Modelling of intelligent training system by a generalized net

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Abstract

A generalized net model of a training process with modified fuzzy marks in answers of the trained objects and their current and total modified fuzzy evaluation, turning themes, estimation criteria and others are discussed and interpreted.

Key Words: Artificial intelligence, Fuzzy set, Fuzzy evaluation, Generalized net, Intelligent tutorial environment, Machine learning, User model.

I. Introduction

The idea for application the apparatus of the Generalized Nets (GNs, see[1]) for processes related to machine learning were discussed in [3,8-10]. Here we shall make the next step, extending some of our previous ideas.

In this paper we discuss the possibility to obtain modified fuzzy evaluations of the students' estimation of their mathematical training in schools and universities. In order to avoid the examiner's subjective evaluating in the traditional estimating systems, we shall modify the computational method of the fuzzy evaluation from [6] and we shall apply modified fuzzy computation over the students' answers.

In the present paper we shall extend the GN-models of the learning processes described in [3,8] with capabilities for alternating training strategy with respect to performance of the trained object and with possibility for estimation and change of the training course. Similarly to [3,8], the GN-models for learning are described in the general case, i.e., for training of abstract objects (neural networks, genetic algorithms, expert systems, intellectual games, GNs, abstract systems, etc.).

The possibility of forgetting pieces of the accumulated information during education has been discussed in [9]. Paper [10] describes the extended GN model now having the opportunity of evaluating and modifying of the tutor material, rendering an account of the educated objects' degree of assimilation. New is also the criterion of estimating the tutoring material. Represented in [3] was the general case of GN models for educational purposes - the abstract objects neuron networks, genetic algorithms, expert systems, intellectual games, abstract systems, etc. An educational GN model working over people, instead of abstract systems, has been discussed in [8] and [9]. These offer different mechanisms for estimating the objects' knowledge and the level of assimilation of the taught material, applying fuzzy sets estimations. On the basis of the information accumulated, we define some techniques of choice and change of the topics and tasks, the difficulty level and the style of stating the questions.

As in our previous models, here we shall use the apparatus of the Intuitionistic Fuzzy sets (IFS, see [2]). IFSs are extensions of the ordinary fuzzy sets with two degrees: of membership (µ) and of non-membership (ν), such that 0≤µ+ν≤1. In this case a degree of uncertainty (π=1−µ−ν) exists. In the ordinary fuzzy set case, π=0.

The real numbers from the set [0, 1] act for the estimations that indicate the level of assimilation of a certain piece of knowledge. In the ordered tuple <µ, ν> µ corresponds to the number of the right answers referred to the total number of the answers, and ν stays for the number of the wrong answers referred to the total number of the answers. The degree of uncertainty π=1−µ−ν stays for the cases when the answers cannot be interpreted definitely or there have been technical mistakes.

At the beginning, when still no information has been derived about the TO, all estimations are assigned value of <0, 0>. The current estimation is calculated on the basis of the preceding estimations (if any) by means of the following formula:

<µₖ₊₁, νₖ₊₁> = <µₖ + mₖ, νₖ + nₖ>ₖ + 1

where <µₖ, νₖ> is the previous estimation and <mₖ, nₖ> is the estimation of the current task for mₖ, nₖ∈[0, 1] and

2. A GN-model.

The GN-model from Fig. 1 gives possibility not only to trace the changes of the parameters of the learning object, but also a possibility for tracing of status of already studied knowledge, i.e., the degree of its remembering and the possibility of the knowledge using. We can evaluate the influence of some individual TO's characteristics, also. Moreover, the effect of training themes and criteria for their changes can be estimated. The GN represents grouping of the objects from a point of view of the different training themes.

Similarly to [8], the GN constructed here is a subnet of the First GN from [3]. We shall discuss only this subnet, that describes the process of learning which traces a concrete theme.

The places of GN \( E \) are of four types: \( a \), \( b \), \( c \), and \( d \) places.

The \( a \) places represent the status of the learning objects, the \( b \) places represent the status of the themes for learning, the \( c \) places - the process of collecting, estimating and reproducing of previous knowledge, and the \( d \) places - the criterion for estimation of the currently used themes. The learning objects are interpreted by \( a \) tokens that transfer in the \( a \) places, the themes for learning are interpreted by \( \beta \) tokens that transfer in the \( b \) places, the levels of the personal knowledge are represent by \( \gamma \) tokens that transfer in the \( c \) places, and the criteria for estimation of training themes are represent by \( \sigma \) tokens that transfer in the \( d \) places.

For simplicity, we shall not formalize the form of token characteristics and transition condition predicates.

Initially, one or more \( a \) tokens enter place \( a_1 \) with initial characteristic \( x^a_1 = \text{"identifier of the object of learning (e.g. the student's name)"} \). This token splits to two tokens: one of them (marked again by \( a \)) enters place \( a_2 \) with a new characteristic "current training course", while the second one (let us mark it by \( a \)) enters place \( c_1 \) with characteristic "level of TO's knowledge".

When predicate \( W_j \) (see below) has a truth-value "true", then the \( a \) token is transferred in place \( a_2 \), where it obtains the characteristic "the theme, that will be a training object.”.

This theme can be related to a set of criteria and the TO's estimations, evaluated on criteria base at the time of the training, but the theme compulsory is related with the level of TO's knowledge, represented by the \( \gamma \) characteristic.

At the initial time-moment of the GN-functioning one \( \beta \) token stays in place \( b_1 \) with initial characteristic \( x^\beta_1 = \text{"list of all themes for learning"} \).

When predicate \( W_1 \) is "true", token \( \beta \) splits to two tokens. One of them (let us mark it again by \( \beta \)) stays in the same place \( b_1 \), while the other one (let us mark it by \( \beta_2 \) is its current number) enters place \( b_2 \) with a characteristic \( x^\beta_2 = \text{"list of themes for training of the object (student) \ a_1"} \).

So, for the \( i \)th object \( a_i \), entering place \( a_2 \), the training themes are determined. Each theme is represented by \( \beta_i \) token (shortly, \( \beta \) token) with characteristic \( x^\beta_i = \langle \text{id}, \text{theme} \rangle \), where \( \text{id} \) is a natural number that represents the theme identifier and

\[
\text{theme} = \{ C | C \in U \},
\]

where \( U \) denotes the set of criteria for estimation of a concrete training object according to a given training theme.

Each of the \( b \) tokens can transfer from place \( b_2 \) to place \( b_3 \), if there exists at least one \( a \) token trained on this theme, i.e., when \( W_3 = \text{true} \). On the contrary, the \( \beta \) token transfers to place \( b_1 \) and leaves the GN.

In place \( d_1 \) token \( \sigma \) with initial characteristic "criterion for estimating the degree of theme acquirement" enters. Then the token enters place \( d_2 \) and stays there by the end of the net functioning, when it transfers to place \( d_2 \).

The \( \gamma \) token in place \( c_2 \) obtain characteristic "level of knowledge after full course of training", and obtain no new characteristic in place \( c_4 \). On entering place \( c_1 \) the token will obtain a new characteristic, namely "current level of knowledge over all training themes after the previous lecturing cycle". We shall note that here just like for the \( a \) and \( \beta \) token estimations we can use intuitionistic fuzzy values. We are going to discuss this matter later on.

The GN \( E \) contains five transitions having the following forms:

\[
T_1 = (a_1, a_2, b_1, c_1, a_1, b_1, b_2, c_4, r_1, \wedge (\forall (a_1, a_2, b_1, c_2))
\]

where

<table>
<thead>
<tr>
<th></th>
<th>( a_2 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( c_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>true</td>
<td>false</td>
<td>false</td>
<td>true</td>
</tr>
<tr>
<td>( W_1 )</td>
<td>true</td>
<td>false</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>false</td>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>false</td>
<td>false</td>
<td>true</td>
<td>true</td>
</tr>
</tbody>
</table>

where \( W_1 = \text{"There is a necessity for determining of a new topic."} \).

The \( a \) tokens enter place \( a_2 \) with no new characteristics, while the \( \beta \) token from place \( b_1 \) splits into two tokens. The first of them is the old token \( \beta \) that continues cycling in place \( b_1 \) without obtaining any new characteristic and a second (current) token \( \beta_\text{cw} \) (let us note it below as a \( \beta \) token) with characteristic

\[
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\]
$x_0^a = \text{"list of the current topics chosen from } x_0^a\text{"}$

\[ T_2 = \langle a_2, a_3, a_4, b_3, b_5, a_3, b_3, b_4, r_2, \langle \forall (a_2, a_3), \forall (b_2, b_5) \rangle \rangle, \]

where

\[
\begin{array}{c|c|c|c}
 & a_2 & b_3 & b_4 \\
\hline
r_2 & a_3 & W_2 & \text{false} \\
a_3 & a_3 & W_2 & \text{false} \\
a_4 & W_2 & \text{false} & \text{false} \\
b_3 & \text{false} & W_2 & \text{false} \\
b_4 & \text{false} & \text{false} & W_3 \\
\end{array}
\]

$W_2 = \text{"there are a learning theme for the current object and an object, to be learnt on the current theme."}$

$W_3 = \text{"the training procedure has finished."}$

The tokens obtain the following current characteristics

\[ x_0^a = \langle \text{id}, \text{id\_theme}, \text{estimations}(a) \rangle \]

in place $a_2$, where $\text{id\_theme}$ is the identifier of the assigned learning theme and

\[ x_0^b = \langle \text{id}, \text{theme}, \text{objects} \rangle \]

in place $b_3$, where

$\text{object} = (a) \ a \in K \ \& \ pr_2(x_0^a) = \text{id}$.  

where here and below $pr_2 X$ will stay for the $2$-th projection of a $\tau$-dimensional set $X$ for $\tau \geq 2\eta$, and an empty set for $\tau > \eta$. $a$- and $\beta$-tokens do not obtain any characteristic in place $b_4$.

Transition $T_2$ represents the process of object training by a given theme. This process is described in more details by the GN $E'$. The connection between both of the nets can be realized by the hierarchical operator $H_1$, applied over GN $E'$, which substitutes place $a_2$ with subnet $E'$.

\[ T_3 = \langle a_3, c_3, c_4, r_3, \forall (a_3) \rangle, \]

where

\[
\begin{array}{c|c|c|c}
r_3 & a_3 & W_4 & \text{false} \\
c_3 & c_3 & W_4 & \text{false} \\
c_4 & c_4 & W_4 & \text{false} \\
d_4 & d_4 & W_4 & \text{false} \\
d_5 & d_5 & W_4 & \text{false} \\
\end{array}
\]

$W_4 = \text{"the time for learning has finished."}$

The tokens obtain characteristic in place $a_4$

\[ x_0^a = \langle \text{id}, \text{id\_theme}, \text{estimations}(a) \rangle \]

where

\[
\text{estimations}(a) = \{ (\mu_2^{c_3}, \nu_2^{c_3}) \mid (\forall j: 1 \leq j \leq |pr_3(x, w_{3-\eta})|) \}
\]

\[
\mu_2^{c_3}, \nu_2^{c_3} \in R^+ \ \& \ \mu_2^{c_3} + \nu_2^{c_3} \leq 1 \}
\]

i.e., it is a set of intuitionistic fuzzy estimations (see [2]) for every associated criterion, where $R^+ = x\ \forall x \in R \ \& \ x \geq 0$.

In place $d_1$ a token delta with initial characteristic "criterion for estimating the degree of theme acquirement" enters. Then the token enters place $d_3$ and stays there by the end of the net functioning, when it transfers to place $d_2$. The g-token in place $c_3$ will obtain characteristic "level of knowledge after the held full course of training", while it is not going to obtain any new characteristic in place $c_4$. On entering place $c_1$ the token will obtain a new characteristic, namely "current level of knowledge over all training themes after the previous lecturing cycle". We shall note that here just like for the $a$- and $\beta$-tokens estimations we can use intuitionistic fuzzy values.

To reach simplicity of the exposition, this set of estimations can be reduced to a single intuitionistic fuzzy estimation (by different manners discussed in [2]) that gives an account of all the elements of the set.

\[ T_4 = \langle a_4, c_2, d_1, d_3, a_5, a_6, a_7, c_3, c_4, d_2, d_3, r_4, \forall (a_4, c_2, \forall (d_1, d_3)) \rangle, \]

where

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
r_4 & a_5 & a_6 & a_7 & c_3 & c_4 & d_2 & d_3 \\
\hline
& a_5 & W_5 & W_6 & W_7 & \text{false} & \text{false} & \text{false} \\
a_6 & a_6 & W_5 & W_6 & W_7 & \text{false} & \text{false} & \text{false} \\
a_7 & a_7 & W_6 & W_7 & \text{false} & \text{false} & \text{false} & \text{false} \\
d_1 & d_1 & W_6 & W_7 & \text{false} & \text{true} & \text{true} & \text{false} \\
d_3 & d_3 & W_5 & W_6 & W_7 & \text{false} & \text{false} & \text{false} \\
\end{array}
\]

where

$W_5 = \text{"the answer of the current test is right."}$

$W_6 = \text{"the answer of the current test is wrong."}$

$W_7 = \text{"the answer of the current test is right and this is the final learning theme of the current object."}$

$\alpha$-token obtain in place $a_3$, the following characteristic:

\[ x_0^a = \langle \text{id}, \text{id\_theme}, \text{estimations}(a) \rangle \]

where
estimations\(_{(a_i)} \setminus \{ (\mu_j^a, \nu_j^a) \mid (\forall j : 1 \leq j \leq |pry(x, ur^a)|)\}
\mu_j^a, \nu_j^a \in \mathbb{R}^+ \& \mu_j^a + \nu_j^a \leq 1}\},
\)
i.e., it is a set of intuitionistic fuzzy estimations (see [2]) for every associated criterion.

To reach simplicity of the exposition, this set of estimations can be reduced to a single intuitionistic fuzzy estimation (by different manners discussed in [2]) that gives an account of all the elements of the set.

With this characteristic the token will return to transition \(T_5\) and it will continues its training with a new theme from the list of the themes determined for it.

The \(a\)–token obtain the characteristic in place \(a_5\):
\"a new problem for solving.\"
and the \(a\)–token obtain in place \(a_7\) the final characteristic:
\"final training estimation.\"

The last estimation can be determined, e.g., as a medium of all current estimations for the different criterions.

The \(\gamma\)–token in place \(c_3\) will obtain characteristic \"level of knowledge after the held full course of training\", while it is not going to obtain any new characteristic in place \(c_4\). On entering place \(c_1\) the token will obtain a new characteristic, namely \"current level of knowledge over all training themes after the previous lecturing cycle\". We shall note that here just like for the \(a\)– and \(\beta\)–tokens' estimations we can use intuitionistic fuzzy values.

\[T_5 = \langle a_5, a_{10}, b_3, a_4, a_5, a_{10}, b_5, \]
\[r_5, \land (\lor (a_4, a_{10}), b_3)\],
where
\[
\begin{array}{cccc}
  a_5 & a_9 & a_{15} & b_5 \\
  r_5 & W_5 & W_9 & W_{10} \\
  a_{10} & W_5 & W_9 & W_{10} \text{false} \\
  b_5 & \text{false} & \text{false} & \text{false} & \text{true}
\end{array}
\]

where
\(W_5\)\: \"the problem is being correctly solved\",
\(W_9\)\: \"the problem is not solved correctly and the TO must be re-trained or trained to other themes\",
\(W_{10}\)\: \"the problem is not solved correctly, but the TO know the previous themes.\"

The \(a\)–token obtain in place \(a_5\) the characteristic:
\[x_5^\gamma = \langle \mu^\gamma, \nu^\gamma \rangle,\]
\text{estimations}(a_j)\};
\]
where \text{estimations} is as above.

The \(a\)–token obtains in place \(a_{10}\) the characteristic:
\"list of other themes which the TO must study (these themes may be both familiar or not familiar for the TO)\"
and it obtains in place \(a_{15}\) the characteristic:
\text{\text{"a new problem for solving."}}

The \(\beta\)–token obtains in place \(b_5\) the current characteristic
\[x_5^\beta = \langle \mu, \nu \rangle,\]
\text{where} \(\mu\) and \(\nu\) are estimations of the levels of assimilation and of non-assimilation of the training themes by the training object, that are calculated for the first degree, as a sum of the characteristics of the tokens in places \(a_5\), \(a_7\) and \(a_9\), and for the second degree - by the sum of the characteristics of the tokens in place \(a_{10}\). Finally, the degree of uncertainty can be obtained by the sum of the characteristics of the tokens in place \(a_{12}\). Of course, we must obtain the last values directly by the previous two values, but using the last tokens characteristics we can hold the control over the estimation process.

The characteristic of the \(\delta\)-token contains four level-values \(M_{\text{max}}, M_{\text{min}}, N_{\text{max}}, N_{\text{min}}\).

If \(\mu > M_{\text{max}}\) and \(\nu < N_{\text{min}}\), then the \(\beta\)-token in place \(b_5\) obtains characteristic \"the course is very easy (elementary).\"
If \(\mu < M_{\text{max}}\) and \(\nu > N_{\text{min}}\), then the \(\beta\)-token in place \(b_5\) obtains characteristic \"the course is very difficult.\"
In the two other cases \(\beta\)-token in place \(b_5\) obtains characteristic \"the course is suitable for the training objects at whole.\"

Therefore, on the basis of the respective \(\beta\)-characteristic, the G-model can realize a decision making about the necessity for a change of the training themes, their volumes, their levels of difficulty etc.

The \(\gamma\)-tokens also obtain intuitionistic fuzzy estimations that correspond to the degrees with which the training object "knows" \(\mu^\gamma\) and, respective "does not know" \(\nu^\gamma\) all themes studied by the moment. Of course, here \(\mu^\gamma + \nu^\gamma \leq 1\) and \(\mu^\gamma, \nu^\gamma \in [0,1]\). The number \(1 - \mu^\gamma - \nu^\gamma\) determines the degree with which the training object is not assured in one's knowledge. The so constructed G-model gives the possibility that each token \(\gamma\) entering place \(c_3\) to actualize its estimations of all themes, studied by the training object to whom/which this token corresponds. Thus, we can account the degree with which the training object has already forgotten the previous studied themes. We can also obtain the estimations of the training object's individual characteristics. This means that the estimation \(\langle \mu^\gamma, \nu^\gamma \rangle\), of the level of knowledge of the \(i\)-th training object about \(j\)-th theme after some time-steps (during which this theme will not be an object of training), may be changed by the estimation \(\langle \mu^\gamma, \nu^\gamma \rangle\), such that
\[\mu^\gamma \geq \mu^\gamma_i \; \text{and} \; \nu^\gamma \leq \nu^\gamma_i.\]

On the other hand, if the last themes, studied by the \(i\)-th training object, are related to the \(j\)-th theme (studied by the training object some time ago), for the new values of \(\mu^\gamma_i\) and \(\nu^\gamma_i\) can be valid the opposite inequalities.
3. Approach for computing of fuzzy estimations with respect to training object's

Now, we will consider modified fuzzy marks (mfms in short) in students' answerscript for performance assessment of mathematic at middle and high schools. In order to avoid any human biasness of teachers with any traditional marking system, we modify the fuzzy evaluation method in the paper [6] and the modified fuzzy evaluation (mfe in short) applies to students' evaluation. And we also discuss some examples of mfe tables.

We consider, for the purpose of an example only, the standard guidelines; (I) Letter grades according to the level of performance judged by teachers are adopted by five fuzzy variables; excellent- $E_1$, very good- $E_2$, good- $E_3$, satisfactory- $E_4$, and unsatisfactory- $E_5$. (II) Each question is constructed by five steps; from 1-step, which is an easy minor question which ask for fundamental concepts, to 5-step, which is the hardest minor question which ask for applications. Using the definition of fuzzy sets, we will define standard fuzzy marks of five fuzzy variables.

Let $X = \{0, 20, 40, 60, 80, 100\}$ be an universal set. If $F$ is one among $A, B, C, D, E$ which are standard fuzzy marks as in the definition 2.4, if the $M$'s are a student's fuzzy marks for $k = 1, 2, \ldots, r$ and $r$ number of rows, and if $e_1 = 1$, $e_2 = 0.9$, $e_3 = 0.7$, $e_4 = 0.6$, $e_5 = 0.5$ are non-fuzzy marks of $A, B, C, D, E$, respectively, then the mfe table of student's fuzzy marks $M_x$ is a $(r + 2) \times 1$ - matrix type structure in the below table 1, where $r$ is the total number of questions and $w_k$ is the weight of the $k$-th question among $r$ questions. At the bottom there is the total mark.

Standard fuzzy marks (smf in short) of five fuzzy variables are defined by

$E_1$ (excellent) = \{0/0, 0/20, 0.8/40, 0.9/60, 1/80, 1/100\}

$E_2$ (very good) = \{0/0, 0/20, 0.5/40, 0.9/60, 0.9/80, 0.9/100\}

$E_3$ (good) = \{0/0, 0.1/20, 0.5/40, 0.9/60, 0.9/80, 0.2/100\}

$E_4$ (satisfactory) = \{0.1/0, 0.4/20, 0.9/40, 0.9/60, 0.2/80, 0/100\}

$E_5$ (unsatisfactory) = \{0.1/0, 0.1/20, 0.1/40, 0.5/60, 0/80, 0/100\}

<table>
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<th>QUESTtion</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>GRADE</th>
<th>$w_k$</th>
<th>MARK</th>
</tr>
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<td>$M_1$</td>
<td></td>
<td></td>
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<td></td>
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<tr>
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<td>...</td>
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<tr>
<td>$M_r$</td>
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</tbody>
</table>

Table 1: The mfe table

A smf like \{(0.0, 0.1/20, 0.8/40, 0.9/60, 0.4/80, 0.2/100)\} in a student's answerscript for a question including five minor questions shows to respondent the degrees 0.1, 0.8, 0.9, 0.4, 0.2 of a teacher's satisfaction for that answer in 0, 20, 40, 60, 80, 100 percent, respectively. For each $i = 1, \ldots, 5$, $a_i \in [0, 1]$ represent a teacher's marking in a student's answerscript for $i$-step minor question. And we also define a mfm in student's answerscript for a question including five steps minor questions. Recall that a fuzzy mark is a fuzzy set of an universal set $\mathcal{X}$ whose graph is any one of the following types: (i) decreasing, (ii) increasing, (iii) at first increasing and then decreasing (see [6], definition 3.3). But we consider a mfm as a fuzzy set satisfying the following condition.

A mfm $M$ in student's answerscript for a question including five steps minor questions is a fuzzy set $M$ of $X$ defined by

$\mu_M(0) = 0.8 - a_1$, $\mu_M(20) = 1 - a_1$, $\mu_M(40) = a_2 - 0.2$, $\mu_M(60) = a_3 - 0.1$, $\mu_M(80) = a_4$, $\mu_M(100) = a_5$,

where $a_i \in \{0, 1\}$ represent a teacher's marking in a student's answerscript for $i$-step minor question, for $i = 1, \ldots, 5$.

Therefore, we can write also:

$M = \{(0.8 - a_1)/0, (1 - a_1)/20, (a_2 - 0.2)/40, (a_3 - 0.1)/60, a_4/80, a_5/100\}$.

Let $X = x_1, x_2, \ldots, x_n$ be a finite universal set and $F,M$ two fuzzy sets of $X$. If $f = (m_{f(x_1)}, \ldots, m_{f(x_n)})$ and $m = (m_{g(x_1)}, \ldots, m_{g(x_n)})$ are the vector representations of the fuzzy sets $F$ and $M$, respectively, then the matching function $S(f, m)$ of $f$ and $m$ is defined by

$S(f, m) = \frac{f \cdot m}{\max\{f \cdot f, m \cdot m\}}$

where $f \cdot m = \sum f(x) \cdot m(x)$, $f \cdot f = \sum f(x) \cdot m(x)$ and $m \cdot m = \sum m(x) \cdot m(x)$. In this case, the value of $S(f, m) \in [0, 1]$ is called the degree of similarity between two fuzzy sets $F, M$.

Using the degree of similarity between a student's mfm and standard fuzzy marks, we define the following mfe of student's mfm.

Let Let $X = x_1, x_2, \ldots, x_n$ be a finite universal set. If $F$ is one among $E_1, E_2, E_3, E_4, E_5$ which are standard fuzzy marks as in the definition 2.4 and if $M$ is a student's fuzzy mark, then the mfe $E_5$, of student's mfm $M$ is a standard fuzzy mark $E_{i_0}$ satisfying the following condition: the number $i_0$ is the smallest number $i$ such that

$S(E_k, M) = \max\{S(E_1, M), S(E_2, M), S(E_3, M), S(E_4, M), S(E_5, M)\}$

Now, we construct the mfe table of student's mfm for $r$ questions including five steps which are 5 minor questions. We denote that $\mu = \mu(0) = m_0$, $\mu(20) = m_1$, $\mu(40) = m_2$, $\mu(60) = m_3$, $\mu(80) = m_4$, $\mu(100) = m_5$.

We discuss the following example of mfe. We consider a student's answerscripts for 3 questions including five minor questions and so take $r = 3$. We note that the mfe table is a $3 \times 6$-type matrix each row of which is a mfe (see Table 2 in the next example).
Let $e_1 = 1$, $e_2 = 0.9$, $e_3 = 0.7$, $e_4 = 0.6$, $e_5 = 0.5$ be non-fuzzy marks of standard fuzzy marks $E_1$, $E_2$, $E_3$, $E_4$, $E_5$, respectively. Actually these non-fuzzy marks may be suitably adopted by the concerned institution. In total there were 3 questions to be answered: TOTAL MARKS = 30. $M_1$ carries $w_1 = 9$ marks, $M_2$ carries $w_2 = 9$ marks, and $M_3$ carries $w_3 = 12$ marks. Then, we consider, for purpose of an example only, a student has the following marks of three questions $\{Q_1, Q_2, Q_3\}$ including five minor questions:

- $Q_1$: $a_1 = 1$, $a_2 = 0.8$, $a_3 = 0.9$, $a_4 = 0.6$, $a_5 = 0.6$
- $Q_2$: $a_1 = 1$, $a_2 = 0.7$, $a_3 = 0.5$, $a_4 = 0.5$, $a_5 = 0$
- $Q_3$: $a_1 = 1$, $a_2 = 0$, $a_3 = 1$, $a_4 = 0.7$, $a_5 = 0.6$

Using above marks, we can find the mfe of a student's mfn:

\[
\max\{S[2E_1,M_1], S[E_2,M_1], S[E_3,M_1], S[E_4,M_1], S[E_5,M_1]\} = S[E_5,M_1]
\]

(1) $M_1 = \{0.0, 0.0, 0.8, 0.8, 0.6, 0.6\}$ and

\[
\max\{S[E_1,M_1], S[E_2,M_1], S[E_3,M_1], S[E_4,M_1], S[E_5,M_1]\} - S[E_5,M_1]
\]

(2) $M_2 = \{0.0, 0.0, 0.4, 0.5, 0.1\}$ and

\[
\max\{S[E_1,M_2], S[E_2,M_2], S[E_3,M_2], S[E_4,M_2], S[E_5,M_2]\} - S[E_5,M_2]
\]

(3) $M_3 = \{0.0, 0.8, 0.9, 0.7, 0.6\}$ and

\[
\max\{S[E_1,M_1], S[E_2,M_1], S[E_3,M_1], S[E_4,M_1], S[E_5,M_1]\} - S[E_5,M_1]
\]

And then we obtain the total marks of the mfe of a student's mfn.

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
<th>$m_5$</th>
<th>GRADE</th>
<th>$e_8$</th>
<th>$w_8$</th>
<th>MARK ($e_8 \times w_8$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0</td>
<td>0</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>C</td>
<td>0.7</td>
<td>9</td>
<td>6.3</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.4</td>
<td>0.5</td>
<td>0</td>
<td>C</td>
<td>0.7</td>
<td>6.3</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
<td>B</td>
<td>0.7</td>
<td>6.3</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23.4</td>
</tr>
</tbody>
</table>

If two students have the following mfn:

Kim: $a_1 = 1$, $a_2 = 0.2$, $a_3 = 0.7$, $a_4 = 0.8$, $a_5 = 0.7$; $\Sigma_k = 3.1$

Park: $a_1 = 1$, $a_2 = 1$, $a_3 = 0.8$, $a_4 = 0.6$, $a_5 = 0$; $\Sigma_p = 3.1$

then they have the same total sum 3.4, but they have different grades each other(see table 3).

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
<th>$m_5$</th>
<th>GRADE</th>
<th>$e_8$</th>
<th>$w_8$</th>
<th>MARK ($e_8 \times w_8$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kim</td>
<td>0</td>
<td>0</td>
<td>0.6</td>
<td>0.8</td>
<td>0.7</td>
<td>B</td>
<td>0.9</td>
<td>9</td>
<td>8.1</td>
</tr>
<tr>
<td>Park</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>C</td>
<td>0.7</td>
<td>9</td>
<td>6.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MARK ($e_8 \times w_8$)</th>
<th>8.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Kim} = 8.1$</td>
<td></td>
</tr>
<tr>
<td>$\text{Park} = 6.3$</td>
<td></td>
</tr>
</tbody>
</table>

4. Conclusion

The proposed model can be considered as one of the possible models of the machine learning process. It gives the opportunity for monitoring the work of training procedures, for analyzing their efficiency at the different stages of the learning process, and for studying their mutual influence in their parallel work.

Many detailizations can be made by using hierarchical operators that replace a given transition or place with a sub-net with the same behavior. More of the model's parameters can be considered as a token's characteristics from an additional contour that models the training process course. In this contour one can accomplish optimizations with respect to a given aim or statistic information can be collected.

References


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