THE ACCELERATION AND TRANSPORT OF COSMIC RAYS
WITH HELIOSPHERIC EXAMPLES

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ABSTRACT

Cosmic rays are ubiquitous in space, and are apparently present wherever the matter density is small enough that they are not removed by collisions with ambient particles. The essential similarity of their energy spectra in many different regions places significant general constraints on the mechanisms for their acceleration and confinement. Diffusive shock acceleration is at present the most successful acceleration mechanism proposed, and, together with transport in Kolmogorov turbulence, can account for the universal spectra. In comparison to shock acceleration, statistical acceleration, invoked in many situations, has significant disadvantages. The basic physics of acceleration and transport are discussed, and examples shown where it apparently works very well. However, there are now well-established situations where diffusive shock acceleration cannot be the accelerator. This problem will be discussed and possible acceleration mechanism evaluated. Statistical acceleration in these cases is possible. In addition, a new mechanism, called diffusive compression acceleration, will be discussed and shown to be an attractive candidate. It has similarities with both statistical acceleration and shock acceleration.

Key words: cosmic rays – transport

I. INTRODUCTION

Cosmic rays are found in many places, including the heliosphere, galaxies and galaxy clusters. Figure 1 shows the observed spectrum of cosmic rays over the energy interval 10^9 – 10^20 eV, compiled from a number of sources (Jokipii, 1991). Apparent at low energies, below ≈ 1 GeV are a turnover and various other features caused by the interaction of cosmic rays with the sun and the solar wind. At low energies, observed only in situ, the spectrum is observed to merge smoothly into the background thermal plasma distribution.

Over the entire 11 decades between 10^9 and 10^20 eV we are presented with a smooth spectrum, with a small change in slope from a spectrum ∝ T^{-2.6} to ∝ T^{-3.1} at some 10^{16} eV (the "knee"), and possibly a small flattening at 10^{19} eV (the "ankle"). To explain the cosmic rays, we must have present both an accelerator which can produce a smooth spectrum over many decades and a transport/confinement mechanism which is also smooth. The transport and confinement are governed by scattering and subsequent diffusion in the turbulent interstellar magnetic field.

The change in energy ΔT in the time interval Δt may be written

\[ \Delta T = q \int_t^{t+\Delta t} \mathbf{w} \cdot \mathbf{E}(r, t) dt, \]  

where the integrand must be evaluated along the actual particle trajectory.

From this we see that in order to evaluate the energy change, we must know the particle trajectory in the electromagnetic field. This leads to the general requirements that acceleration and spatial transport be intimately coupled and both \(\mathbf{E}(r, t)\) and \(\mathbf{B}(r, t)\) must be considered together.

II. GENERAL CONSIDERATIONS

Quite generally, a cosmic-ray particle, as it gyrates in the turbulent magnetic field, interacts primarily with turbulent irregularities having scales close to its gyroradius. For galactic cosmic rays in the interstellar magnetic field the relevant scales range from a fraction of an AU to greater than a parsec. Over this range of scales, there is good observational evidence (Armstrong, et al., 1978) that the spectrum of the turbulence is a Kolmogorov power law over many decades in wavenumber, covering the range of scales relevant to cosmic rays, and this presumably is the cause of the smooth transport and confinement (Jokipii, 2001). The Kolmogorov power law is ubiquitous in fluid turbulence and this, presumably, is the mechanism producing the required small turbulence.

This leaves the acceleration mechanism to be determined. A number of different mechanisms have been proposed. Of these, diffusive shock acceleration of charged particles at collisionless astrophysical shocks has emerged as the most viable, primarily because it quite naturally and robustly produces a near-universal spectrum close to that observed, and because it is efficient. Such acceleration at supernova blast waves has emerged as the most-attractive accelerator for the bulk of the observed galactic cosmic rays below the knee.
the fluctuation spectrum of the magnetic field. Kolmogorov turbulence in the interstellar medium, which is smooth over a broad range of relevant length scales and which seems to be ubiquitous in large-scale systems, is probably responsible for the transport which is a smooth function of energy.

(b) Acceleration

A variety of acceleration mechanisms have been proposed. If the turbulent irregularities which scatter the cosmic rays move randomly relative to the flow (i.e. forward and backward moving Alfvén waves), acceleration is introduced caused by the associated diffusion in momentum as the particles gain and lose energy. This is essentially the well-known 2nd-order Fermi acceleration, introduced by Fermi (1949). Fermi acceleration (currently also called statistical acceleration) may be added as an additional term in equation (4), which may be written

$$\left( \frac{\partial f}{\partial t} \right)_{\text{2nd-order Fermi}} = \frac{1}{p^2} \frac{\partial}{\partial p} \left\{ p^2 D_{pp} \frac{\partial f}{\partial p} \right\}$$

(3)

and which represents momentum diffusion with coefficient $\Delta_{pp}$. This kind of acceleration is often invoked as an alternative to diffusive shock acceleration where shocks are believed not to be present. However, it is very important to note that a model which uses this mechanism to accelerate particles quite generally produces a spectrum which, far from being the observed universal power law, does not in general even produce a power law. Moreover, if parameters are chosen to produce a power law, the index of the power law in many cases has a sensitive dependence on poorly known quantities which, moreover, must vary considerably from location to location and event to event (see, e.g., Syrovatsky, 1961).

IV. DIFFUSIVE SHOCK ACCELERATION OF ENERGETIC CHARGED PARTICLES

The transport equation (2) may be applied to a collisionless shock. The resulting spectrum has the remarkable property that, quite generally, it produces a power law spectrum in momentum with an index which is close to that observed and which is essentially independent of the parameters.

Consider a steady, plane shock propagating in a uniform medium. Define the $x$-direction as the direction of propagation and let particles be introduced uniformly and steadily at the shock, at an injection momentum $p_0$. Work in the shock-normal coordinate system, with the shock at the fixed position $x = x_{sh}$. The shock ratio $r$ is defined as the ratio of upstream to downstream flow speed $U_1/U_2$. It is readily found that the steady solution to the Parker equation in this case is given by

$$f(p) = A p^{-\frac{\nu}{r-\nu}} H(p - p_0) F(x, p)$$

(4)
where \( H(p) \) is the Heaviside step function and the function \( F(x, p) \) is independent of \( p \) at the shock, is independent of \( x \) behind the shock, and decreases exponentially upstream as \( \exp(-U_{1}(x, p_{s})/\kappa_{xx}(p)) \). Note that in the limit of a strong shock, where \( r \to 4 \) the momentum dependence becomes \( f(p) \propto p^{-4} \), which corresponds to an energy spectrum \( dJ/dE \propto p^{2} f \propto p^{-2} \) which, if steepened somewhat due to energy-dependent transport and loss from the galaxy, is not far from the observed spectrum at relativistic energies (e.g., figure 1). This power law spectrum is independent of shock speed, diffusion coefficients and other parameters. In general, there will be a high-energy cutoff and, in some cases, the accelerated particles will modify the shock. Since the shocks of interest in cosmic-ray acceleration are generally strong, \( r \) is not far from 4. This mechanism therefore provides a universal power-law energy spectrum.

V. THE ACCELERATION RATE

The acceleration by the shock is not instantaneous, of course. The time to accelerate particles to a given energy is finite and, in contrast to the power-law index, depends significantly on the various parameters such as \( \kappa_{xx} \), etc. Hence, the cutoff energy may vary considerably in different places and for different sources. Solving the time-dependent version of equation (4), with the injection at momentum \( p_{0} \) turned on at a time \( t_{0} \), reveals that the spectrum above \( p_{0} \) is still the universal power law given in equation (6), but with a high-momentum cutoff, \( p_{c} \) which increases at a rate (Forman and Mollif, 1979)

\[
dp{p_{c}}{t} \approx 4U_{1}^{2} p_{c}/\kappa_{xx}.
\]  

Clearly, a larger shock speed or a smaller \( \kappa_{xx} \) will accelerate particles faster.

The diffusion coefficient normal to the shock front is \( \kappa_{xx} = \kappa_{||} \cos^{2}(\theta_{B}) + \kappa_{\perp} \sin^{2}(\theta_{B}) \), where \( \theta_{B} \) is the angle between the shock normal and the magnetic vector. Hence, quasi-perpendicular shocks will in general accelerate particles faster than will quasi-parallel shocks, since \( \kappa_{\perp} \) is usually significantly smaller than \( \kappa_{||} \).

Consider first a parallel shock. From (3) we may write

\[
1 \frac{dp_{c}}{dt} = \frac{U_{sh}^{2}}{4\kappa_{||}} = 3U_{sh}^{2}/4\lambda_{||} w
\]  

Clearly, \( \lambda_{||} \approx r_{g} \), in which case we have,

\[
\left( \frac{1}{p_{c}} \frac{dp_{c}}{dt} \right)_{B} = \frac{U_{sh}^{2}}{4r_{g} w}
\]  

which is known as the “Bohm limit” for the rate of acceleration.

Next consider the perpendicular shock, for which \( \kappa_{xx} \) becomes \( \kappa_{\perp} \). In the case of simple “billiard-ball” scattering we have

\[
\frac{\kappa_{\perp}}{\kappa_{||}} = \frac{1}{1 + (\lambda_{||}/r_{g})^{2}}
\]  

so that as \( \lambda_{||} \) becomes larger, \( \kappa_{\perp} \) becomes smaller and the rate of acceleration becomes larger. Hence, for a perpendicular shock, a longer scattering mean free path may give more rapid acceleration, in contrast to case of a parallel shock, where a short mean free path leads to more rapid acceleration. This makes the point that the physics of acceleration at quasi-perpendicular shocks is quite different than that at quasi-parallel shocks, as illustrated in figure (2). Note that the simple billiard ball scattering model is used to illustrate the general effect. Transport in a turbulent plasma is generally more complex (e.g., Giacalone and Jokipii, 1999).

One important aspect of perpendicular shocks is that \( \lambda_{||} \) cannot become too large (or \( \kappa_{\perp} \) become too small). If this were to occur, the diffusion approximation would become invalid. Hence, in contrast to quasi-parallel shocks where the particles must be scattered to return to the shock, acceleration at perpendicular shocks is not so clearly related to scattering, and one must impose an additional constraint to ensure that...
the diffusion approximation is valid. The associated maximum allowable value of \( \lambda_\parallel \) can be obtained from a number of different considerations Jokipii (1987), all of which lead to the same conclusion. One may require that the particle scatter often enough to keep the distribution function isotropic at the shock or that the particles diffuse upstream fast enough to stay ahead of the shock. Each of these leads to the condition

\[
\mathcal{T} = U_{\text{shock}} \frac{r_g}{\kappa_L} \ll 1, \quad (9)
\]

Hence, we can have a significant enhancement above the Bohm rate, which is the limit for parallel shocks. For particles whose speed is nearer the shock speed, which is true for nearly thermal particles, the possible enhancement in acceleration rate will be small.

Diffusive shock acceleration has so many attractive aspects — it is quite fast (especially at quasi-perpendicular shocks), it naturally produces a power-law energy spectrum which is quite close to that observed in many places, and the shocks which can do the acceleration are quite common — that it is regarded by many as possibly the only important acceleration mechanism. Certainly, it is highly likely that most galactic cosmic rays are accelerated at supernova shock waves by this mechanism.

VI. THE LIMITING ENERGIES IN DIFFUSIVE SHOCK ACCELERATION

Related to the acceleration rate is the question of the maximum energy attainable. For example, can supernovae accelerate cosmic rays to the knee?

This aspect of shock acceleration was explored in detail by Lagage and Cesarsky (1983). They considered the maximum energy which might be obtained by this mechanism in a supernova blast wave, but only for the case of a parallel shock. They suggested that the minimum value of the diffusion coefficient for a particle of speed \( w \) and gyroradius \( r_g \) is the Bohm value, so that the acceleration rate may be approximated by that in equation (7). They then considered equation (5) for a supernova blast wave using the modified Sedov solution for \( U_1 \). In this procedure they found a “firm upper limit” of a few times \( 10^{14} \) eV on the maximum energy obtainable in a typical supernova blast wave. This clearly would be a very severe constraint on the allowed \( Z_\star \), as conventional wisdom states that the particles below the knee at \( \approx 3 \times 10^{15} \) eV, where the slope of the spectrum changes, comes from supernova shock waves. This picture clearly has difficulties in the above scenario, particularly since the mean free path is likely to exceed the gyro-radius by a considerable amount.

However, this analysis does not apply to quasi-perpendicular shocks, and since a spherical shock is quasi-perpendicular over much of its area, \( \kappa_L \) plays a more-important role over much of the shock than \( \kappa_\parallel \). The acceleration can be much faster than the Bohm limit, alleviating the above problem. Hence the maximum energy at a supernova shock wave can be significantly larger than the \( 10^{14}Z \) discussed by Lagage and Cesarsky.

Consider equation (5) for a perpendicular shock. To obtain the maximum energy, we set \( \kappa_L \) equal to its minimum value by setting \( \mathcal{T} \) from equation (8) at about 3. This leads to a revised maximum energy for diffusive shock acceleration at the perpendicular supernova blast wave at about

\[
T_{\text{max}} \approx 5 \times 10^{16} \text{ZeV}. \quad (10)
\]

This is ample large enough to accommodate the interpretation in which the knee occurs when the supernova acceleration mechanism ceases. The actual energy at which this occurs at a supernova will be determined by the scattering, and will in general be significantly less than \( T_{\text{max}} \).

VII. COMPRESSION ACCELERATION, AN ALTERNATIVE

It may be shown (Giacalone, et al, 2001, 2004) that accelerations at compressions in the plasma flow which are more gradual than shocks can accelerate particles quite effectively.

Consider one-dimensional flow having a gradual compression with a characteristic length scale \( L_c \). In the limit where the ratio of the diffusive skin depth \( L_d = \kappa_{\text{eff}} / U_z \) to the length scale \( L_c \) is large, or, equivalently,

\[
\xi = \frac{L_d}{L_c} = \frac{k_{\text{eff}}}{U_d L_c} \gg 1 \quad (11)
\]

the solution to equation (1) for the cosmic-ray distribution \( f \) goes over to the standard diffusive shock solution.

In the opposite limit \( \xi \ll 1 \), the cosmic rays are closely tied to the convecting fluid, and simply compress adiabatically by the compression factor with the rest of the gas, and the acceleration negligible.

However, for a broad range of parameters \( \xi \) of the order or greater than unity, and we may expect significant acceleration.

This concept was shown by Giacalone, et al (2002) to explain quite naturally some puzzling observations in the inner Heliosphere, where significant acceleration was found in association with co-rotating gradual (non-shock) compressions in the solar wind. A numerical model incorporating gradual compressions chosen to fit the observed solar wind at the time of the observations fit the time profile and energy spectrum very well, with essentially no free parameters. The evidence against shock acceleration in this case was quite convincing. Statistical acceleration has also been invoked here (Schwadron, et al, 1996), but with a significant number of free parameters.
VIII. TURBULENT COMPRESSIONS

This compression acceleration may be regarded as a new acceleration mechanism, sharing properties with both diffusive shock acceleration and statistical acceleration. I suggest the term "diffusive compression acceleration".

To illustrate the nature of diffusive compression acceleration, consider the simple periodic one-dimensional velocity profile \( U_x(x) = U_0(1 + a \sin(kx)) \). This corresponds to a uniform velocity \( U_0 \) upon which is superimposed a sinusoidal compression variation with relative amplitude \( a \). For simplicity, we assume that the spatial diffusion coefficient, \( \kappa_{xx} \), associated with the particle motions is independent of \( x \) or \( p \). We define the dimensionless variables \( \chi = (U_0/\kappa_{xx})x \), \( \tau = (U_0^2/\kappa_{xx})t \) and \( \eta = (\kappa_{xx}/U_0)k \). Equation 1 then becomes,

\[
\frac{\partial f}{\partial \tau} = \frac{\partial^2 f}{\partial \chi^2} - (1 + a \sin(\eta \chi)) \frac{\partial f}{\partial \chi} + a \frac{\eta}{3} \cos(\eta \chi) \frac{\partial f}{\partial \ln(p)} + Q - L. \tag{12}
\]

This is simple to solve numerically, and the solutions depend only on the parameters \( \eta \) and \( a \). The solutions are clearly periodic in \( \chi \) with a period \( 2\pi/\eta \).

Because of the sinusoidal velocity, compression is exactly balanced by expansion, so the average of the acceleration rate is zero at any given momentum \( p \). Nonetheless, there is a net acceleration of the particles.

Illustrated in Figure 3 is the initial rate of acceleration \( d\ln(p)/d\tau \), averaged over \( x \) and plotted as a function of normalized wavenumber \( \eta \), for the case where the parameter \( a = 0.6 \), which corresponds to a ratio of maximum density (or velocity) to minimum density (or velocity) of 4.

It is apparent from Figure 3 that the acceleration rate decreases rapidly for wavenumber less than 1 (when the particles are tightly coupled to the local flow), and asymptotically approaches a constant which is about 0.25 for larger wavenumbers (when the diffusion becomes more important). A net acceleration occurs in spite of the balancing of compression and expansion.

The acceleration rate is of order unity in the case where \( \eta \) is approximately unity or larger, and is small for small values of \( \eta \) because then the diffusion is too slow. The fact that the rate approaches a constant for large \( \eta \) can be understood as follows. The particles spend a time of the order of \( 1/(k^2 \kappa_{xx}) \) in a region of compression and the acceleration rate there is of order \( U_0^2 a/\kappa_{xx} \). The mean-square change of the logarithm of momentum in one interaction is then \( (U_0^2 a/(k \kappa_{xx}))^2 \).

Then the appropriate Fokker-Planck coefficient is

\[
\frac{<(\Delta \ln(p))^2>}{\Delta t} \approx U_0^2 a^2 / \kappa_{xx}. \tag{13}
\]

which is independent of \( k \) and is of the order of \( a^2 \) in the normalized time defined above.

Figure 4 shows the energy spectrum obtained when the system is not periodic, but a simple "leaky box", where there are diffusive loss boundaries at \( x = \pm 15 \) and \( \eta = 2 \). The velocity is now of the form \( U(x,t) = a \sin(\eta x - t) \), where \( a = 0.6 \), which is essentially the same as the periodic system used above, but which corresponds to a propagating wave. Because the loss is due to the same diffusion which is involved in the acceleration, the spectral slope is less dependent on the parameters.

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**Fig. 3.**—Plot of the dimensionless acceleration rate for periodic compressions, defined in equation 12, plotted vs dimensionless wave number \( \eta \), for the case \( 1 = 0.6 \).

**Fig. 4.**—Plot of the steady-state energy spectrum \( dj/dt \) in arbitrary units vs. momentum in units of the injection momentum, for the case where the dimensionless wavenumber \( \eta = 2 \) and the loss is due to diffusion through a free-loss boundary at \( x = \pm 15 \). The dotted line shows a \( 1/p^2 \) spectrum for comparison.
(a) High-Energy Tails in Heliospheric Pickup-Ion Distributions

Gloeckler et al. (2000) reported on observations of interstellar pickup ions and found that there is always a high-energy tail connected to the freshly-ionized portion of the distribution. The origin of these tails is poorly understood. Here we suggest that acceleration by compression acceleration in the turbulent solar wind may help explain this. We present only a qualitative argument in this paper. A future paper will address this issue more quantitatively.

Gloeckler et al. (1995) reported on observations of pickup-ion distributions in the solar wind and found that their scattering in the turbulent magnetic fluctuations was considerably weaker than theoretical predictions. The inferred mean-free path is of the order of 2 AU. Thus, the associated parallel diffusion coefficient of the pickup ions (moving in the fast solar wind) may be as large as $\kappa_{\parallel} \approx 8 \times 10^{20} \text{cm}^2/\text{s}$. Thus, the associated diffusive skin depth is $\approx 10^{11} \text{cm}$. This is considerably larger than the scale of turbulent fluctuations in the solar wind. Thus, the pickup ions should be accelerated via diffusive compression acceleration as discussed in section 3.2. This may be responsible for the suprathermal tails seen in the pickup-distributions reported by Gloeckler et al. (2000).

The time scale for acceleration may be approximated

$$\tau_{\text{acc}} \approx \frac{\kappa}{U^2} \quad (14)$$

where $U_c$ is the speed of the compression in the frame co-moving with the solar wind. We assume that turbulent velocity fluctuations in the solar wind move at the magnetosonic speed which is about 60 km/s at 1 AU. Moreover, taking kappa to be $8 \times 10^{20} \text{cm}^2/\text{sec}$ for pickup ions, we obtain $\tau_{\text{acc}} \approx 240$ days. This is clearly to long to accelerate pickup ions at 1 AU since the cooling time is considerably shorter (a few days).

(b) Galactic Cosmic Rays

In the interstellar medium, where the diffusion coefficient is typically $\gtrsim 10^{26} \text{cm}^2/\text{sec}$, and typical fluid velocities are $\approx 100$ km/sec or so, scales of several parsecs to tens of parsecs can correspond to $\xi \gtrsim 1$. Again, we expect significant compressive fluid fluctuations on these scales. So these variations may contribute to the acceleration of energetic particles.

Compressive variations on these scale are clearly not shocks, but the above arguments suggest that disturbances having these scales may be efficient accelerators. Any resulting acceleration, to be of interest, must balance competing processes such as loss from the system and overall adiabatic cooling such as that in the solar wind.

In the interstellar medium, we expect turbulent velocity fluctuations to move at a speed of about 30 km/s. For a diffusion coefficient of $\kappa \approx 10^{26} \text{cm}^2/\text{sec}$, Equation 6 gives for the acceleration time $\tau_{\text{acc}} \approx 350,000$ years. This is shorter than the time scale associated with escape from the Galaxy. Thus, diffusive compression acceleration should be an effective acceleration mechanism in the interstellar medium. This possibility has not yet been explored quantitatively.

IX. SUMMARY AND DISCUSSION

Shock acceleration provides a natural explanation of most observed cosmic rays. Acceleration also occurs where shocks cannot do the job. Statistical acceleration (2nd-order Fermi) is less attractive because it can depend significantly on varying and unknown parameters. Compression acceleration is a new mechanism which might accelerate particles in the absence of shocks.

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