Terrain-Based Localization using Particle Filter for Underwater Navigation†

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Abstract

Underwater localization is a crucial capability for reliable operation of various types of underwater vehicles including submarines and underwater robots. However, sea water is almost impermeable to high-frequency electromagnetic waves, and thus absolute position fixes from Global Positioning System (GPS) are not available in the water. The use of acoustic telemetry systems such as Long Baseline (LBL) is a practical option for underwater localization. However, this telemetry network system needs to be pre-deployed and its availability cannot always be assumed. This study focuses on demonstrating the validity of terrain-based localization techniques in a GPS-denied underwater environment. Since terrain-based localization leads to a nonlinear estimation problem, nonlinear filtering methods are required to be employed. The extended Kalman filter (EKF) which is a widely used nonlinear filtering algorithm often shows limited performance under large initial uncertainty. The feasibility of using a particle filter is investigated, which can improve the performance and reliability of the terrain-based localization.

Keywords: Terrain-based localization, Underwater navigation, Particle filter, Extended Kalman filter

1. Introduction

Localization is a crucial capability for reliable operation of manned or unmanned vehicles including aerial vehicles, ground vehicles, surface ships and underwater vehicles. For vehicles operated in open outdoor environments (e.g., aerial vehicles, surface ships), position information can be acquired from the global positioning system (GPS), which provides absolute position fixes to correct drift errors from the inertial navigation system (INS). This integrated INS/GPS system is an ideal sensor combination and has been successfully applied to vehicle navigation applications. However, GPS signals are not available in the water, since sea water is almost impermeable to high-frequency electromagnetic waves. Instead, acoustic telemetry systems such as Long-Baseline (LBL) can be used for underwater localization and navigation. However, the use of an LBL system requires pre-deployment of a set of baseline transponders on the sea floor, and its coverage is relatively limited compared with that of GPS.

In this study, terrain-based localization that utilizes subsea terrain information is addressed, which enables accurate and reliable localization and navigation in a GPS-denied underwater environment without using acoustic telemetry systems. The terrain-based localization algorithm focuses on minimizing drift errors due to dead-reckoning or inertial navigation by using drift-free position fixes with respect to the terrain. A key limitation of this terrain-based approach is that it is a map-based approach and thus requires the availability of a database of seabed terrain.

The seminal work of this kind is the Terrain Contour Matching (TERCOM) algorithm [1] which was originally developed for cruise missiles and aircraft in the 1970s before GPS was fully opera-
ational. The algorithm is still regarded to be important since GPS is vulnerable to jamming, spoofing and other electronic interference.

Terrain-based localization requires representing an undulating terrain surface as a mathematical function, which results in a highly nonlinear estimation problem. The extended Kalman filter (EKF) which is perhaps the most widely used nonlinear filtering algorithm can be applied to the problem. However, it is well known that the EKF may not show satisfactory performance if a given system is highly nonlinear or has a large initial uncertainty.

This study suggests using a particle filter (PF) [2] in order to improve the performance and reliability of terrain-based localization, particularly under large initial uncertainty. To demonstrate the feasibility of the proposed approach, a set of numerical simulations were performed.

This paper is organized as follows. In Section 2, the particle filter algorithm is described. Then, in Section 3, the terrain-based localization problem is formulated into a nonlinear estimation problem. The results of numerical simulation are shown in Section 4, and finally the conclusion is given in Section 5.

2. Particle Filter

The particle filter is an approximation algorithm based on the Monte-Carlo method that describes uncertainty and a variety of hypothesis by many particles. The Monte-Carlo method requires lots of calculation to obtain reliable information. Thus, the use of particle filter was limited to relatively simple problems or offline applications. But as computing power increases through the development of computer technology, particle filter can be used for various areas. Also, particle filters are widely used as real-time estimator for the navigation of unmanned vehicles.

Above all, the strength of the particle filter is that the estimator can be constructed without mathematical approximations for the given system model and sensor model. For example, in case of EKF which is a widely used nonlinear filtering algorithm, the system model is linearized and its uncertainty probability distributions such as disturbance and noise are approximated to normal distributions. Some system characteristics may disappear due to this model simplification and approximation. Therefore, the EKF often shows unsatisfied performance when nonlinearity is strong and uncertainty cannot be effectively described as a normal distribution [3].

On the other hand, the particle filter does not need the system and measurement model simplification if those are defined mathematically. Thus, it is possible to construct a more realistic estimator model. However, the particle filter may be unsuitable for complex systems operating at high update frequencies because the number of particles needed for the filter is rapidly increasing with the system’s state dimension.

A pseudocode indicating the algorithm of the standard particle filter is shown in Table 1 [4].

| Algorithm Particle Filter ( | $\{\hat{x}_{k-1}^{i}, i\}_{i=1}^{N}, u_{i}, z_{k} \}$ |
|-----------------------------|
| Input posterior sample set at k-1: $\{\hat{x}_{k-1}^{i}, i\}_{i=1}^{N}$ |
| Input control and measurement at k: $u_{k}, z_{k}$ |
| N = number of particles |
| for $i = 1$ to $N$ |
| sample $\hat{x}_{k}^{i} = p(x_{k} | x_{k-1}^{i}, u_{k})$ |
| $w_{k}^{i} = p(z_{k} | \hat{x}_{k}^{i})$ |
| end |
| Normalize weights: |
| calculate total weight $W = \sum_{i=1}^{N} w_{k}^{i}$ |
| for $i = 1$ to $N$ |
| normalize $w_{k}^{i} := w_{k}^{i} / W$ |
| end |
| Resampling step: |
| for $i = 1$ to $N$ |
| draw $\hat{x}_{k}^{i}$ with probability $\propto w_{k}^{i}$ |
| end |
| Return posterior sample set at k: $\{\hat{x}_{k}^{i}, i\}_{i=1}^{N}$ |

Real-time filtering basically calculates a predicted value by integrating the system model at every time step and performs correction of prediction with given measurements. In case of particle
filter, the filter integrates the particle states to obtain the prior probability density function first,

\[ p_k^\sim = p(x_k \mid z_{k-1}, u_k) \]  

(1)

The likelihood of each particle is evaluated with the measurements.

\[ l_k = p(z_k \mid x_k) \]  

(2)

Based on the likelihood, the importance factor is assigned to each particle, and then the posterior probability density function is obtained through resampling.

\[ p_k^+ = p(x_k \mid z_k, u_k) \]  

(3)

Recursive filtering is performed by repeating the above steps.

The estimation is representative value calculated based on posterior probability density function. The weighted average is commonly used as the representative value.

3. Terrain-Based Localization

To design an estimator for terrain-based localization, a mathematical model of the system and measurement should be defined. In this section, the mathematical model is derived in state-space form.

3.1 System model

Terrain-based localization techniques can be used in various types of vehicles. In particular, in underwater applications the techniques are more useful owing to the difficulty of position information acquisition under the water surface.

In principle, underwater vehicles undergo six degrees-of-freedom (DOF) motions, so a series of procedures including hydrodynamics and Euler angle transformation are necessary for a complete description of the motion. In this study, focusing on the feasibility of using a particle filter, a 3 DOF kinematic model in the horizontal plane is used to describe the vehicle motions. The equation of motion at this state space is expressed as follows.

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi}
\end{bmatrix} = 
\begin{bmatrix}
V \cos\psi \\
V \sin\psi \\
r
\end{bmatrix} + w
\]  

(4)

where \( x \) and \( y \) are the position coordinates of the vehicle in the horizontal plane, \( \psi \) is the yaw angle, \( r \) is the yaw rate and \( V \) is the longitudinal velocity of the vehicle. \( u_1 \) and \( u_2 \) denote the control inputs of yaw angular acceleration and longitudinal acceleration, and \( n_w \sim \mathcal{N}(0, \sigma_w^2) \) is the zero-mean Gaussian process noise. This noise is assumed to contain uncertainties of the system model, control inputs and environmental disturbance such as wave and current.

3.2 Measurement model

Relative position of the underwater vehicle from terrain surface in underwater environment can be measured using acoustic sensors such as Multibeam Echosounder (MBE), Doppler Velocity Log (DVL) and altimeters [5]. This study supposes using a sonar altimeter which is the simplest one among those. Unlike MBE, a sonar altimeter can measure the relative distance from a single point on the seabed. Although the amount of information is minimal, the use of an altimeter is relatively easy due to its configurational simplicity. Basically, if the technique can be applied to a sonar altimeter, higher performance is expected with using MBE or DVL. The use of sonar altimeter is, therefore, a fair and conservative assumption in this study.

The employed measurement model is given below.

\[
\begin{bmatrix}
z_d \\
z_f \\
z_r
\end{bmatrix} = 
\begin{bmatrix}
h(x, y) \\
V \\
r
\end{bmatrix} + v
\]  

(5)

where \( z_d \) is the measured relative distance, which is a vertical distance from the vehicle to the seabed, and \( h \) is the function that represents the relation between the relative distance to the bottom and the horizontal position of the vehicle. \( z_f \) and
denote the measured longitudinal velocity and angular velocity. $n_v \sim N(0, \sigma_v^2)$ is the measurement noise which is assumed to be a zero-mean Gaussian distribution.

The function $h$ will have a complicated form and serious nonlinearity to express the curve of a sea bottom created by nature. Therefore, the linear approximation of this system depresses the reliability of estimate, and the filter may diverge or converge to a wrong value if the initial estimate is not sufficiently accurate in particular. The usefulness of the filter in terrain-based localization depends on how well such nonlinearity is dealt with.

### 3.3 Estimator application

An estimator is designed on the assumption of real-time operation and uses the particle filter algorithm introduced in Section 2. A control block diagram which represents the overall organization of the vehicle operation system is shown in Fig. 1.

The part shown in dotted line in this figure indicates the estimator which is the main subject in this study. As shown in the block diagram, the control input is obtained from the calculation between the output of the estimator and the reference input, which controls the motion of the vehicle. Because the control input directly affects the vehicle’s motion, the accuracy and reliability of the estimator output determine the overall control capacity of the vehicle.

### 4. Numerical Simulation

Numerical simulations were performed in order to quantitatively and qualitatively verify the validity of this study. The EKF simulation result is also presented for verification and comparison of the performance of the particle filter.

#### 4.1 Generating seabed terrain

A 200 X 200 grid map of virtual seabed terrain was generated for numerical simulation. The shape of generated sea bottom is shown in Fig. 2.

In the figure, artificial seabed terrain is generated to have a complex surface.

#### 4.2 Simulation scenario

In order to understand the robustness of filtering performance depending on the reliability and accuracy of the initial condition, two simulation scenarios are considered for both the EKF and the particle filter. One is under the small initial uncertainty and the other is under the large initial uncertainty.

Initial settings and experimental parameters are shown in Table 2 and Table 3.

#### Table 2 Initial settings and process noise parameters

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>$y$ (m)</th>
<th>$\psi$ (deg)</th>
<th>$r$ (deg/s)</th>
<th>$V$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Condition</td>
<td>40.0</td>
<td>40.0</td>
<td>45.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma _i n_i$: Initial Uncertainty</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$\sigma _w$: Process Noise</td>
<td>0.1</td>
<td>0.1</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

#### Table 3 Measurement noise parameters

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$V$ (m/s)</th>
<th>$r$ (deg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma _v$: Measurement Noise</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>
4.3 Experimental results

Estimated vehicle trajectories obtained using the proposed terrain-based localization algorithm are shown in Figs. 3-5. The results compare the simulation results of the EKF and the particle filter. Since different results can be obtained depending on which random numbers are used for simulations, each simulation uses the same pre-defined random number sequence for an objective performance comparison. These results are shown in Fig. 3 and Fig. 4. The vehicle moves from the initial position on bottom left ($x=40m$, $y=40m$) to top right in the figure.

First, actual and estimated vehicle trajectories when the initial position reliability is large are shown in Fig. 3. As shown, both estimates by the EKF and the particle filter show satisfactory results. The linear approximation of the EKF is reasonable under the small initial uncertainty.

However, the big difference in the result between the EKF and the particle filter under the large initial uncertainty can be found in Fig. 4. In Figure 4, the EKF yields wrong estimate which is different from the actual value and does not converge, which is believed to be due to the excessively large initial uncertainty. On the other hand, the particle filter, which first shows somewhat unstable result due to the large initial uncertainty, converges to the actual one after a while.

Figure 5 compares the position errors between the EKF and the PF for the simulation results with the large initial uncertainty, which clearly shows that the PF outperforms the EKF in terms of the filter’s robustness against initial uncertainty.

5. Conclusions

This study highlights the fact that terrain-based localization is a core technology for underwater vehicle operation and focuses on demonstrating the feasibility and validity of using the particle filter algorithm to improve the performance of terrain-based localization. It is confirmed that the particle filter can significantly improve the reliability of terrain-based localization techniques, particularly in the presence of large initial uncertainty.

The major effectiveness of the particle filter in contrast with the EKF comes from nonlinearity of the seabed terrain. Furthermore, if the measuring
characteristics (e.g., measurement errors with non-Gaussian distributions) of acoustic sensors used in terrain-based localization are considered, the advantage of particle filter will be greater.

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References


