Compact and Wideband Coupled-Line 3-dB Ring Hybrids

Coupled Line으로 구성된 작고 넓은 대역폭을 가지는 3-dB Ring Hybrids

Hee-Ran Ahn · Jungjoon Kim · Bumman Kim

Abstract

In this paper, two types of wideband 3-dB ring hybrids are compared and discussed to show the ring hybrid with a set of coupled-line sections better. However, the better one still has a realization problem that perfect matching can be achieved only with −3 dB coupling power. To solve the problem, a set of coupled-line sections with two shorts is synthesized using one- and two-port equivalent circuits and design equations are derived to have perfect matching, regardless of the coupling power. Based on the design equations, a modified II-type of transmission-line equivalent circuit is newly suggested. It consists of coupled-line sections with two shorts and two open stubs and can be used to reduce a transmission-line section, especially when its electrical length is greater than π. Therefore, the 3λ/4 transmission-line section of a conventional ring hybrid can be reduced to less than π/2. To verify the modified II-type of transmission-line equivalent circuit, two kinds of simulations are carried out; one is fixing the electrical length of the coupled-line sections and the other fixing its coupling coefficient. The simulation results show that the bandwidths of resulting small transmission lines are strongly dependent on the coupling power. Using modified and conventional II-types of transmission-line equivalent circuits, a small ring hybrid is built and named a compact wideband coupled-line ring hybrid, due to the fact that a set of coupled-line sections is included. One of compact ring hybrids is compared with a conventional ring hybrid and the compared results demonstrate that the bandwidth of a proposed compact ring hybrid is much wider, in spite of being more than three times smaller in size. To test the compact ring hybrids, a microstrip compact ring hybrid, whose total transmission-line length is 220°, is fabricated and measured. The measured power divisions (S31, S41, S32, and S43) are −2.78 dB, −3.34 dB, −2.8 dB, and −3.2 dB, respectively at a design center frequency of 2 GHz, matching and isolation less than −20 dB in more than 20 % fractional bandwidth.

요약

두 종류의 넓은 대역폭을 갖는 ring hybrids (하나는 coupled line이 포함되어 있고, 다른 하나는 left-handed transmission line을 포함한 ring hybrids)가 비교되었으며, 비교 결과로부터 coupled line을 포함한 ring hybrid가 모든 면에서 우수한 특성을 가짐을 보여주었다. 그러나, coupled line을 포함한 ring hybrid는 −3 dB coupling power를 가질 경우에 한해서만이 perfect matching이 이루어지기 때문에, perfect matching을 갖는 coupled line ring hybrid는 2차원으로 구현하기는 거의 불가능하다. 이 문제를 해결하기 위해서 coupled line을 해석했고, 그 해석 결과로부터 coupling coefficient에 관계없이 어느 경우에도 perfect matching을 이룰 수 있는 설계 식을 유도했다. 이 설계식을 이용하여, transmission line의 길이가 π보다 큰 경우에도 적용될 수 있는 크기를 줄이기 위한 새로운
Compact and Wideband Coupled-Line 3-dB Ring Hybrids

I. Introduction

The ring hybrids are indispensable and fundamental components which can be used for various applications such as balanced amplifiers, balanced mixers, multipliers, phase shifters and attenuators, power amplifiers, antenna feeding networks and so on. Since the first ring hybrid was introduced in 1947 by Tyrrel [1], a number of engineers discussed performance and realization of the ring hybrids [2][3]. However, the conventional ring hybrid, consisting of only transmission-line sections, is inherently of narrow bandwidth and large in size. To overcome this disadvantage, S. March [4] suggested a wideband ring hybrid having a set of coupled-line sections but the problem still remained: perfect matching can be achieved only with $-3$ dB coupling coefficient. The ring hybrids suggested by S. March will be named coupled-line ring hybrids, hereafter. However, the coupled-line sections with $-3$ dB coupling coefficient can not be easily realized and many efforts have been done to solve this problem; using broadside [5] and vertical coupling [6], uniplanar structures [7][8][9][10], left-handed transmission-line section [11] and so on. However, the left-handed transmission-line section should be realized with lumped elements, multisectons are used for wideband performance [6], and in any case where the coupled-line sections are used, $-3$ dB coupling is not changed [10][11][12][13] for perfect matching.

To get compact ring hybrids, two design methods have been applied; using arbitrary transmission-line sections [7][12][13] and transmission-line equivalent circuits [14][15]. However the compact ring hybrids designed by the first method are perfectly matched at a frequency where transmission-line sections become 90°. Therefore, size reduction effect can not be expected. For the second method, lumped-element [16][17], T-type [14][15] or T-type of transmission-line equivalent circuits are used. However, in the T-type of equivalent circuit, the characteristic impedance of transmission-line section becomes very high with small size reduction. Using the lumped-element equivalent circuit, the bandwidth of resulting small transmission lines is very small. The conventional T-type of transmission-line equivalent circuit can be used only when the transmission-line length is less than $\pi$.

In this paper, a wideband coupled-line ring hybrid is compared with a ring hybrid having a left-handed transmission-line section [11] and the compared results show that the coupled-line ring hybrid is better in any case. The coupled-line sections of the ring hybrid is a kind of impedance transformer [15] and can be obtained from a four-port directional coupler for impedance transforming, very recently introduced [10][12]. However, in this case, perfect matching can be achieved only with $-3$ dB coupling. To have perfect matching with any coupling coefficient, the coupled-line sections with two
shorts are synthesized using one- and two-port equivalent circuits and design equations are derived to design and fabricate the wideband coupled-line ring hybrids in planar structure, without any restriction of coupling power.

Also, to reduce the ring hybrid size more, modified \( \Pi \)-type of transmission-line equivalent circuit is newly proposed based on the design equations of the coupled-line sections with two shorts. The modified equivalent circuit can be used for any transmission-line section whose electrical length is greater than \( \pi \) and therefore the 3 \( \lambda/4 \) transmission-line section of a ring hybrid can be reduced more. Using both modified and conventional \( \Pi \)-types of equivalent circuits, compact wideband coupled-line ring hybrids are newly constructed and one of them is compared with the conventional ring hybrid in terms of power division. The compared results demonstrate that the compact ring hybrid proposed in this paper shows more bandwidth in spite of being three times smaller in size. To test the compact ring hybrids, one microstrip ring hybrid is fabricated and measured. The measured power divisions (\( S_{21}, S_{41}, S_{23} \), and \( S_{43} \)) are \(-2.78 \) dB, \(-3.34 \) dB, \(-2.8 \) dB, and \(-3.2 \) dB, respectively at a design center frequency of 2 GHz and matching and isolation less than \(-20 \) dB in more than 20 \% fractional bandwidth.

This paper is constructed with six sections. Section I gives brief introduction of conventional wideband ring hybrids and contents of this paper. Section II compares conventional wideband ring hybrids to know which one has better performance and which problem the better one has. In section III, coupled-line sections with two shorts are synthesized to solve the problem, that is, to realize them without any restriction of coupling power. In section IV, using the design equations of the coupled-line sections with two shorts, wideband coupled-line ring hybrids are simulated to show perfect matching can be achieved, independently of coupling power. Then, to reduce the ring hybrid size more, a modified \( \Pi \)-type of transmission-line equivalent circuit is newly proposed and how to get compact wideband coupled-line ring hybrids is discussed in section V. Finally, this paper concludes with section VI.

II. Conventional Ring Hybrids

2-1 Conventional Ring Hybrids

Three 3-dB conventional ring hybrids terminated in equal impedances \( Z_0 \) are depicted in Fig. 1. The conventional ring hybrid in Fig. 1(a), consisting of three \( \lambda/4 \) transmission-line sections and one 3 \( \lambda/4 \) transmission-line section, is inherently of narrow bandwidth and large in size. The shortcoming is mainly due to that the bandwidth, where \( \lambda/4 \) and 3 \( \lambda/4 \) transmission-line sections have 180° phase difference, is very narrow. To increase the bandwidth, that is, to have 180° phase difference in wider bandwidth, the 3 \( \lambda/4 \) transmission-line section is replaced by a set of coupled-line sections with two shorts or a left-handed transmission-line section, as described in Fig. 1(b) and (c), respectively. Because of the reason, the ring hybrid in Fig. 1(b) is named “a coupled-line ring hybrid” and that in Fig. 1(c)

![Diagram](image-url)

(a) A ring hybrid with a 3 \( \lambda/4 \) transmission-line section between ports 1 and 3

(b) A coupled-line ring hybrid

(c) A left-handed ring hybrid

Fig. 1. Conventional ring hybrids.
“a left-handed ring hybrid.” However, the two ring hybrids have realization problems; in the coupled-line ring hybrid in Fig. 1(b), the perfect matching can be achieved only with −3 dB coupling coefficient, and the left-handed transmission-line section may be realized only with lumped-elements which may cause unwanted frequency performance.

2-2 Conventional Wideband Ring Hybrids

To compare the two ring hybrids in Fig. 1(b) and (c), the two are simulated at a center frequency of 1 GHz and the simulation results are plotted in Fig. 2. In this case, the even- and odd-mode impedances of the coupled-line sections in Fig. 1(b) are 171.4 Ω and 29.3 Ω, respectively when Z₀ is 50 Ω [14,15]. The power excited at port 1 or 3 in Fig. 1 is divided equally between ports 2 and 4 and isolated from port 3 or 1, respectively. The divided waves are in phase or out of phase, depending on the input port chosen. Considering these points, the ratios of S₂₁ to S₄₁ and S₂₃ to S₄₃, and phase differences of |∠S₂₁ − ∠S₄₁| and |∠S₂₃ − ∠S₄₃| are plotted in Fig. 2 where solid lines indicate the frequency responses of the coupled-line ring hybrid and the dotted ones are those of the left-handed ring hybrid.

In 100 % fractional bandwidth, S₂₁/S₄₁ of the coupled-line ring hybrid exists from 0 dB to 0.567 dB and S₂₃/S₄₃ from 0 dB to 0.724 dB as shown in Fig. 2(a). On the other hand, in the left-handed ring hybrid, S₂₁/S₄₁ exists between 0 dB and 0.9 dB and S₂₃/S₄₃ from −0.9 dB to 0 dB as displayed in Fig. 2(a). The less power division ratios, the better. Therefore, the coupled-line ring hybrid is better than the left-handed ring hybrid in terms of power divisions.

In phase responses in Fig. 2(b), out-of-phase response, |∠S₂₁ − ∠S₄₁| of the coupled-line ring hybrid exists between 170.4° and 188.3° while that of the left-handed ring hybrid between 137.6° and 228.3°. In Fig. 2(c), in-phase response, |∠S₂₃ − ∠S₄₃| of the coupled-line ring hybrid is between 0° and 9.4° and that of the left-handed ring hybrid between 0° and 25.66°. Since the ideal phase differences of |∠S₂₁ − ∠S₄₁| and |∠S₂₃ − ∠S₄₃| are equal to 180° and 0°, respectively, the coupled-line ring hybrid is much better than the left-handed ring hybrid in terms of phase response. Exact simulation results are written in Table 1 where the coupled-line ring hybrid show the better performance than those of the left-handed ring hybrid, in every point.

Even though the performance of the coupled-line ring hybrid is much better than that of the left-handed ring hybrid in every point, how to realize high even-mode impedance of 171.4 Ω and low odd-mode impedance of 29.3 Ω in microstrip technology is a big problem. To solve the problem, the coupled-line sections with two shorts connected between ports 1 and 3 in Fig. 1(b) will be discussed in more detail.

### Table 1. Simulation results within a 100 % fractional bandwidth (CPL and LH stand for coupled-line and left-handed ring hybrids, respectively)

<table>
<thead>
<tr>
<th></th>
<th>dB(S₂₁/S₄₁)</th>
<th>dB(S₂₃/S₄₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPL</td>
<td>0 ~ 0.567 dB</td>
<td>0 ~ 0.724 dB</td>
</tr>
<tr>
<td>LH</td>
<td>0 ~ 0.902 dB</td>
<td>−0.902 ~ 0 dB</td>
</tr>
<tr>
<td></td>
<td>∠S₂₁ − ∠S₄₁</td>
<td>∠S₂₃ − ∠S₄₃</td>
</tr>
<tr>
<td>CPL</td>
<td>170.4 ~ 188.3°</td>
<td>0 ~ 9.4°</td>
</tr>
<tr>
<td>LH</td>
<td>137.6 ~ 228.3°</td>
<td>0 ~ 25.66°</td>
</tr>
</tbody>
</table>

### III. Coupled Lines with Two Shorts

3-1 Coupled Lines with Two Shorts

As discussed in the literatures [14,17], the coupled-line sections with two shorts connected between ports 1 and 3 in Fig. 1(b) is used as an impedance transformer to transform 2Z₀ into Z₀, when power is fed into port 1. The coupled-line sections with two shorts may be derived from a four-port impedance transforming directional coupler discussed in the literatures [20,21]. The directional coupler for an impedance transformer and a two-port coupled-line sections with two shorts are depicted in Fig. 3(a) and (b), respectively where an input-im-
and the remainder of the input power is delivered to port ④. Theoretically, no power is delivered to port ③, which is called an isolated port. In this case, the even- and odd-mode impedances\(^{(20)}\)\(^{(21)}\) are

\[
Z_{0e} = \sqrt{Z_r Z_L} \frac{1 + C}{1 - C}
\]

\[
Z_{0o} = \sqrt{Z_r Z_L} \frac{1 - C}{1 + C}
\]

where \(C\) is a coupling coefficient.

Terminating ports ② and ④ in Fig. 3(a) in two shorts results in the coupled-line sections with two shorts in Fig. 3(b). Applying the short boundary condition gives its admittance matrix as

\[
[Y] = \begin{bmatrix}
-j \frac{Y_{0e} + Y_{0o}}{2} \cot \Theta & -j \frac{Y_{0e} - Y_{0o}}{2} \csc \Theta \\
-j \frac{Y_{0e} - Y_{0o}}{2} \csc \Theta & -j \frac{Y_{0e} + Y_{0o}}{2} \cot \Theta
\end{bmatrix}
\]

where \(Y_{0e} = 1/Z_{0e}\), \(Y_{0o} = 1/Z_{0o}\) and a pure TEM (transverse electromagnetic) propagation is assumed.

Based on the admittance matrix in (2), the scattering parameters of coupled-line sections in Fig. 3(b) are derived as

Fig. 2. Two wideband ring hybrids are compared and CPL and LH indicate coupled-line and left-handed ring hybrids, respectively.

When \(\Theta = \pi/2\) in Fig. 3(a), the power excited at port ① is coupled to port ② with a certain coupling power

Fig. 3. Impedance transforming directional coupler and coupled-line sections with two shorts.
$S_{11} = \frac{1}{D}[(Y_{o}\!-\!Y_{o})^{2} \sec^{2}\Theta - (Y_{o} + Y_{o})^{2} - 4Y_{o}Y_{L}\tan^{2}\Theta - j2(Y_{o} + Y_{o})(Y_{L} - Y_{o})\tan\Theta]$  
\hspace{1cm} (3a)

$S_{22} = \frac{1}{D}[(Y_{o}\!-\!Y_{o})^{2} \sec^{2}\Theta - (Y_{o} + Y_{o})^{2} - 4Y_{o}Y_{L}\tan^{2}\Theta + j2(Y_{o} + Y_{o})(Y_{L} - Y_{o})\tan\Theta]$  
\hspace{1cm} (3b)

$S_{21} = \frac{1}{D}[-j4Y_{o}(Y_{o}\!-\!Y_{o})\sin\Theta\sec^{2}\Theta]$  
\hspace{1cm} (3c)

where $D = (Y_{o} + Y_{o})^{2} - (Y_{o} - Y_{o})^{2} \sec^{2}\Theta$

$-4Y_{o}Y_{L}\tan^{2}\Theta + j2(Y_{o} + Y_{o})(Y_{o} + Y_{L})\tan\Theta$, 
$Y_{r} = 1/Z_{r}$ and $Y_{L} = 1/Z_{L}$.

Using the calculation results in (3), the coupled-line sections with two shorts were simulated by use of a mathematical software Matlab 6.1 and simulation results are plotted in Fig. 4 where the termination impedances, $Z_{r}$ and $Z_{L}$ are 50 $\Omega$ and 100 $\Omega$, respectively. Depending on the coupling coefficients, coupling characteristics are classified as critical coupling($C = -3$ dB), over coupling($C > -3$ dB) and under coupling($C < -3$ dB)[22]. Fig. 4 shows that perfect matching appears only with the critical coupling, and that ripples with no perfect matching exist in the over coupling case. The power excited at port (1) in Fig. 3(b) is transmitted into port (2), and how much power can be transmitted is dependent on the coupling structure. In order that the perfect matching at a design center frequency can be achieved regardless of the coupling coefficient, design equations will be derived using one- and two-port equivalent circuits.

3-2 One-Port Equivalent Parallel Resonant Circuit

How much power excited at port (1) in Fig. 3(b) can be transmitted into port (2) is dependent on the coupling structure. So, the coupled-line sections terminated in $Z_{L}$ in Fig. 3(b) may be equivalent to a one-port parallel resonant circuit[22] as described in Fig. 5 where input impedance and input reference impedance $Z_{in}$ and $Z_{r}$ are indicated.

The input impedance $Z_{in}$[23] in Fig. 3(b) and Fig. 5 is calculated by use of the following equation;

$Z_{in} = [Y_{in}]^{-1} = \left[ Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22} + Y_{L}} \right]^{-1}$  
\hspace{1cm} (4)

which gives frequency dependent $R_{o}$, $L_{o}$ and $C_{o}$ in Fig. 5 as follows;

$R_{o} = \frac{Y_{L}(1-C^{2}) + Y_{o}\cot^{2}\Theta}{Y_{o}Y_{L}C^{2}\csc^{2}\Theta}$  
\hspace{1cm} (5a)

for $\cot\Theta \geq 0$.
The input impedance $Z_{in}$ may be displayed on an impedance Smith chart and Fig. 6 illustrates coupling to a parallel resonant circuit. In this case, $Z_r = 50 \, \Omega$ and $Z_L = 100 \, \Omega$ in Fig. 3(b) are fixed, and coupling coefficient and electrical length $\Theta$ are varied as shown in Fig. 6. The input impedance loci cut the real axis two times, and when $\Theta = \pi/2$, the input impedance normalized to $Z_r$ is exactly unity with the critical coupling but the two others are more or less than unity. It means that a perfect matching appears only with the critical coupling; these results coincide with those in Fig. 4. For any set of coupled-line sections with two shorts in Fig. 3(b) to have perfect matching, regardless of the coupling coefficient, the even- and odd-mode impedances need to be modified in such a manner that the over- coupled input impedances are increased and those of the under coupled ones are decreased. When $\Theta = \pi/2$ in (5), only $R_o$ appears and its value is

$$R_o = Z_r \frac{1-C^2}{C^2}$$

From (6), $R_o$ is $Z_r$, when $C = 1/\sqrt{2}$ (critical coupling), which agrees with the simulation results in Figs. 4 and 6. For the coupled-line sections to be perfectly matched at a design center frequency with any coupling coefficient, the even- and odd-mode impedances should be modified so that the value of $R_o$ is always $Z_r$, regardless of the coupling coefficient. The $Z_r$ in (6) comes from the even- and odd-mode impedances in (1) and can be modified to have a constant value of input impedance. The solution for this is to replace $Z_r$ in (1) by $Z_r \frac{C^2}{(1-C^2)}$.

In such a manner, the modified even- and odd-mode impedances $Z_{oe}^m$ and $Z_{oo}^m$ are derived as

$$Z_{oe}^m = \frac{C}{1-C} \sqrt{Z_r Z_L}$$

$$Z_{oo}^m = \frac{C}{1+C} \sqrt{Z_r Z_L}$$

With $Z_{oe}^m$ and $Z_{oo}^m$, the input impedance $Z_{in}$ always becomes $R_o = Z_r$ at resonant frequency, that is, at a center frequency, regardless the coupling coefficient.

### 3.3 Two-Port Equivalent Circuit

A set of coupled-line sections with electrical length $\Theta$ and termination admittances $Y_r$ and $Y_L$ in Fig. 3(b) may be equivalent to a circuit, consisting of a transmission-line section with electrical length $\pi + \Theta$ and two stubs as depicted in Fig. 7 where the characteristic admittances of transmission-line section and stubs are $(Y_{oo} - Y_{oe})/2$ and $Y_{oe}$, respectively. When $\Theta = \pi/2$, only the transmission-line section is connected between the two termination admittances and should be a quarter wave admittance transformer. Therefore, its characteristic admittance is a geometric mean value of the two

---

**Fig. 6.** Input impedances are illustrated on an impedance Smith chart depending over, critical and under coupling.
Fig. 7. Two-port equivalent circuit of coupled-line sections with two shorts.

termination admittances and related with the coupling coefficients\[^3\]\[^4\]. So, the relative relations are given as

\[
\frac{Y_{oo} - Y_{0e}}{2} = \sqrt{Y_r Y_L} \quad (8a)
\]

\[
Y_{oo} = \frac{1 + C}{1 - C} Y_{0e} \quad (8b)
\]

The even- and odd-mode admittances satisfying (8) are different from the original ones in (1) and calculated as

\[
Y_{0e}^m = \frac{1 - C}{C} \sqrt{Y_r Y_L} \quad (9a)
\]

\[
Y_{oo}^m = \frac{1 + C}{C} \sqrt{Y_r Y_L} \quad (9b)
\]

which are the same as those in (7).

As shown in the equivalent circuit in Fig. 7, the transmission-line section can considered as a circuit where a transmission-line section with \( \Theta \) is connected with a frequency independent 180° phase shifter. Therefore, \( \lambda/4 \) transmission-line section and \( \lambda/4 \) coupled-line sections in Fig. 1 can have 180° phase difference in wider frequency band. That is the reason the coupled-line ring hybrid in Fig. 1(b) can have wideband performance.

3-4 Coupled-Line Section Measurements

To verify the design equations in (7) and (9), a microstrip coupled-line sections with two shorts terminated in 100 \( \Omega \) and 50 \( \Omega \) was fabricated on a substrate\((H = 0.76 \text{ mm and } \varepsilon_r = 3.4)\) and tested at a center frequency of 2 GHz. Table 2 gives \( Z_{0e}^m \) and \( Z_{oo}^m \) depending on the coupling coefficient where the even- and odd-

mode impedances with -3 dB coupling coefficient are 171.4 \( \Omega \) and 29.3 \( \Omega \), respectively, which are almost impossible to realize with two-dimensional microstrip lines.

For the measurement, a microstrip coupled-line sections with \( C = -7 \) dB was fabricated and its even- and odd-mode impedances are \( Z_{0e}^m = 57.1 \Omega \) and \( Z_{oo}^m = 21.8 \Omega \) as given in Table 2. The even- and odd-mode impedances required above can be realized without any problem using a stand PCB(printed circuit board) technology but an easy method, with which the odd-mode impedance can be fabricated in a school, will be introduced.

The even-mode impedance of 57.1 \( \Omega \) can be realized without any problem, but the odd-mode impedance is somewhat difficult because the given substrate has a low dielectric constant. Therefore, a three-dimensional structure or a set of three coupled-line sections is needed to get the odd-mode impedance. In our case, the three-dimensional structure\[^2\] was used.

If a TEM propagation of two coupled transmission lines is assumed, then the characteristics of coupled transmission lines can be completely determined from capacitances and propagation velocities on the transmission lines. Two-dimensional capacitance equivalent network of a pair of coupled transmission lines is shown in Fig. 8(a) where \( C_{12} \) represents the capacitance per unit length between the two conductor lines in the absence of the ground conductor, while \( C_{11} \) and \( C_{22} \) denote the capacitances per unit length between one conductor and ground, in the absence of the other conductor line. If the coupled transmission lines are identical in size, then \( C_{11} = C_{22} \). For the even-mode excitation, no current flows between the two trans-

<table>
<thead>
<tr>
<th>Table 2. Even and odd-mode impedances with ( Z_r = 100 \Omega ) and ( Z_L = 50 \Omega ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C ) [dB]</td>
</tr>
<tr>
<td>( Z_{0e}^m ) [( \Omega )]</td>
</tr>
<tr>
<td>( Z_{oo}^m ) [( \Omega )]</td>
</tr>
</tbody>
</table>
plunging, the odd-mode impedance is not easy to realize.

To realize the low odd-mode impedance with three-dimensional structure as shown in Fig. 8(b), a pair of coupled transmission lines, with which only a required even-mode impedance can be realized, is first fabricated with a space of \( h \), here \( h \) is an assumed thickness of a given substrate. Then, the width \( w_h \) of a conductor vertically constructed in Fig. 8(b) is determined to have the required odd-mode impedance. In this case, the even-mode impedance is connected in parallel with the impedance produced by the vertical conductor.

Design data for the microstrip coupled-line sections with \( C = -7 \text{ dB} \) are given in Table 3 where \( Z_T \) is a characteristic impedance of a quarter wave impedance transformer to transform 100 \( \Omega \) into 50 \( \Omega \). For the given substrate, a pair of coupled transmission lines is first realized, fixing its space at the thickness of the given substrate in order that only the even-mode impedance of 57.1 \( \Omega \) can be obtained. For the odd-mode excitation, since a voltage null exist between two coupled transmission lines, the even-mode impedance of 57.1 \( \Omega \) is connected in parallel with an impedance produced between a vertically constructed conductor and the voltage null between the two coupled lines. This results in the required odd-mode impedance \( Z_{oo} = 21.8 \Omega \). Therefore, the impedance produced from the vertical conductor is 35.3 \( \Omega \), symbolized as a marked \( Z_v \) in Table 3. The characteristic impedance of \( Z_v \) is easily realized using a commercial simulation tool but an important point is its effective thickness is half of the

![Diagram](image)

Table 3. Fabrication data for a microstrip coupled line with two shorts.

<table>
<thead>
<tr>
<th>( Z_{oo} )</th>
<th>( w_v = 1.34 \text{ mm}, \quad \ell = 22.9 \text{ mm}, \quad s = 0.76 \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>57.1 ( \Omega )</td>
<td>(vertical conductor)</td>
</tr>
<tr>
<td>( Z_{oo} = 21.8 \Omega )</td>
<td>( w_h = 1.407 \text{ mm}, \quad \ell = 22.9 \text{ mm} )</td>
</tr>
<tr>
<td>21.8 ( \Omega )</td>
<td>(even mode)</td>
</tr>
<tr>
<td>( Z_T )</td>
<td>( w = 0.895 \text{ mm}, \quad \ell = 23.28 \text{ mm} )</td>
</tr>
<tr>
<td>50 ( \Omega )</td>
<td>( w = 1.678 \text{ mm} )</td>
</tr>
</tbody>
</table>
Fig. 9. Results measured and simulated are compared.

given substrate thickness. In this way, the microstrip
coupled transmission lines were fabricated as shown in
Fig. 8(c). Fig. 9 compares the measured and predicted
results and they show good agreement between them.

IV. Wideband Coupled-Line Hybrids

Using the analyzed coupled lines, wideband coupled-
line ring hybrids can be realized without any restriction
of coupling coefficient. As mentioned before, the cou-
pelled-line sections between ports 1 and 4 in Fig. 1(b)
is a kind of impedance transformer to transform 100 Ω
into 50 Ω, when $Z_0 = 50$ Ω. Therefore, the data given
in Table 2 may be used and several coupled-line ring
hybrids are compared in terms of port 1 excitation.
The compared results are plotted in Fig. 10 where per-
fected matching at a center frequency is achieved regard-
less of the coupling coefficients and the bandwidth is
proportional to the coupling power.

V. Compact Wideband Coupled-Line Ring Hybrids

5-1 Small Transmission Lines

The coupled-line and left-handed ring hybrids in Fig.
1(b) and (c) are somewhat smaller than the conventional
ring hybrids in Fig. 1(a) but they are still large in size.
To reduce the size of ring hybrids, transmission-line
sections are required to be reduced. For this purpose,

Fig. 10. Several coupled-line ring hybrids.

conventional $\Pi$-type of transmission-line equivalent
circuit was suggested but it can be used only when its
electrical length is less than $\pi$. To reduce the size of
ring hybrids more, modified $\Pi$-type of transmis-
sion-line equivalent circuit is newly suggested. It can be
used to reduce the 3 $\lambda$/4 transmission-line section of a
conventional ring hybrids to less than $\pi/2$. Transmi-
sion-line sections with characteristic impedance $Z_0$, 
their conventional and modified $\Pi$-types of transmis-
sion-line equivalent circuits are depicted in Fig. 11
where $\Theta$ and $\Theta_s$ are less than $\pi$ and $\pi/2$, respec-
tively. Therefore, the equivalent circuit in Fig. 11(b) is
used when $\Theta \leq \pi$ and that consisting of coupled-line
sections in Fig. 11(d) for transmission-line sections with
more than $\pi$.

The even- and odd-mode impedances $Z_{oe}^*$ and $Z_{bo}^*$
in Fig. 11 are computed, applying $Z_r = Z_L$ in Fig. 3(b),
(7) and (9) and the relations between $\Theta$, $Z_0$, $Z_s$, $\Theta_s$,
$Y_{op}$, $\Theta_{op}$, $Z_{oe}^*$, $Z_{bo}^*$, $Y_{cop}$ and $\Theta_{cop}$ in Fig. 11 are de-

871
where \( Y_0 = 1/Z_0 \) and \( C \) is coupling coefficient.

When \( Z_0 = 70.71 \ \Omega \) and \( \Theta = \pi/2 \) in Fig. 11(c), it is equal to the 3/4 transmission-line section between

![Diagram](image)

Table 4. Design data for small transmission lines.

| \( \Theta_s \) = 60°, \( \Theta_{cop} \) = 30° |
|------------------|------------------|------------------|
| \( C \) [dB]     | 5                | 7                | 9                |
| \( Z_{oe} \)     | 104.9            | 65.9             | 44.9             |
| \( Z_{oo} \)     | 29.4             | 25.2             | 21.4             |
| \( Z_{cop} \)    | 45.9             | 36.5             | 29               |

Table 5. Design data for small transmission lines.

| \( \Theta_s \) = 30° |
|------------------|------------------|------------------|
| \( C \) = -7 dB, \( \Theta_{cop} \) = 30° |
| \( \Theta_s \) | 40° | 50° | 60° |
| \( Z_{oe} \) | 88.8 | 74.5 | 65.9 |
| \( Z_{oo} \) | 33.97 | 28.5 | 25.2 |
| \( Z_{cop} \) | 23.8 | 28.4 | 36.5 |

ports ① and ④ of the conventional ring hybrid in Fig. 1(a). Based on the equations in (10) and (11), when \( Z_0 = 70.71 \ \Omega \), \( \Theta = \pi/2 \), with \( \Theta_{cop} = 30° \) and \( \Theta_s = 60° \) fixed, \( Z_{oe}, Z_{oo} \) and \( Z_{cop} \) are calculated as the coupling coefficients are varied and the calculation results are given in Table 4. When \( Z_0 = 70.71 \ \Omega \), \( \Theta = \pi/2 \), with \( \Theta_{cop} = 30° \) and \( C = -7 \) dB fixed, \( Z_{oe}, Z_{oo} \) and \( Z_{cop} \) are calculated as the electrical lengths of \( \Theta_s \) are varied and the calculation results are written in Table 5. Based on Tables IV and V, several small transmission-line sections in Fig. 11(d) are simulated and the simulation results are plotted in Fig. 12 where one can know that the more coupling power gives more bandwidth but the bandwidth is not strongly dependent on the electrical length \( \Theta_s \) when the coupling coefficient is fixed.

5-2 Compact Wideband Coupled-Line Ring Hybrids

Using the relations in Fig. 11, a compact wideband coupled-line ring hybrid can be built as depicted in Fig. 13 where the following relations hold:

\[
Y_{r1} \tan \Theta_{r1} = 2Y_{op} \tan \Theta_{op} \quad (11a)
\]

\[
Y_{r2} \tan \Theta_{r2} = Y_{op} \tan \Theta_{op} + Y_{cop} \tan \Theta_{cop} \quad (11b)
\]

where \( Y_{r1} = 1/Z_{r1} \) and \( Y_{r2} = 1/Z_{r2} \).
Compact and Wideband Coupled-Line 3-dB Ring Hybrids

The compact wideband coupled-line ring hybrids are designed at a center frequency of 1 GHz using Table 6 and compared with a conventional ring hybrid in Fig. 1(a). The compared simulation results are plotted in Fig. 14 where only power division frequency response is plotted. In this case, the total transmission-line length of the compact wideband coupled-line ring hybrids is 160°, whereas that of the conventional one is 540°. From the compared results of the power divisions, the proposed compact ring hybrid shows wider bandwidth, in spite of being more than three times smaller in size. The compact ring hybrids are designed using the equations in (10), (11) and (12) and three types of data with variable $\Theta_s$ are listed in Tables 6-8 where $\Theta_s$, $\Theta_{r1}$ and $\Theta_{r2}$ are fixed and coupling coefficient is varied from −5 dB to −11 dB.

![Diagram](image)

Table 6. Design data of compact wideband coupled-line ring hybrids for $\Theta_s = 40^\circ$.

<table>
<thead>
<tr>
<th>$\Theta_s = 40^\circ$</th>
<th>$Z_{r1} = 110 , \Omega$, $Z_{r2} = 38.73 , \Omega$, $\Theta_{r1} = 40^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$-5 , \text{dB}$</td>
</tr>
<tr>
<td>$Z_{\theta_{01}}$</td>
<td>141.3 $\Omega$</td>
</tr>
<tr>
<td>$Z_{\theta_{02}}$</td>
<td>39.6 $\Omega$</td>
</tr>
<tr>
<td>$Z_{\theta_{03}}$</td>
<td>27.9 $\Omega$</td>
</tr>
<tr>
<td>$\Theta_{r1}$</td>
<td>40°</td>
</tr>
<tr>
<td>$\Theta_{r2}$</td>
<td>40°</td>
</tr>
</tbody>
</table>

![Diagram](image)

Fig. 14. Compared power division frequency responses.

Fig. 13. Proposed compact wideband coupled-line ring hybrid.
Table 7. Design data of compact wideband coupled-line ring hybrids for $\Theta_s = 50^\circ$.

<table>
<thead>
<tr>
<th>$\Theta_s = 50^\circ$</th>
<th>$Z_s = 92.3 , \Omega$, $Z_{r1} = 65.5 , \Omega$, $\Theta_{r1} = 50^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$-5 , \text{dB}$</td>
</tr>
<tr>
<td>$Z_{0e}^r$</td>
<td>118.6 $\Omega$</td>
</tr>
<tr>
<td>$Z_{0o}^r$</td>
<td>33.2 $\Omega$</td>
</tr>
<tr>
<td>$Z_{r2}$</td>
<td>47.2 $\Omega$</td>
</tr>
<tr>
<td>$\Theta_{r2}$</td>
<td>50$^\circ$</td>
</tr>
</tbody>
</table>

Table 8. Design data of compact wideband coupled-line ring hybrids for $\Theta_s = 60^\circ$.

<table>
<thead>
<tr>
<th>$\Theta_s = 60^\circ$</th>
<th>$Z_s = 81.6 , \Omega$, $Z_{r1} = 59.3 , \Omega$, $\Theta_{r1} = 40^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$-5 , \text{dB}$</td>
</tr>
<tr>
<td>$Z_{0e}^r$</td>
<td>104.9 $\Omega$</td>
</tr>
<tr>
<td>$Z_{0o}^r$</td>
<td>29.4 $\Omega$</td>
</tr>
<tr>
<td>$Z_{r2}$</td>
<td>60.7 $\Omega$</td>
</tr>
<tr>
<td>$\Theta_{r2}$</td>
<td>50$^\circ$</td>
</tr>
</tbody>
</table>

5-3 Compact Coupled-Line Ring Hybrid Measurement

To test the compact ring hybrids, one microstrip ring hybrid was fabricated on a substrate ($H = 0.76 \, \text{mm}$ and $\varepsilon_r = 3.4$) as shown in Fig. 15 where port numbers are indicated. Fabrication data are written in Table 9 where the transmission-line section of $\Theta_s$ is fixed at $55^\circ$ and $Z_{0e}$ and $Z_{0o}$ are chosen as $64.21 \, \Omega$ and $25.8 \, \Omega$, respectively.

The microstrip ring hybrid was tested at a center frequency of 2 GHz and the measured and predicted results are compared in Figs. 16 and 17 where measured frequency response is expressed as solid lines and dotted ones are prediction. When the power is excited at port $\odot$ in Fig. 15, the measured power division response is compared with simulation ones in Fig. 16(a) where the measured $S_{21}$ and $S_{41}$ show $-3.34 \, \text{dB}$ and $-2.78 \, \text{dB}$, respectively.

When the power is excited at port $\ominus$, the power division characteristics are measured and plotted in Fig. 16(b) where measured $S_{23}$ and $S_{43}$ at the design center frequency of 2 GHz are $-2.8 \, \text{dB}$ and $-3.2 \, \text{dB}$, respectively. The measured matching performance is plotted in Fig. 17(a) where the return loss is less than $-15 \, \text{dB}$ in the frequency range of 1.66 GHz to 2.18 GHz.

In the proposed compact ring hybrids case, $|S_{11}| = |S_{44}|$, $|S_{22}| = |S_{33}|$, $|S_{13}| = |S_{24}|$ and the measured isolation is also compared in Fig. 17(b) where almost perfect isolation is achieved in more than 20 % fractional bandwidth.

VI. Conclusions

In this paper, two types of wideband ring hybrids
Fig. 16. Results measured and predicted are compared.

(one is composed of a set of coupled-line sections and another of a left-handed transmission-line section) were discussed and compared. The compared results showed the coupled-line ring hybrid was better than the left-handed ring hybrid in every point.

However, the coupled-line ring hybrid had a realization problem: perfect matching could be achieved only with \(-3\) dB coupling coefficient. To solve this problem, a set of coupled-line sections with two shorts was synthesized and design equations, with which the coupled-line ring hybrids could be designed without any restriction of coupling power, were derived. Based on the derived design equations, modified \(\pi\)-type of transmission-line equivalent circuit was newly suggested. The modified one was similar to the conventional one but a set of coupled-line sections was replaced with a transmission-line section. Using the modified transmission-line equivalent circuit, a \(3\lambda/4\) transmission-line section of a conventional ring hybrid could be reduced to less than \(\pi/2\) and compact wideband coupled-line ring hybrid was suggested using both modified and conventional ones.

To verify the compact wideband coupled-line ring hybrids, one with the total transmission-line length 160° was compared with a conventional ring hybrid and the compared results showed that the bandwidth of the proposed compact ring hybrid was much wider than that of the conventional one, in spite of being three times smaller in size.

Using the suggested transmission-line equivalent circuit, all the passive components including the ring hybrids can be reduced more. Using the proposed compact wideband coupled-line ring hybrids, all the elec-
tical equipments consisting of ring hybrids can be reduced.

References

Compact and Wideband Coupled-Line 3-dB Ring Hybrids
