M)}{1 + k(\alpha M_{an} - M)} + c \frac{dM_{an}}{dt} $$  \hspace{1cm} (5)

The parameter $\delta$ has no physical meaning and equals +1 for $dH/dt > 0$ and -1 otherwise. Introducing a domain function in the Jiles–Atherton hysteresis model [6].

2.2 Simulated annealing

The name and inspiration of simulated annealing (SA) come from annealing in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects. The heat causes the atoms to become unstuck from their initial positions (local minimums of the internal energy) and wander randomly through states of higher energy; the slow cooling gives them more chances of finding configurations with lower internal energy than the initial one.

By analogy with this physical process, each step of the SA algorithm replaces the current solution with a random “nearby” solution, chosen with a probability that depends both on the difference between the corresponding function values and also on a global parameter $T$ (called the temperature), that is gradually decreased during the process. The dependency is such that the current solution changes almost randomly when $T$ is large, but the changes are increasingly “downhill” (for a minimization problem) as $T$ goes to zero. The allowance for “uphill” moves saves the method from potentially becoming stuck at local optima—which are the bane of greedier methods.

In the SA method, each point of the search space is analogous to a state of some physical system, and the function $E(s)$ to be minimized is analogous to the internal energy of the system in that state. The goal is to bring the system from an arbitrary initial state to a state with the minimum possible energy.

2.3 The process of parameter identification

In this paper, the J-A hysteresis model is mapped into a Simulink dynamic system, and a Matlab m-program controls the search for the best fitting parameters. The differential equation describing the J-A hysteresis model is given by Eq. (5). The anhysteretic magnetization $M_{an}$ is given by Eq. (4) using the Langevin function. For a given set of values of the J-A parameters ($M_s$, $a$, $\alpha$, $c$, $k$), a simulation is conducted and the calculated pairs ($B$, $H$) are collected with an appropriate sampling rate so as to allow for the comparison with the experimental data [5]. Fig. 1 shows the Simulink block diagram for the generation of hysteresis curves using the J-A hysteresis model.

![Simulink block diagram to generate hysteresis curves using the Jiles-Atherton model](image_url)
Identification of the Jiles-Atherton Model Parameters Using Simulated Annealing Method

As previously mentioned, the simulated annealing method is based on the principle of annealing. The basic idea of the simulated annealing method includes four parts:

① The optimization problem is compared to metal objects.
② The target function of the optimization problem is analogous to the energy of a metal object.
③ The solution of the optimization problem is regarded as the state of the matter (metal object).
④ The optimum solution is equivalent to the state with the lowest energy.

In this paper, the optimization problem is to find the best values for the five parameters of the J-A hysteresis model, which is used to describe the hysteresis characteristics of the 30ZH120 electrical steel sheet.

Eq. (6) shows the error function that is used. It is regarded as the target function of the optimization problem. It indicates the successful/unsuccessful moves based on the quadratic error between the experimental pairs (B, H) and the pairs obtained from the J-A hysteresis model:

\[
Error = \left[ \frac{1}{N} \sum_{\text{samples}} \left( B_{\text{experimental}} - B_{\text{model}} \right)^2 \right]^{\frac{1}{2}}
\]

where \( N \) is the number of experimental samples, and \( B_{\text{experimental}} \) is the experimental value of \( B \), and \( B_{\text{model}} \) is the corresponding value of \( B \) calculated by the J-A hysteresis model, for each sample. The solution of the problem is

3. Results

First, we need to generate a (B, H) curve by the J-A hysteresis model itself, and then run the parameter identification program using this “experimental hysteresis curve” in order to check if the extracted parameters match the given parameters well. We used a known vector of parameters, shown in the second column of Table 2, of a type of soft magnetic material to generate the (B, H) pairs. Then, we considered these pairs to represent the “experimental hysteresis curve” to be satisfied by the automatic Simulink program. The five parameters extracted by the program are shown in the third column of Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Original values</th>
<th>Extracted values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_s )</td>
<td>1.5e6</td>
<td>1.51e6</td>
</tr>
<tr>
<td>( a )</td>
<td>350</td>
<td>350.9</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>7.0e-4</td>
<td>7.05e-4</td>
</tr>
<tr>
<td>( c )</td>
<td>1.0e-3</td>
<td>0.998e-3</td>
</tr>
<tr>
<td>( k/\mu_0 )</td>
<td>265</td>
<td>264.86</td>
</tr>
</tbody>
</table>

Fig. 3 displays a high degree of consistency between the experimental curve and the fitted curve, and it shows that the parameters obtained by the program are in excellent agreement with the given parameters. All this demonstrates that the method in the paper is feasible and the Simulink program is effective.

![Fig. 2. Simulation diagram](image)

Fig. 2. Simulation diagram

![Fig. 3. Measured hysteresis curve and the best fitting curve generated by the Jiles-Atherton model](image)
In the experiment, the core material is 30ZH120 electrical steel sheet, its size is 300mm×30mm×0.3mm, and there are 112 slices, 14 slices in each solenoid. We take the Epstein-square to get a set of hysteresis loops, including a saturated hysteresis loop. A picture of the experimental site is shown in Fig. 4.

![Experimental site picture](image)

**Fig. 4.** Experimental site picture

This paper studies the effect of the joint number on the identification of parameters for the J-A hysteresis model, and discusses two lap ways of electrical steel sheets, which are shown in Fig. 5. Fig. 5(a) shows a 30°, 60° joint; and Fig. 5(b) shows a direct oblique lap, in which there is a seam lap in one side and no seam laps in the other three sides.

![Two kinds of overlapping forms of silicon steel](image)

**Fig. 5.** Two kinds of overlapping forms of silicon steel

Fig. 6 shows the modeled hysteresis loop calculated by the Simulink program that used the experimental saturated hysteresis loop for a magnetic core with 30°, 60° joints. The error is about four percent, which is acceptable.

![Fitted hysteresis loop for 30°, 60° joints](image)

**Fig. 6.** Fitted hysteresis loop for 30°, 60° joints

Fig. 7 shows the experimental saturated hysteresis loop when the magnetic core has a direct oblique lap. The upper left corner and lower right corner of the hysteresis loop are raised. This is caused by the modeling error increasing, but the result is still good.

![Experimental saturated hysteresis loop for direct oblique lap](image)

**Fig. 7.** Experimental saturated hysteresis loop for direct oblique lap

4. Conclusions

The objective of this paper was to show how the hysteresis parameters (M_s, a, α, c, k) can be determined from experimental hysteresis measurements, and then used to model the hysteresis curves using simulated annealing techniques, which were successfully implemented and tested. The identification program can be applied to an experimental (B, H) curve for 30°, 60° joints or for direct oblique lap, and tested to see whether the extracted parameters generate a curve close to the experimental one. It has been shown that the method described is capable of determining the values of these parameters within an error of a few percent. A comparison of measured and modeled hysteresis loops has also been given, and the comparison showed excellent agreement between the modeled and measured curves when the magnetic core had 30°, 60° joints. Though the result for direct oblique lap is not as good, direct oblique lap is not a commonly used way method, so there is little need for performing numerical calculations for applications that use direct oblique lap.

References


Baodong Bai received Doctor’s degree in electric machines and electric apparatus from Shenyang University of Technology. His research interests include electromagnetic fields and electromagnetic compatibility.

Jiayin Wang is studying in electric machines and electric apparatus of Shenyang University of Technology for doctor degree. Her research interests are magnetostriction and vibration and noise of transformer.