FINITE TEMPERATURE EFFECTS ON SPIN POLARIZATION OF NEUTRON MATTER IN A STRONG MAGNETIC FIELD

ALEXANDER A. ISAYEV and JONGMANN YANG

1 Kharkov Institute of Physics and Technology, Academicheskaya Street 1, Kharkov, 61108, Ukraine
1 Kharkov National University, Svobody Sq., 4, Kharkov, 61077, Ukraine
E-mail: isayev@kipt.kharkov.ua
2 Department of Physics and the Institute for the Early Universe, Ewha Womans University, Seoul 120-750, Korea
E-mail: jyang@ewha.ac.kr
(Received August 16, 2010; Revised September 03, 2010; Accepted September 09, 2010)

ABSTRACT

Magnetars are neutron stars possessing a magnetic field of about $10^{14}$-$10^{15}$ G at the surface. Thermodynamic properties of neutron star matter, approximated by pure neutron matter, are considered at finite temperature in strong magnetic fields up to $10^{18}$ G which could be relevant for the inner regions of magnetars. In the model with the Skyrme effective interaction, it is shown that a thermodynamically stable branch of solutions for the spin polarization parameter corresponds to the case when the majority of neutron spins are oriented opposite to the direction of the magnetic field (i.e. negative spin polarization). Moreover, starting from some threshold density, the self-consistent equations have also two other branches of solutions, corresponding to positive spin polarization. The influence of finite temperatures on spin polarization remains moderate in the Skyrme model up to temperatures relevant for protoneutron stars. In particular, the scenario with the metastable state characterized by positive spin polarization, considered at zero temperature in Phys. Rev. C 80, 065801 (2009), is preserved at finite temperatures as well. It is shown that, above certain density, the entropy for various branches of spin polarization in neutron matter with the Skyrme interaction in a strong magnetic field shows the unusual behavior, being larger than that of the nonpolarized state. By providing the corresponding low-temperature analysis, we prove that this unexpected behavior should be related to the dependence of the entropy of a spin polarized state on the effective masses of neutrons with spin up and spin down, and to a certain constraint on them which is violated in the respective density range.

Key words : Neutron star models — magnetar — neutron matter — Skyrme interaction — strong magnetic field — spin polarization — finite temperature

1. INTRODUCTION

Magnetars are strongly magnetized neutron stars (Duncan & Thompson 1992) with the magnetic field strength at the surface of about $10^{14}$-$10^{15}$ G (Duncan & Thompson 1996; Ibrahim et al. 2002). Such a huge magnetic field can be inferred from observations of magnetar periods and spin-down rates, or from hydrogen spectral lines. Among various classes of neutron stars, soft gamma-ray repeaters and anomalous X-ray pulsars are believed to be the most probable candidates for these ultrastrong magnetized astrophysical bodies (Woods & Thompson 2006). Magnetars are relatively frequent objects in the Universe, and comprise about 10% of the whole population of neutron stars (Kouveliotou et al. 1998). In the interior of a magnetar, the magnetic field strength may be even larger, reaching values of about $10^{18}$ G (Chakrabarty, Bandyopadhayay, & Pal 1997; Broderick, Prakash, & Lattimer 2000; Cardall, Prakash, & Lattimer 2001; Perez-Garcia 2008; Isayev & Yang 2009), which are inaccessible in the terrestrial laboratories.

In a recent study by Perez-Garcia (2008), neutron star matter was approximated by pure neutron matter, in a model with the effective nuclear forces. In the high density region, and in the presence of a strong magnetic field, it has been shown that the behavior of the neutron matter spin polarization depends crucially on whether neutron matter develops a spontaneous spin polarization (in the absence of a magnetic field) at several times nuclear matter saturation density, or the appearance of a spontaneous polarization is not allowed at the considered densities (or delayed to much higher densities). The first case is usual for the Skyrme forces (Rice 1969; Silverstein 1969; Østgaard 1970; Viduarre, Navarro, & Bernabeu 1984; Reddy et
al. 1999; Akhiezer, Laskin, & Peleinsky 1996; Marcos et al. 1991; Maruyama & Tatsumi 2001; Beraudo et al. 2004; Kutschera & Wojcik 1994; Isayev 2003; Isayev & Yang 2004a; Rios, Polls, & Vidaña 2005; Isayev 2006), while the second one is characteristic for the realistic nucleon-nucleon (NN) interaction (Pandharipande, Garde, & Srivastava 1972; Bäckman & Källman 1973; Haensel 1975; Vidaña, Polls, & Ramos 2002; Fantoni, Sarsa, & Schmidt 2001; Sammarruca & Krastev 2007; Bordbar & Bigdeli 2007). In the former case, a ferromagnetic transition to a totally spin polarized state occurs, while in the latter case a ferromagnetic transition is excluded at all relevant densities and the spin polarization remains quite low, even in the high density region. If a spontaneous ferromagnetic transition is allowed, in a subsequent model consideration with the Skyrme effective forces (Isayev & Yang 2009) it has been shown that the self-consistent equations for the spin polarization parameter, at nonzero magnetic field, have not only solutions corresponding to negative spin polarization (with the majority of neutron spins oriented opposite to the direction of the magnetic field), but also solutions with positive spin polarization (because of the strong spin-dependent medium correlations in the high-density region). In the last case, the formation of a metastable state with the majority of neutron spins oriented along the magnetic field is possible in the high-density interior of a neutron star.

In this study, we extend our previous consideration of neutron matter with Skyrme forces in a strong magnetic field, given at zero temperature (Isayev & Yang 2009), to finite temperatures up to a few tens of MeV being relevant for protoneutron stars. We use a Fermi liquid approach for the description of nuclear matter (Akhiezer et al. 1994, 1997; Isayev & Yang 2006) as a framework for consideration. One of the goals of the research is to study the impact of finite temperatures on spin polarized states in neutron matter in a strong magnetic field, and, in particular, to clarify whether or not the formation of a metastable state with positive spin polarization is possible in the high-density region of neutron matter. Moreover, we provide a fully self-consistent finite temperature calculation of the basic thermodynamic functions of spin polarized neutron matter in a strong magnetic field. It will be shown that, starting from some density, the entropy for various branches of spin polarization in neutron matter in a strong magnetic field shows an unusual behavior: it is larger than that of nonpolarized neutron matter. We relate this unexpected result to the dependence of the entropy on the effective masses of neutrons with spin up and spin down, and to the violation of a certain constraint on them in the corresponding density range.

Note that we consider thermodynamic properties of spin polarized states in neutron matter in a strong magnetic field up to the high density region relevant for astrophysics. However, we take into account the nucleon degrees of freedom only, although other degrees of freedom, such as pions, hyperons, kaons, or quarks could be important at high densities.

2. BASIC EQUATIONS

In what follows, we only present the basic equations useful in this work; more details concerning a Fermi-liquid approach to neutron matter in a strong magnetic field can be found in our earlier article (Isayev & Yang 2009). The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons \( f_{\epsilon,\sigma} = \text{Tr} \, g_{\epsilon,\sigma} \), where \( \epsilon \equiv (\mathbf{p}, \sigma) \), \( \mathbf{p} \) is the momentum, \( \sigma \) is the projection of spin on the third axis, and \( \rho \) is the density matrix of the system. Later on we will assume that the third axis is directed along the external magnetic field \( \mathbf{H} \). Given the possibility for alignment of neutron spins along or opposite to the magnetic field \( \mathbf{H} \), the normal distribution function of neutrons and single particle energy can be expanded in Pauli matrices \( \sigma_i \) in spin space as

\[
\begin{align*}
    f(\mathbf{p}) &= f_0(\mathbf{p}) \sigma_0 + f_3(\mathbf{p}) \sigma_3, \\
    \epsilon(\mathbf{p}) &= \epsilon_0(\mathbf{p}) \sigma_0 + \epsilon_3(\mathbf{p}) \sigma_3.
\end{align*}
\]

Expressions for the distribution functions \( f_0, f_3 \) in terms of the quantities \( \epsilon \) are (Isayev 2003; Isayev & Yang 2004a)

\[
\begin{align*}
    f_0 &= \frac{1}{2} \{ n(\omega_+) + n(\omega_-) \}, \\
    f_3 &= \frac{1}{2} \{ n(\omega_+) - n(\omega_-) \}.
\end{align*}
\]

Here \( n(\omega) = \{ \exp(\gamma \omega) + 1 \}^{-1} \) and

\[
\begin{align*}
    \omega_\pm &= \xi_0 \pm \xi_3, \\
    \xi_0 &= \epsilon_0 - \mu_0, \xi_3 = \epsilon_3,
\end{align*}
\]

\( \mu_0 \) being the chemical potential of neutrons. The branches \( \omega_\pm \) of the quasiparticle spectrum correspond to neutrons with spin up and spin down.

The distribution functions \( f \) should satisfy the normalization conditions

\[
\begin{align*}
    \frac{2}{\mathcal{V}} \sum_{\mathbf{p}} f_0(\mathbf{p}) &= \varrho, \\
    \frac{2}{\mathcal{V}} \sum_{\mathbf{p}} f_3(\mathbf{p}) &= \varrho_\uparrow - \varrho_\downarrow \equiv \Delta \varrho,
\end{align*}
\]

where \( \varrho = \varrho_\uparrow + \varrho_\downarrow \) is the total density of neutron matter, \( \varrho_\uparrow \) and \( \varrho_\downarrow \) are the neutron number densities with spin up and spin down, respectively. The quantity \( \Delta \varrho \) may be regarded as the neutron spin order parameter. It determines the magnetization of the system \( M = \mu_0 \Delta \varrho \), \( \mu_0 \) being the neutron magnetic moment. The magnetization may contribute to the internal magnetic field \( B = H + 4\pi M \). However, as we discussed earlier (Isayev & Yang 2009), and, as in Perez-Garcia
(2008) and Broderick, Prakash, & Lattimer (2000), we will assume that the contribution of the magnetization to the magnetic field $B$ remains small for all the relevant densities and magnetic field strengths; hence, $B \approx H$.

The self-consistent equations for the components of the single-particle energy have the form (Isayev & Yang 2009)

\[ \xi_0(p) = \xi_0(p) + \epsilon_0(p) - \mu_0, \]
\[ \xi_2(p) = -\mu_n H + \epsilon_3(p). \]  

Here $\xi_0(p) = \frac{p^2}{2m_0}$ is the free single particle spectrum, $m_0$ is the bare mass of a neutron, and $\epsilon_0, \epsilon_3$ are the Fermi liquid (FL) corrections to the free single particle spectrum; these are related to the normal FL amplitudes $U_0^n(k), U_1^n(k)$ by

\[ \tilde{\epsilon}_0(p) = \frac{1}{2\nu} \sum_q U_0^n(k)f_0(q), \quad k = \frac{p-q}{2}, \]
\[ \tilde{\epsilon}_3(p) = \frac{1}{2\nu} \sum_q U_1^n(k)f_3(q). \]  

To obtain numerical results, we utilize the effective Skyrme interaction (Vautherin & Brink 1972). Expressions for the normal FL amplitudes in terms of the Skyrme interaction (Vautherin & Brink 1972). Expressions for the distribution functions \( f_0 \), \( f_3 \), we obtain the self-consistent equations (6), (7) for the components of the single-particle energy $\xi_0(p)$ and $\xi_3(p)$, which should be solved together with the normalization conditions (4), (5).

Note that the spin ordering in neutron matter can be characterized by the neutron spin polarization parameter

\[ \Pi = \frac{\varphi_1 - \varphi_1}{\varphi_3} = \frac{\Delta \rho}{\rho}. \]

The number densities of neutrons with spin up and spin down are related to the spin polarization parameter $\Pi$ by

\[ \varphi_1 = \frac{\rho}{2}(1 + \Pi), \quad \varphi_3 = \frac{\rho}{2}(1 - \Pi). \]  

To examine the thermodynamic stability of different solutions of the self-consistent equations, it is necessary to compare the corresponding free energies $F = E - TS$, where the energy functional $E$ is characterized by two FL amplitudes $U_0^n, U_1^n$ (Isayev & Yang 2006) and the entropy is given by

\[ S = -\sum_p \sum_{\sigma = \pm} \left\{ n(\omega_\sigma) \ln n(\omega_\sigma) \right\}, \quad n(\omega) = 1 - n(\omega). \]  

3. SOLUTIONS OF SELF-CONSISTENT EQUATIONS AT FINITE $T$. THERMODYNAMIC FUNCTIONS

In the model with SLy4 and SLy7 Skyrme effective forces (Chabanat et al. 1998), the self-consistent equations were analyzed at zero temperature by Isayev & Yang (2009) for the magnetic field strengths up to $H_{\text{max}} \sim 10^{18}$ G, allowed by a scalar virial theorem (Lai & Shapiro 1991). These Skyrme parametrizations were originally constrained to reproduce the results of microscopic neutron matter calculations (pressure-versus-density curve) and give neutron star models in broad agreement with observables (such as the minimum rotation period, gravitational mass-radius relation, the binding energy, released in supernova collapse, etc., see Rikovska Stone et al. (2003)). It was shown that a thermodynamically stable branch of solutions for the spin-polarization parameter, as a function of density, corresponds to a negative spin polarization when the majority of neutron spins are oriented opposite to the direction of the magnetic field. Moreover, starting from some threshold density which depends on the magnetic field strength, the state with positive spin polarization can be realized as a metastable state in the high-density region of neutron matter, in the model with the Skyrme effective forces.

In this section, we study the impact of finite temperatures on spin polarization of neutron matter in a strong magnetic field. In particular, we are interested in whether or not the metastable state with positive spin polarization will survive at temperatures about several tens of MeV typical of protoneutron stars. To this end, we directly find solutions of the self-consistent equations at nonzero temperature. Since the results of calculations with SLy4 and SLy7 Skyrme forces are very similar, here we present only the obtained dependences for the SLy7 Skyrme interaction.

Fig. 1 shows the neutron spin polarization parameter as a function of density, normalized to the nuclear saturation density $\varrho_0$ (for SLy7 force, $\varrho_0 = 0.158\,\text{fm}^{-3}$), for a set of fixed values of the magnetic field strength at the temperature $T = 20$ MeV. The branches of spontaneous polarization $\Pi_0^\sigma, \Pi_0^\sigma$, corresponding to the vanishing magnetic field, are shown by the solid curves. As one can see, the obtained dependences are qualitatively similar to those obtained at zero temperature (Isayev & Yang 2009). The branches of spontaneous polarization are modified differently by the magnetic field: the branch $\Pi_0^\sigma(\varrho)$ turns to the branch $\Pi_1(\varrho)$ with negative spin polarization, while the branch $\Pi_2(\varrho)$ splits into two branches, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, corresponding to positive spin polarization. For the lower branch $\Pi_1(\varrho)$, there are three characteristic density domains. At low densities $\varrho \leq 0.5\varrho_0$, the magnitude of the spin polarization parameter increases with decreasing density. At intermediate densities $0.5\varrho_0 \leq \varrho \leq 3\varrho_0$, there is a plateau in the $\Pi_1(\varrho)$ dependence, whose characteristic
value depends on $H$ (at the given temperature). At densities $\varrho \gtrsim 3\varrho_0$, the absolute value of the spin polarization parameter increases with density and tends to unity. Note that, in the low-density domain, the possibility of the appearance of a nuclear magnetic pasta and its impact on the neutrino opacities in the protoneutron star early cooling stage should be explored in a more elaborated analysis as discussed in detail by Perez-Garcia (2008).

The upper branches $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, corresponding to a positive spin polarization, appear stepwise at the same threshold density $\varrho_{th}$ dependent on the magnetic field (at the given temperature), and being larger than the critical density of spontaneous spin instability in neutron matter. For the branch $\Pi_2(\varrho)$, the spin polarization parameter decreases with density and tends to zero, while for the branch $\Pi_3(\varrho)$ it increases with density and approaches unity. Because of the negative value of the neutron magnetic moment, the magnetic field tends to orient the neutron spins opposite to the magnetic field direction. As a result, the spin polarization parameter for the branches $\Pi_2(\varrho)$, $\Pi_3(\varrho)$ with positive spin polarization is smaller than that for the branch of spontaneous polarization $\Pi^s_0$; vice versa, the magnitude of the spin polarization parameter for the branch $\Pi_1(\varrho)$ with negative spin polarization is larger than the corresponding value for the branch of spontaneous polarization $\Pi^s_0$.

Thus, at densities larger than $\varrho_{th}$, we have three branches of solutions: one, $\Pi_1(\varrho)$, with negative spin polarization and the remaining two, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, with positive polarization. In order to establish which branch is thermodynamically preferable, we should compare the corresponding free energies. Fig. 2 shows the free energy per neutron as a function of density at $T = 20$ MeV and $H = 10^{18}$ G for these three branches, compared with the free energy per neutron for a spontaneously polarized state [the branches $\Pi^s_0(\varrho)$]. The state with the majority of neutron spins, oriented opposite to the direction of the magnetic field [the branch $\Pi_1(\varrho)$], has a lowest free energy. This result is intuitive, since the magnetic field tends to direct the neutron spins opposite to the magnetic field, as mentioned earlier. However, the state described by the branch $\Pi_3(\varrho)$ with positive spin polarization has the free energy very close to that of the thermodynamically stable state. This means that, despite the presence of a strong magnetic field $H \sim 10^{18}$ G, the state with the majority of neutron spins directed along the magnetic field can be realized as a metastable state in the dense core of a neutron star, in the model with the Skyrme effective interaction. In this scenario, since such states exist only at densities $\varrho \gtrsim \varrho_{th}$, under decreasing density (from the inner to the outer regions of a magnetar) a metastable state with positive spin polarization at the threshold density $\varrho_{th}$ changes to a thermodynamically stable state with negative spin polarization.

Note that the possibility of the appearance of a metastable state, with the majority of neutron spins oriented along the magnetic field, was missed in the finite temperature calculations by Perez-Garcia (2008). This is because only one branch of solutions for the spin polarization parameter, corresponding to the thermodynamically stable state, was considered in that study for neutron matter in a strong magnetic field, with the
is the Fermi-Dirac integral of the order $\nu$, $\rho_0$ is the effective chemical potential of neutrons with spin up ($\sigma = +$) and spin down ($\sigma = -$), whose explicit expression was derived by Isayev & Yang (2000). In the low-temperature limit $\eta_\sigma \gg 1$, and using the corresponding expansion for Fermi-Dirac integrals in Eq. (12), we obtain

$$s = \sum_{\sigma = +, -} s_\sigma, \quad s_\sigma = \frac{\pi^2}{2eF_{\sigma}} T. \quad (13)$$

In the previous equation $\varepsilon_\sigma = \frac{\hbar^2 k_\sigma^2}{2m_\sigma}$ is the Fermi energy of neutrons with spin up and spin down, and $k_\sigma = (6\pi^2 \rho_\sigma)^{1/3}$ is the respective Fermi momentum. The low-temperature expansion (13) is valid till $T/\varepsilon_{F,\sigma} \ll 1$. Then, requiring the difference between the entropies of spin polarized and nonpolarized states to be negative, we derive the constraint on the effective masses $m_{n1}$ and $m_{n1}$ of neutrons with spin up and spin down in a spin polarized state:

$$D \equiv \frac{m_{n1}}{m_n} (1 + \Pi_1)^{3/4} + \frac{m_{n1}}{m_n} (1 - \Pi_1)^{3/4} - 2 < 0. \quad (14)$$

Here $m_n$ is the effective mass of a neutron in nonpolarized neutron matter (Isayev & Yang 2004a),

$$\frac{\hbar^2}{2m_n} = \frac{\hbar^2}{2m_0} + \frac{g}{8} [t_1 (1 - x_1) + 3t_2 (1 + x_2)], \quad (15)$$

and the effective masses $m_{n1}, m_{n1}$ are given by (Isayev & Yang 2009)

$$\frac{\hbar^2}{2m_{1(l)}} = \frac{\hbar^2}{2m_0} + \frac{g_{1(l)}}{2} \left[ t_2 (1 + x_2) + \frac{g_{1(l)}}{4} [t_1 (1 - x_1) + t_2 (1 + x_2)]. \quad (16)$$

In Eqs. (15), (16), $t_i$ and $x_i$ are phenomenological parameters, which specify a given parametrization of the Skyrme interaction. According to Eq. (10), the number densities of neutrons with spin up and spin down are determined by the spin polarization parameter $\Pi$, which, after the self-consistent determination, is a function of the thermodynamic parameters $\rho, T, H$. Hence, (14) determines the region in the domain of admissible values of the thermodynamic parameters, where the entropy of a spin polarized state exhibits the expected behavior.

Fig. 4 shows the left-hand side $D$ of constraint (14) for the branches $\Pi_1-\Pi_3$ of spin polarization, as a function of density at $H = 10^{18}$ G and temperature $T = 5$ MeV, when the condition $\varepsilon_{F,\sigma}/T \gg 1$ holds true. For the branch $\Pi_1$, the difference $D$ is negative up to the density $\rho_\sigma \approx 0.41 \rho_0$ (see the insert in Fig. 4), where it changes sign and remains positive for all larger densities. Besides, for the branches $\Pi_2$ and $\Pi_3$, the difference $D$ is positive for all densities at which the corresponding solutions of the self-consistent equations
We have studied the impact of finite temperatures on the spin structure in the magnetar interior. We approximated the neutron star matter by pure neutron matter, and considered the Skyrme effective interaction as a first step towards a more realistic description of neutron stars, taking into account a finite fraction of protons with the charge neutrality and beta equilibrium conditions. In particular, some admixture of protons can affect the onset densities of enhanced polarization in neutron matter with the Skyrme interaction (Isayev & Yang 2004a). Nevertheless, at such strong magnetic fields one can expect that the proton fraction is relatively small, and even can completely disappear in the dense interior of a magnetar (Mao et al. 2003).

It is worth noticing that we approximated neutron star matter with pure neutron matter. This should be considered as a first step towards a more realistic description of neutron stars, taking into account a finite fraction of protons with the charge neutrality and beta equilibrium conditions. In particular, some admixture of protons can affect the onset densities of enhanced polarization in neutron matter with the Skyrme interaction (Isayev & Yang 2004a). Nevertheless, at such strong magnetic fields one can expect that the proton fraction is relatively small, and even can completely disappear in the dense interior of a magnetar (Mao et al. 2003).

4. CONCLUSIONS

We have studied the impact of finite temperatures on the spin structure in the magnetar interior. We approximated the neutron star matter by pure neutron matter, and considered the Skyrme effective interaction as a first step towards a more realistic description of neutron stars, taking into account a finite fraction of protons with the charge neutrality and beta equilibrium conditions. In particular, some admixture of protons can affect the onset densities of enhanced polarization in neutron matter with the Skyrme interaction (Isayev & Yang 2004a). Nevertheless, at such strong magnetic fields one can expect that the proton fraction is relatively small, and even can completely disappear in the dense interior of a magnetar (Mao et al. 2003).

It is worth noticing that we approximated neutron star matter with pure neutron matter. This should be considered as a first step towards a more realistic description of neutron stars, taking into account a finite fraction of protons with the charge neutrality and beta equilibrium conditions. In particular, some admixture of protons can affect the onset densities of enhanced polarization in neutron matter with the Skyrme interaction (Isayev & Yang 2004a). Nevertheless, at such strong magnetic fields one can expect that the proton fraction is relatively small, and even can completely disappear in the dense interior of a magnetar (Mao et al. 2003).

ACKNOWLEDGMENTS

J. Y. was supported by grant 2010-0011378 from Basic Science Research Program through NRF of Korea funded by MEST and by grant R32-2009-000-10130-0 from WCU project of MEST and NRF through Ewha Womans University.

REFERENCES

Akhiezer, A. I., Krasil’nikov, V. V., Peletminsky, S. V., & Yatsenko, A. A. 1994, Research on Superfluidity and Superconductivity on the Basis of the Fermi Liquid Concept, Phys. Rep., 245, 1

Fig. 4.— (Color online) The difference $D$ in constraint (14) for various branches $\Pi_1-\Pi_3$ of spin polarization as a function of density at $H=10^{18}\text{ G}$ and $T=5\text{ MeV}$ for the SLy7 interaction.
FINITE TEMPERATURE EFFECTS ON SPIN POLARIZATION


Isayev, A. A. 2003, Competition of Ferromagnetic and Antiferromagnetic Spin Ordering in Nuclear Matter, JETP Letters, 77, 251


