Obstacle Avoidance Methods in the Chaotic Mobile Robot with Integrated some Chaos Equation

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Abstract

In this paper, we propose a method to avoid obstacles that have unstable limit cycles in a chaos trajectory surface. We assume all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When a chaos robot meets an obstacle in an Arnold equation or Chua’s equation trajectory, the obstacle reflects the robot.

We also show computer simulation results of Arnold equation and Chua’s equation and random walk chaos trajectories with one or more Van der Pol obstacles and compare the coverage rates of each trajectory. We show that the Chua’s equation is slightly more efficient in coverage rates when two robots are used, and the optimal number of robots in either the Arnold equation or the Chua’s equation is also examined.

Key words: chaos, mobile robot, Chua’s equation, Arnold equation.

1. Introduction

CHAOS theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], Chemistry [8], Biology [9] and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods.

In this paper, we propose a method to avoid obstacles using unstable limit cycles in the chaos trajectory surface. We assume that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When chaos robots meet obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Arnold equation or Chua’s equation, the obstacles reflect the chaos robots.

Computer simulations also show multiple obstacles can be avoided with an Arnold equation or Chua’s equation. The rate of coverage from random walk, Arnold equation and Chua’s equation trajectories was also compared and the amount of time required for n robots to reach 90% coverage was computed.

2. Chaotic Mobile Robot’s Equation

A. Mobile Robot

As the mathematical model of mobile robots, we assume a two-wheeled mobile robot as shown in Fig. 1.

![Fig. 1 two-wheeled mobile robot](image)

Let the linear velocity of the robot $v$ [m/s] and angular velocity $\omega$[rad/s] be the inputs in the system. The state equation of the two-wheeled mobile robot is written as follows:

$$
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & 0 & v \\
\sin \theta & 0 & 0 \\
0 & 1 & \omega
\end{pmatrix}
$$

(1)

where $(x,y)$ is the position of the robot and $\theta$ is the angle of the robot.

B. Chaos Equations

In order to generate chaotic motions for the mobile robot, we apply chaos equations such as an Arnold equation or Chua’s circuit equation.
1) Arnold equation [10]

We define the Arnold equation as follows:

\[
\begin{align*}
\dot{x}_1 &= A \sin x_1 + C \cos x_2 + B \cos x_3, \\
\dot{x}_2 &= B \sin x_1 + A \cos x_2 + B \cos x_3, \\
\dot{x}_3 &= C \sin x_2 + B \cos x_3,
\end{align*}
\]

(2)

where A, B, C are constants. The Arnold equation describes a steady solution to the three-dimensional (3D) Euler equation

\[
\frac{\partial v_i}{\partial t} + \sum_{k=1}^{3} v_k \frac{\partial v_i}{\partial x_k} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + f_i
\]

(3)

which express the behaviors of noncompressive perfect fluids on a 3D torus space. \((x_1, x_2, x_3)\) and \((v_1, v_2, v_3)\) denote the position and velocity of particle and \(p\) \(\rho\) denote the pressure, external force, and density, respectively. It is known that the Arnold equation shows periodic motion when one of the constant, for example \(C\), is 0 or small and shows chaotic motion when \(C\) is large[14].

2) Chua’s Circuit Equation (2-Double Scroll)

Chua’s circuit is one of the simplest physical models that has been widely investigated by mathematical, numerical and experimental methods. One of the main attractions of Chua’s circuit is that it can be easily built with less than a dozen standard circuit components, and has often been referred to as the “poor man’s chaos generator.” Since the Chua’s circuit is endowed with an unusually rich repertoire of nonlinear dynamical phenomena, it has become a universal paradigm for chaos. The Chua’s circuit and their nonlinear resistor are shown on Fig. 2(a),2(b) respectively.

We can derive the state equation of Chua’s circuit following as from Fig. 2(a) and 2(b) and then we also can get the phase plane looks like Fig. 3

\[
\begin{align*}
\dot{x}_1 &= \alpha (x_2 - g(x_1)) \\
\dot{x}_2 &= x_1 - x_2 + x_3 \\
\dot{x}_3 &= -\beta x_2
\end{align*}
\]

(5)

where

\[
g(x) = m_{x_i} x + \frac{1}{2} \sum_{i=1}^{3} (m_{i+1} - m_i)(|x+c_i| - |x-c_i|)
\]

C. Embedding of Chaos circuit in the Robot

In order to embed the chaos equation into the mobile robot, we define and use the Arnold equation and Chua’s circuit equation as follows.

1) Arnold equation

We define and use the following state variables:

\[
\begin{align*}
\dot{x}_1 &= D \dot{y} + C \cos x_2, \\
\dot{x}_2 &= D \dot{x} + B \sin x_1, \\
\dot{x}_3 &= \theta
\end{align*}
\]

(6)

where B, C, and D are constant.

Substituting (1) into (2), we obtain a state equation on \(\dot{x}_1\), \(\dot{x}_2\), and \(\dot{x}_3\) as follows:

\[
\begin{align*}
\dot{x}_1 &= Dv + C \cos x_2 \\
\dot{x}_2 &= Dv + B \sin x_1, \\
\dot{x}_3 &= \omega
\end{align*}
\]

(7)

We now design the inputs as follows [10]:

\[
\begin{align*}
\nu &= A / D \\
\omega &= C \sin x_2 + B \cos x_1
\end{align*}
\]

(8)

Finally, we can get the state equation of the mobile robot as follows:

Fig. 2. Chua circuit (a), Nonlinear resistor (b)
\[ \begin{align*}
\dot{x}_1 &= A \sin x_3 + C \cos x_2 \\
\dot{x}_2 &= B \sin x_1 + A \cos x_3 \\
\dot{x}_3 &= C \sin x_2 + B \cos x_1 \\
\dot{x} &= V \cos x_3 \\
\dot{y} &= V \sin x_3
\end{align*} \]  \tag{9}

Equation (9) includes the Arnold equation. Fig.4 and 5 show the phase plane of the gradients of the mobile robot of Arnold equation in x-y plane and in 3D plane respectively.

In the Nakamura et al.[10], they used phase plane components such as \((x, y), (y, z), (z, x)\) in the equation (9), but we used gradients of each variables such as,

\[ \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t} \]

for convenience in computation of chaotic path of the mobile robot.

Fig. 4 Phase plane of gradient \(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\) of Arnold equation in x-y plane and in 3D (\(v=1, \ A=1, \ B=0.5, \ C=0.5\))

Fig. 5. Trajectory of the mobile robot of Arnold equation, when there is no boundary.

2) Chua’s Equation

Using the methods explained in equations (6)-(9), we can obtain equation (10) with Chua’s equation embedded in the mobile robot.

\[ \begin{align*}
\dot{x}_1 &= a (x_2 - g(x_1)) \\
\dot{x}_2 &= x_1 - x_2 + x_3 \\
\dot{x}_3 &= -\beta x_2 \\
\dot{x}_4 &= V \cos x_3 \\
\dot{y}_4 &= V \sin x_3
\end{align*} \]  \tag{10}

Using equation (10), we obtain the embedding chaos robot trajectories with Chua’s equation. Fig.6 and 7 show the phase plane of the gradients of Chua’s equation, which is used for the computational convenience as in the Arnold equation.

Fig. 6. Phase portrait of gradient vector \(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\) in Chua’s circuit.

Fig. 7. Trajectory of the mobile robot of Chua’s equation, when there is no boundary.

D. Mirror Mapping.

Equation (9) and (10) assume that the mobile robot moves in a smooth state space without boundaries. However, real robots move in space with boundaries like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the robots approach walls or obstacles using Eq. (11) and (12). Whenever the robots approach a wall or obstacle, we calculate the robots’ new position by using Eq. (11) or (12).

\[ A = \begin{pmatrix}
\cos \theta & \sin \theta \\
\sin \theta & -\cos \theta
\end{pmatrix} \]  \tag{11}

\[ A = 1 / 1 + m \begin{pmatrix}
1 - m^2 & 2m \\
2m & 1 + m^2
\end{pmatrix} \]  \tag{12}

We can use equation (11) when the slope is infinity, such as \(\theta = 90^\circ\), and use equation (12) when the slope is not infinity.
3. Numerical Analysis of the Behavior of the Chaos Robot

We investigated by numerical analysis whether the mobile robot with the proposed controller actually behaves in a chaotic manner. In order to computer simulation, we applied mirror mapping and have shown it in fig. 7. The parameters and initial conditions are used as follows:

**A. Arnold equation case**

Coefficients:
\[ v = 1 \text{[m/s]}, A = 0.5 \text{[1/s]}, B = 0.25 \text{[1/s]}, C = 0.25 \text{[1/s]} \]

Initial conditions:
\[ x_1 = 4, \ x_2 = 3.5, \ x_3 = 0, \ x = 0, \ y = 0 \]

**B. Chua's equation case**

Coefficients:
\[ \alpha = 9, \ \beta = 14.286 \]
\[ m_0 = -\frac{1}{7}, \ m_1 = \frac{2}{7}, \ m_2 = -\frac{4}{7}, \ m_3 = m_1 \]
\[ c_1 = 1, \ c_2 = 2.15, \ c_3 = 3.6 \]

Initial conditions:
\[ x_1 = 4, \ x_2 = 3.5, \ x_3 = 0, \ x = 0, \ y = 0 \]

Fig. 8 shows the trajectories in which mirror mapping was applied only on the outer wall. In this case, the chaos robot has no obstacles, and we can confirm that the robot is adequately meandering along the trajectories of Arnold and Chua's equation and are covering the whole space in their chaotic manner.

4. The Chaotic Behavior of Chaos Robot with mirror MAPPING and Obstacle

In this section, we will study avoidance behavior of a chaos trajectory with obstacle mapping, relying on the Arnold equation and Chua's equation respectively.
5. The mobile Robot with Van der Pol equation obstacle.

In this section, we will discuss the mobile robot’s avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the mobile robot cannot move close to the obstacle and the obstacle is avoided.

A. VDP equation as an obstacle

In order to represent an obstacle of the mobile robot, we employ the VDP, which is written as follows:

\[ \begin{align*}
\dot{x} &= y \\
\dot{y} &= (1 - y^2) y - x
\end{align*} \]  \( (13) \)

From equation (11), we can get the following limit cycle as shown in Fig. 11.

![Fig. 11. Limit cycle of VDP](image)

B. Magnitude of Disturbing force from the obstacle

We consider the magnitude of disturbing force from the obstacle as follows:

\[ D = \frac{0.325}{(0.2D_k + 1)e^{3(0.2D_k - 1)}} \]  \( (14) \)

where \( D_k \) is the distance between each effective obstacle and the mobile robot.

We can also calculate the VDP obstacle direction vector as follows:

\[ \begin{bmatrix}
\tilde{x}_k \\
\tilde{y}_k
\end{bmatrix} = \begin{bmatrix}
x_o - y \\
0.5(1 - (y_o - y)^2)(y_o - y) - (x_o - x)
\end{bmatrix} \]  \( (15) \)

where \((x_o, y_o)\) are the coordinates of the center point of each obstacle. Then we can calculate the magnitude of the VDP direction vector \( L \), the magnitude of the moving vector of the virtual robot \( I \) and the enlarged coordinates \((1/2L)\) of the magnitude of the virtual robot in VDP\((x_k, y_k)\) as follows:

\[ \begin{align*}
L &= \sqrt{\left(\frac{x_k}{L}\right)^2 + \left(\frac{y_k}{L}\right)^2} \\
I &= \sqrt{\left(\frac{x_o}{L}\right)^2 + \left(\frac{y_o}{L}\right)^2} \\
x_{\circ k} &= \frac{x_k}{L} \frac{I}{2}, \quad y_{\circ k} = \frac{y_k}{L} \frac{I}{2}
\end{align*} \]  \( (16) \)

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation.

\[ \begin{bmatrix}
\sum_k \left(1 - \frac{D_k}{D_{\circ k}}\right) \tilde{x}_k \\
\sum_k \left(1 - \frac{D_k}{D_{\circ k}}\right) \tilde{y}_k
\end{bmatrix} \]  \( (17) \)

Using equations (14)-(17), we can calculate the avoidance method of the obstacle in the Arnold equation and Chua's equation trajectories with one or more VDP obstacles.

In Fig. 12, the computer simulation result shows that the chaos robot has two robots and a total of 5 VDP obstacles, including two VDP obstacles at the origin in the Arnold equation trajectories. We can see that the robot sufficiently avoided the obstacles in the Arnold equation trajectories.

![Fig. 12. Computer simulation result of obstacle avoidance with 2 robots and 5 obstacles in Arnold equation trajectories.](image)

In Fig. 13, the computer simulation result shows that the chaos robot surface has two robots and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the Chua’s equation trajectory. We can see that the robot sufficiently avoided the obstacles in the Chua’s equation trajectory.

![Fig. 13. Computer simulation result of obstacle avoidance with 2 robots and 5 obstacles in Chua’s equation trajectory.](image)

C. The relationship between two mobile robots

At this point, we consider two mobile robots that have VDP trajectories. If we do not consider the distance of the two
mobile robots, they may happen to collide. So we embedded the VDP equation in the movements of the mobile robot.

If the two robots approach each other, because they have a VDP equation with an unstable limit cycle, the two robots repel each other. As a result, the two robots never happen to collide.

We assume that if the distance between the two robots is less than 0.5 m, the possibility of collision is higher than over 0.5 m. Thus, we can say that two robots with less than 0.5 m between them have collided.

So, in order to avoid collision between the two robots, we can also apply a VDP equation in the Chua’s equation chaos robot system.

1) Arnold equation

In Fig 15(a), we can see that when no other action was taken, to the Arnold equation trajectory of each robot, the robots collide at several instances (700S, 1800S, 2800S, 4000S, 6000S, 6300S etc.).

In order to avoid collision, we applied a VDP equation to the Arnold equation trajectory of each robot. In Fig 15(b), we can see that there is no point when the robots collided. So, in order to avoid collision between the two robots, we need to apply a VDP equation in the Arnold equation trajectories.

2) Chua’s equation

In Fig 16(a), we can see that when VDP trajectories are not applied to the Chua’s equation trajectory of each robot, the robots collided at several instances (1600S, 2300S, 3200S, 4500S etc.). In order to avoid collision between the two robots, we applied VDP trajectories in each robot with a Chua’s equation. In Fig 16(b), we can see that there were no collisions.

Fig. 14. Robot to avoid collision according to VDP equation

Fig. 15. Inter-robot distance (a) when no action taken, (b) when VDP equation applied to the Arnold equation trajectories of each robot

Fig. 16. Inter-robot distance in which were not applied (a) and were applied (b) VDP trajectories with Chua’s equation

6. The calculation of rate of coverage

In order to calculate the rate of coverage upon the surface, we employed a random walk, an Arnold equation and a Chua’s equation trajectory and then, we compared the coverage rate.

A. The comparison of robot trajectories

In Fig. 17, we can see the robot trajectories in the random walk (a), the Arnold equation (b), and the Chua’s equation (c) respectively.

Fig. 18 plots the position of the robot at regular interval during 7500 second runtime in the random walk (a), the Arnold equation (b), and the Chua’s equation (c) respectively. The chaotic mobile robots with the Arnold and Chua’s equation trajectories in Fig 18(b) and (c) were able to cover the work space more efficiently compared to the random walk robot(a)

B. Comparison of coverage rate in random walk, Arnold equation and Chua’s equation

Several initial values were used in each run of the random walk, the Arnold equation and the Chua’s equation. To compare the coverage rates of each equation, a Monte-Carlo method was applied to obtain an average coverage rate.

In Fig. 19, we can see that the rate of coverage of the Chua equation (2- Double Scroll) and Arnold equation trajectories are superior to the random walk, and among those, the Chua’s equation is slightly superior to the Arnold equation in the coverage rate. Until about 2500 seconds, the coverage rate of the Arnold equation is slightly higher, but after 2500 seconds, this situation is reversed.
C. Comparison of coverage rate in Arnold equation and Chua's equation with two robots.

In this section, we examine the rate of coverage in the Arnold equation and Chua's equation using two robots.

As explained before, a Monte-Carlos was applied to obtain an average coverage rate for each trajectory. As seen in Fig. 20, there is little difference in the efficiency of coverage between the two trajectories, but overall the Chua's equation trajectory was more efficient in covering the work space.
D. Optimal robot number

In order to verify the optimal robot number in the search area surface, we calculated the amount of time required to reach 90% coverage for each set of robots.

![Graph](https://via.placeholder.com/150)

Fig. 21. Number of robots versus the time require to reach 90% coverage

As seen in Fig. 21, in either trajectory the optimal robot number was three. Also, we can see that the Chua’s circuit equation was slightly superior in efficiency.

7. Conclusion

In this paper, we proposed a chaotic mobile robot, which employs a mobile robot with Arnold equation and Chua’s equation trajectories, and also proposed an obstacle avoidance method in which we assume that the obstacle has a Van der Pol equation with an unstable limit cycle.

We designed robot trajectories such that the total dynamics of the mobile robots was characterized by an Arnold equation or Chua’s equation, and we also designed the robot trajectories to include an obstacle avoidance method. By the numerical analysis, it was illustrated that obstacle avoidance methods with a Van der Pol equation that has an unstable limit cycle gave the best performance.

In order to make a chaotic behavior in the robot system, we applied the Arnold equation and Chua’s equation. As a result, we realized that the rate of coverage of Chua’s equation is superior to the Arnold equation. We also showed that the optimal robot number in each trajectory was three.

Reference

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