Decode-and-Forward Relaying Systems with $N^{th}$ Best-Relay Selection over Rayleigh Fading Channels

Tran Trung Duy · Hyung-Yun Kong

Abstract

In this paper, we evaluate performances of dual-hop decode-and-forward relaying systems with the $N^{th}$ best-relay selection scheme. In some schemes, such as scheduling or load balancing schemes, the best relay is unavailable and hence the system must resort the second best, third best, or generally the $N^{th}$ best relay. We derive the expressions of the outage probability and symbol error rate (SER) for this scenario over Rayleigh fading channels. Monte-Carlo simulations are presented to verify the analytical results.

Key words: Cooperative Communication, Decode-and-Forward Relaying, Outage Probability, Rayleigh Fading Channel.

I . Introduction

Cooperative communication is well known to improve channel capacity and achieve higher diversity gain under fading environments when compared to direct transmission [1], [2]. Cooperative diversity protocols utilize dual-hop relaying scheme when the source has no direct link to the destination. For amplify-and-forward relaying schemes [3], [4], the relay just amplifies the signal received from the source and forwards the amplified signal to the destination. Decode-and-forward relaying schemes can use either the fixed relaying scheme [5] or the adaptive relaying scheme [6]. In the first strategy, the relay always detects and forwards the received signal to the destination. In contrast, the relay in the second scheme just sends the signal to the destination if it decodes it successfully; otherwise, it keeps silent.

Relay selection is an important issue for the improvement of system performance. Therefore, most of the methods related to relay selection have mainly focused on the best-relay selection. However, in some cases, the system cannot choose the best relay due to some scheduling or load balancing conditions. In [7], [8], S.S. Ikki et al have proposed cooperative schemes in which the $N^{th}$ best relay would be used to forward the source’s information to the destination.

In this paper, we investigate dual-hop decode-and-forward relaying systems using the $N^{th}$ best-relay selection scheme over Rayleigh fading channels. We first derive the cumulative density function (CDF) and probability density function (PDF) of the $N^{th}$ best order statistics of the equivalent instantaneous signal to noise ratio (SNR). Relying on these distributions, we derive the exact expression of the outage probability for the adaptive relay scheme and the approximate expressions of symbol error rate (SER) for the fixed relay scheme. Monte-Carlo simulations are presented to confirm the derived expressions.

The rest of the paper is organized as follows. The system model description is described in Section II and performance analysis is discussed in Section III. In Section IV, we will show the simulation results and Section V concludes the paper.

II . System Model Description

As shown in Fig. 1, we assume that no direct link exists between the source and the destination; hence the source’s data must be sent to the destination via a relay. We also assume that $M$ relays are available for cooperation. We further assume that all nodes are equipped with a single antenna and operate in a half-duplex mode.

The data received by relay $R_i$, $i \in \{1, 2, ..., M\}$, and the destination $D$ due to the transmission of the source and relay $R_i$ can be respectively presented as follows:

$$y_{SR_i} = \sqrt{P_i h_{SR_i}} s_i + n_i,$$  \hspace{1cm} (1)

$$y_{RD} = \sqrt{P h_{RD}} s_i + n_D,$$  \hspace{1cm} (2)

where $s_i$ and $s_2$ are signals transmitted by $S$ and relay

Manuscript received November 2, 2011; Revised January 5, 2012; Accepted January 13, 2012. (ID No. 20111102-028J)

Department of Electronic Engineering, University of Ulsan, Ulsan, Korea.

Corresponding Author: Hyung-Yun Kong (e-mail: hkong@mail.ulsan.ac.kr)

This is an Open-Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License (http://creativecommons.org/licenses/by-nc/3.0) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

ⓒ Copyright The Korean Institute of Electromagnetic Engineering and Science. All Rights Reserved.
\( R_i \), respectively, \( P \) is transmit power of the source and relay \( R_i \), \( h_{SR} \) and \( h_{RD} \) are Rayleigh channel coefficients between the source and relay \( R_i \) and between relay \( R_j \) and the destination, respectively, \( n_s \) and \( n_d \) are zero-mean additive white Gaussian noise (AWGN) with variance \( \sigma_0^2 \) at relay \( R_i \) and the destination, respectively.

From (1), (2), the instantaneous received SNR at relay \( R_i \) and the destination are respectively calculated by

\[
y_i = \frac{\gamma_i h_{SR}^2}{\sigma_0^2}, \\
y_j = \frac{\gamma_j h_{RD}^2}{\sigma_0^2},
\]

where \( \gamma_i = P / \sigma_0^2 \) is the transmit SNR.

Since \( h_{SR} \) and \( h_{RD} \) are identical independent Rayleigh distributed random variables (RVs), the cumulative density function (CDF) of \( |h_{SR}|^2 \) and \( |h_{RD}|^2 \) can be respectively expressed as

\[
F_{|h_{SR}|^2}(x) = 1 - \exp(-\Omega_1 x), \\
F_{|h_{RD}|^2}(x) = 1 - \exp(-\Omega_2 x),
\]

where \( \Omega_1 \) and \( \Omega_2 \) are expected values of \( |h_{SR}|^2 \) and \( |h_{RD}|^2 \), respectively.

From (3)–(6), the CDF of \( \gamma_i \) and \( \gamma_j \) can be respectively given by

\[
F_{\gamma_i}(x) = 1 - \exp(-\lambda_1 x), \\
F_{\gamma_j}(x) = 1 - \exp(-\lambda_2 x),
\]

where \( \lambda_1 = \Omega_1 / \gamma \) and \( \lambda_2 = \Omega_2 / \gamma \).

Setting \( \gamma_i = \min(\gamma_u, \gamma_2) \), due to the independence of \( \gamma_u \) and \( \gamma_2 \), the CDF of \( \gamma_i \) is given by [5]

\[
F_{\gamma_i}(x) = \Pr[\min(\gamma_u, \gamma_2) < x] = 1 - \left(1 - F_{\gamma_u}(x)\right)\left(1 - F_{\gamma_2}(x)\right).
\]

In this paper, the relay having the \( N^{th} \) largest value of \( \gamma_i \) would be chosen to transmit the data. The corresponding order statistics are then obtained by arranging random variables \( \gamma_i \) in a non-increasing order, denoted by \( X_1, X_2, ..., X_{M-1}, X_M \), where \( X_1 \) has the largest value, \( X_M \) has the smallest value, and \( X_N \) has the \( N^{th} \) largest value. Assuming that \( \gamma_1, \gamma_2, ..., \gamma_M \) are independent and identical random variables, the CDF of the \( N^{th} \) order statistic is given by [7, eq.(22)]:

\[
F_{X_N}(x) = \sum_{j=1}^{M} C_M^{j-1} F_{\gamma_j}(x)^{M-j+1} \left[1 - F_{\gamma_j}(x)^{j-1}\right],
\]

where \( C_M^n = \frac{b!}{a!(b-a)!} \) with \( a \) and \( b \) to be non-negative integers and \( a \leq b \).

Substituting (9) into (10), we get the CDF \( F_{X_N}(x) \) as follows:

\[
F_{X_N}(x) = \sum_{j=1}^{M} \sum_{i=0}^{M-j+1} (-1)^i C_M^{i-1} C_{M-j+1}^{j-1} \times \exp(-(i+j-1)(\lambda_1 + \lambda_2)x).
\]

From (11), we get the probability density function (PDF) \( f_{X_N}(x) \) as

\[
f_{X_N}(x) = \sum_{j=1}^{M} \sum_{i=0}^{M-j+1} (-1)^i C_M^{i-1} C_{M-j+1}^{j-1} (i+j-1)(\lambda_1 + \lambda_2) \exp(-(i+j-1)(\lambda_1 + \lambda_2)x).
\]

### III. Performance Evaluation

#### 3-1 Outage Probability

We assume that each node (including relays and the destination) decodes the data successfully if the instantaneous received SNR at this node is larger than a predetermined threshold, \( \gamma_o \). If the chosen relay, e.g., \( R_i \), decodes the source’ signal successfully, it will forward the decoded signal to the destination. Otherwise, it keeps silent. Therefore, the outage probability in this case can be formulated as

\[
P_{out} = \Pr[\gamma_u < \gamma_o] \Pr[\gamma_2 < \gamma_o] = \Pr[\gamma_i < \gamma_o]
\]
\[ = F_{X_{h}}(\gamma_{th}) \]  

3-2 Symbol Error Rate

In fixed decode-and-forward relaying scheme, the equivalent SNR for two hops of each relay can be approximated tightly as [5]

\[ \gamma_{eq,i} \approx \min(\gamma_{1}, \gamma_{2}) = \gamma_{i} \]  

For \( V \)-ary PAM, by averaging the SER in AWGN channel [9, Eq. (5.2.45)] over the PDF \( f_{X_{h}}(x) \), the SER over Rayleigh fading channel can be calculated approximately by

\[ \text{SER}_{\text{PAM}} \approx \int_{0}^{\frac{V-1}{V}} 2(\frac{V-1}{V}) Q\left(\sqrt{\frac{6x}{V^2-1}}\right) f_{X_{h}}(x) dx \]  

where \( Q(.) \) is Q-function defined in [9].

With the help of Appendix A, the average SER in (15) can be calculated approximately by

\[ \text{SER}_{\text{PAM}} \approx \sum_{j=1}^{N} \sum_{i=0}^{M-1} (-1)^{i+1} \frac{(V-1)}{V} C_{M}^{i} C_{M-1}^{i} \times \left(1 - \frac{3}{\sqrt{3+(V^2-1)(i+j-1)(\lambda_{i}+\lambda_{j})}}\right) \]  

For \( V \)-ary PSK \((V>4)\), utilizing [9, Eq. (5.2.61)], the average SER over the distribution of the instantaneous SNR can be approximated by

\[ \text{SER}_{\text{PSK}} \approx \int_{0}^{\frac{\pi}{V}} 2Q\left(\frac{2\sin^{2}(\frac{\pi}{V})}{V} x\right) f_{X_{h}}(x) dx \]  

Substituting (12) into (17) and with the help of Appendices A and B, the high transmit SNR approximation of SER is calculated by

\[ \text{SER}_{\text{QAM}} \approx \sum_{j=1}^{N} \sum_{i=0}^{M-1} (-1)^{i+1} C_{M}^{i} C_{M-1}^{i} \left[ 4\left(\frac{\sqrt{V}-1}{\sqrt{V}}\right) Q\left(\frac{3x}{V-1}\right) - 4\left(\frac{\sqrt{V}-1}{V}\right)^{2} Q^{2}\left(\frac{3x}{V-1}\right) \right] \]

where \( \phi = \frac{3}{3+2(M-1)(\lambda_{i}+\lambda_{j})} \).

IV. Simulation Results

In this section, we provide some numerical results that have been developed in Section III and we verify these results with Monte-Carlo simulations. In all simulations, we assume that the transmit power of the source and relays are same and the expected values \( 1 \) and \( 2 \) equal to 1.

Fig. 2 shows the outage probability of the system as a function of the transmit SNR \( \gamma \) in dB. In this simulation, we assign the value of \( M \) and \( \gamma_{th} \) as 3 and 2, respectively, while the value of \( N \) varies from 1 to 3. As shown in Fig. 2, the outage performance is smallest when the best relay \((N=1)\) is chosen. This is due to the fact that the obtained diversity order decreases with the increasing of \( N \).

In Fig. 3, we investigate the effect of threshold \( \gamma_{th} \)
on the outage performance. In this figure, we assume that \( M=4 \) and \( N=2 \). As expected, the outage performance decreases with decreasing \( \gamma_g \). Figs. 2–3 show that the theoretical and simulation results are in good agreement, which verifies the accuracy of our analyses.

In Fig. 4, we investigate the average SER performance when different modulation techniques are used. In this figure, the number of relays is fixed at 3 and the second best relay is chosen for the cooperation. The performance is best when 16-QAM is used and the worst performance is obtained when 16-PAM is used. In addition, the theoretical and simulation results match well at high SNR region. The small gap between them comes from the approximate method which is made by (14).

V. Conclusion

In this paper, we evaluate the performance of decode-and-forward relaying scheme with \( N^{th} \) best relay selection by deriving expressions of outage probability and symbol error rate over Rayleigh fading channels. Monte Carlo simulations are also presented to verify our derivations. The simulation results show that the simulation and theoretical results match very well at high signal to noise ratio (SNR).

Appendix A

Considering the following integral:

\[
A(a,b,c) = \int_0^{\infty} x^a \exp(-bx) Q\left(\sqrt{cx}\right) dx
\]  

(A.1)

Substituting the \( Q \) function [10, Eq. (4.2)], 

\[
Q(x) = \frac{1}{\pi} \int_0^{\infty} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta
\]

into (A.1), and after some manipulation, we have

\[
A(a,b,c) = \frac{\Gamma(a+1)}{\pi b^{a+1}} \int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta+c/2b}\right)^{1+a} d\theta
\]  

(A.2)

In case that \( a=0 \), after some manipulation, \( A(0,b,c) \) can be calculated as

\[
A(0,b,c) = \frac{1}{2b} \left(1-\sqrt{c/2b}\right)
\]  

(A.3)

Appendix B

Considering the following integral:

\[
B(a,b,c) = \int_0^{\infty} x^a \exp(-bx) Q^2\left(\sqrt{cx}\right) dx
\]  

(B.1)

Substituting the \( Q^2 \) function [10, Eq. (4.9)], 

\[
Q^2(x) = \frac{1}{\pi} \int_0^{\pi/4} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta
\]

into (B.1), and after some manipulation, we have

\[
B(a,b,c) = \frac{\Gamma(a+1)}{\pi b^{a+1}} \int_0^{\pi/4} \left(\frac{\sin^2\theta}{\sin^2\theta+c/2b}\right)^{1+a} d\theta
\]  

(B.2)

For case of \( a=0 \), substituting [10, Eq. (5A.13)] into (B.2) yields

\[
B(0,b,c) = \frac{1}{4b} \left[1-\frac{c/2b}{1+c/2b}\left(\frac{4}{\pi} \tan^{-1}\left(\frac{1+c/2b}{c/2b}\right)\right)\right]
\]  

(B.3)

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0006043).
Tran Trung Duy

He received Telecommunications Engineering from Ho Chi Minh City University of Technology, Vietnam, in 2007. He is currently working toward the Master degree in the Department of Electrical Engineering, University of Ulsan, Korea. His major research interests are mobile ad-hoc networks, wireless sensor networks, cooperative communications, cooperative routing, cognitive radio, combining techniques.

Hyung-Yun Kong

He received M.E. and Ph.D. degrees in electrical engineering from Polytechnic University, Brooklyn, New York, USA, in 1991 and 1996, respectively. He received a BE in electrical engineering from New York Institute of Technology, New York, in 1989. Since 1996, he has been with LG electronics Co., Ltd., in the multimedia research lab developing PCS mobile phone systems, and from 1997 the LG chairman's office planning future satellite communication systems. Currently he is a Professor in electrical engineering at the University of Ulsan, Korea. His research area includes channel coding, detection and estimation, cooperative communications, cognitive radio and sensor networks. He is a member of IEEK, KICS, KIPS, IEEE, and IEICE.

References