A Fault Detection System Design for Uncertain Fuzzy Systems

Seog-Hwan Yoo, Byung-Jae Choi

School of Electronic Engineering, Daegu University
Gyungsan, Gyungbuk, Korea

Abstract

This paper deals with a fault detection system design for uncertain nonlinear systems modelled as T-S fuzzy systems with the integral quadratic constraints. In order to generate a residual signal, we used a left coprime factorization of the T-S fuzzy system. From the filtered signal of the residual generator, the fault occurrence can be detected effectively. A simulation study with nuclear steam generator level control system shows that the suggested method can be applied to detect the fault in actual applications.

Key Words: T-S fuzzy systems, Left coprime factorization, Steam generator, Linear matrix inequality, Fault detection

1. Introduction

Recently, fault detection of control systems has been an active research area due to the growing complexity of modern automatic control systems. A fault can be defined as an unexpected change in a system, such as a component malfunction, which tends to cause undesirable behaviour in the overall system performance. The purpose of the fault detection system design is to detect the occurrence of a fault as fast as possible.

A lot of works for the fault detection system design have been developed based on generation of the residual signal [1-5]. In order to design the residual generator, the use of robust parity equations [1], eigen-structure assignment [2], unknown input observers [3] and \( H_\infty \) optimal estimation approaches [4,5] have been intensively investigated. But these works are mainly focused on linear systems. Comparatively little work has been reported for the fault detection system design of nonlinear systems.

In the past few years, there has been growing interest in fuzzy control of nonlinear systems and there have been many successful applications. Among them, a controller design method for nonlinear dynamic systems modelled as a T-S (Takagi-Sugeno) fuzzy model has been intensively addressed [6-8]. Unlike a single conventional model, this T-S fuzzy model usually consists of several linear models to describe the global behaviour of the nonlinear system. Typically the T-S fuzzy model is described by fuzzy IF-THEN rules.

In this paper, using the coprime factorization approach we develop a fault detection scheme for T-S fuzzy systems with the integral quadratic constraints (IQC’s). The IQC’s can be used conveniently to describe uncertain parameters, time delays, unmodeled dynamics, etc [9]. An application to fault detection of a nuclear steam generator control system is presented in order to demonstrate the efficacy of the proposed scheme. We think that the primary contribution of this work is that the fault detection system design is extended from linear systems toward nonlinear systems using T-S fuzzy approach.

2. Uncertain Fuzzy Systems

We consider the following fuzzy dynamic system with the IQC’s.

Plant Rule \( i (i=1,\cdots,r) : \)

IF \( \rho_i (\theta) \) is \( M_{\bar{A}_i} \) and \( \cdots \) and \( \rho_i (\theta) \) is \( M_{\bar{A}_i} \), THEN

\[
\begin{align*}
\dot{x}(t) &= A_i x(t) + \sum_{j=1}^{r} F_{ij} w_j(t) + B_i u(t) \\
z_j(t) &= H_{ij} x(t) + J_{ij} u(t) \\
y(t) &= C_i x(t) + \sum_{j=1}^{r} G_{ij} w_j(t) + D_i u(t) \\
w_j(t) &= \theta_j z_j(t),
\end{align*}
\]

(1)

where \( r \) is the number of fuzzy rules, \( \rho_i (\theta) \) and \( M_{\bar{A}_i} \) \((k=1,\cdots,g)\) are the premise variables and the fuzzy set respectively. \( x(t) \in \mathbb{R}^n \) is the state vector, \( u(t) \in \mathbb{R}^m \) is the input, \( y(t) \in \mathbb{R}^p \) is the output variable and \( w_j(t) \in \mathbb{R}^{k_j} \), \( z_j(t) \in \mathbb{R}^{k_j} \), are variables related to uncertainties. \( A_i, B_i, \ldots, D_i \) are real matrices with compatible dimensions. \( \theta_j \) is an uncertain operator described by the following IQC’s.

\[
\int_{0}^{\infty} w_j(t)^T w_j(t) dt \leq \int_{0}^{\infty} z_j(t)^T z_j(t) dt, \quad j=1,\cdots,q
\]

(2)

Let \( \mu_{\nu_i}, i=1,\cdots,r \), be the normalized membership function defined as follows:
3. Fault Detection System

In this section, we discuss a fault detection system design method for the fuzzy system (5). We construct a residual generator using the left coprime factors. Let \( \mathcal{M}_\mu(\mu) \) and \( \mathcal{N}_\mu(\mu) \) be the nominal system of \( \mathcal{M}(\mu) \) and \( \mathcal{N}(\mu) \). Thus

\[
\begin{bmatrix}
\hat{\mathcal{M}}_\mu(\mu) \\
\hat{\mathcal{N}}_\mu(\mu)
\end{bmatrix} =
\begin{bmatrix}
A_{\mu} + L_\mu C_{\mu} & L_\mu B_\mu + L_\mu D_\mu \\
C_{\mu} & 0
\end{bmatrix}
\begin{bmatrix}
I \\
D_{\mu}
\end{bmatrix},
\]  

(8)

Let \( u(t) \) and \( y(t) \) be the actuator fault and the measurement sensor fault signals respectively. Then we construct a residual signal as follows:

\[
e(t) = Q(\mu)\left(\hat{\mathcal{M}}_\mu(\mu)y(t) - \hat{\mathcal{N}}_\mu(\mu)u(t)\right),
\]  

(9)

where \( Q(\mu) \) is a fuzzy filter which will be used as a design parameter and \( y(t) = y_*(t) + y_*(t) \) is an output sensor measurement. Note that

\[
0 = \hat{\mathcal{M}}_\mu(\mu)y(t) - \hat{\mathcal{N}}_\mu(\mu)(u(t) + u_*(t)).
\]  

(10)

Using (10), the residual signal \( e(t) \) in (9) can be expressed as

\[
e(t) = Q(\mu)(e(t) + e_*(t)),
\]  

(11)

where

\[
e_*(t) = \left(\hat{\mathcal{N}}_\mu(\mu) - \hat{\mathcal{N}}_\mu(\mu)\right)u(t) - \left(\hat{\mathcal{M}}_\mu(\mu) - \hat{\mathcal{M}}_\mu(\mu)\right)y(t) + y_*(t),
\]

\[
e_*(t) = \hat{\mathcal{N}}_\mu(\mu)u_*(t) + \hat{\mathcal{M}}_\mu(\mu)y_*(t).
\]  

In (11), \( e(t) \) and \( e_*(t) \) correspond to signals due to the model uncertainties and the fault signals respectively. Even if no fault occurs, the residual signal \( e(t) \) is not zero due to the system model uncertainties.

A state space realization of the coprime factor uncertainty can be expressed as

\[
\begin{bmatrix}
\hat{\mathcal{M}}_\mu(\mu) \\
\hat{\mathcal{N}}_\mu(\mu)
\end{bmatrix} =
\begin{bmatrix}
A_{\mu} + L_\mu C_{\mu} & L_\mu B_\mu + L_\mu D_\mu \\
C_{\mu} & 0
\end{bmatrix}
\begin{bmatrix}
I \\
D_{\mu}
\end{bmatrix},
\]  

(7)

We define

\[
\begin{bmatrix}
P_{1\mu}(\mu) \\
P_{2\mu}(\mu)
\end{bmatrix} =
\begin{bmatrix}
A_{\mu} + L_\mu C_{\mu} & 0 \\
0 & A_\mu + L_\mu C_{\mu}
\end{bmatrix}
\begin{bmatrix}
L_\mu B_\mu + L_\mu D_\mu \\
C_{\mu}
\end{bmatrix},
\]  

(13)
With the definition of (13), the coprime factor uncertainty (12) also can be expressed as
\[
\begin{bmatrix}
\tilde{M}(\mu) - \tilde{M}_f(\mu) \\
\tilde{N}(\mu) - \tilde{N}_f(\mu)
\end{bmatrix} = P_{21}(\mu)\Theta(I - P_{11}(\mu)\Theta)^{-1}P_{12}(\mu).
\] (14)

When no fault occurs the residual signal \( e(t) \) becomes
\[
e(t) = Q(\mu)P_{12}(\mu)w_2(t),
\] (15)

where
\[
w_2(t) = \Theta(I - P_{11}(\mu)\Theta)^{-1}P_{12}(\mu)\begin{bmatrix} -y(t) \\ u(t) \end{bmatrix}.
\] (16)

In (15), \( Q(\mu) \) is chosen to minimize the effect of disturbances or model uncertainties in the frequency range \( (w_1, w_2) \) that \( w_2(t) \) manifests itself. For the above purpose, we design \( Q(\mu) \) such that \( || W_s(s) - Q(\mu) P_{21}(\mu) ||_{\infty} < \epsilon \) where \( W_s(s) \) is a stable transfer function for frequency shaping.

The faults can thus be detected using a simple thresholding logic:
\[
J = \|e(t)\| \begin{cases} 
\leq J_n \text{ normal} \\
> J_n \text{ faulty}
\end{cases}
\] (17)

where
\[
J_n = \max_{\omega \in (\omega_1, \omega_2)} \mathcal{S}(W_s(\omega)) + \gamma \|A\|_2 \|z_r(t)\|.
\]

Note that \( A \) is a system with feedback connection of \( \Theta \) and \( P_{11}(\mu) \). Hence, a state space realization of \( A \) can be expressed as
\[
A = \begin{bmatrix}
A_p + L_u C_p & 0 & 0 \\
0 & A_g + L_u G_p & 0 \\
H_p & 0 & I
\end{bmatrix}
\] (18)

By using the well developed LMI (Linear Matrix Inequality) tools and S-procedure, we can compute \( \|A\|_2 \).

4. S/G Fault Detection System

The dynamics of a nuclear steam generator is described in terms of the feedwater flowrate, the steam flowrate, and the steam generator water level. Irving[10] derived the following fourth order Laplace transfer function model based on the step response of the steam generator water level for step change of the feedwater flowrate and the steam flowrate:

\[
y(s) = \frac{g_2}{s} (u(s) - d(s)) - \frac{g_2}{1 + \tau_2 s} (u(s) - d(s)) - \frac{G_s}{s} \left( 1 + \frac{g_2 s}{\tau_2 + 4\pi^2 \tau_1^2 + 2\tau_1 s + s^2} \right) u(s),
\] (19)

where
\( \tau_1 \) and \( \tau_2 \) : damping time constants,
\( T \) : period of the mechanical oscillation,
\( g_1 \) : magnitude of the mass capacity effect,
\( g_2 \) : magnitude of the swell or shrink due to the feedwater or steam flowrates,
\( g_3 \) : magnitude of the mechanical oscillation.

This plant has a single input(feedwater flowrate \( u(s) \), a single output(water level \( y(s) \)) and a measurable known disturbance(steam flowrate \( d(s) \)). The parameter values of a steam generator at several power levels are given in Table 1, and the parameters are very different according to the power levels. The third term of the right hand side in (19) is extremely small in affecting the water level response [11].

<table>
<thead>
<tr>
<th>Power (%)</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
<th>( g_2 )</th>
<th>( g_3 )</th>
<th>( g_1 )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>43.4</td>
<td>4.5</td>
<td>1.83</td>
<td>0.310</td>
<td>0.058</td>
<td>17.7</td>
</tr>
<tr>
<td>50</td>
<td>34.8</td>
<td>3.6</td>
<td>1.05</td>
<td>0.215</td>
<td>0.058</td>
<td>14.2</td>
</tr>
<tr>
<td>100</td>
<td>28.6</td>
<td>3.4</td>
<td>0.47</td>
<td>0.105</td>
<td>0.058</td>
<td>11.7</td>
</tr>
</tbody>
</table>

By neglecting the third term in (19), we have the following reduced order state space model:

\[
\dot{x}(t) = \begin{bmatrix}
0 & 0 \\
0 & -\tau_1^{-1}(1 + \beta_1 \theta_1) \\
0 & -\tau_2^{-1}(1 + \beta_2 \theta_2)
\end{bmatrix} x(t) + \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} u(t)
\] (20)

\[
y(t) = \begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix} x(t),
\]

where \( \theta_1 \) and \( \theta_2 \) are introduced to describe possible parameter uncertainties and satisfy \( | \theta_1 | \leq 1, | \theta_2 | \leq 1 \). Thus, we assume that \( \tau_i^{-1} \) is within \( \beta_i \% \) of the nominal value given in Table 1. We treat the third term as an additive uncertain dynamics \( \delta_s(s) \) such that \( \delta_s(s) = W_s(s) \theta_2 \), \( \theta_2 \leq 1 \), where \( W_s(s) \) is a frequency weight. For simulation, let \( \beta_1 = \beta_2 = 0.1 \) and \( W_s(s) = \frac{0.62s}{s^2 + 0.46s + 0.126} \).

Then we have the following T-S fuzzy model:

Plant Rule \( i (i = 1,2,3) \):
IF the power level is \( M_4 \), THEN
\[
\dot{x}(t) = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & a_i & 0 & 0 \\
0 & -0.0461 & -0.1265 \\
0 & 0 & 1 & 0
\end{bmatrix} x(t)
\]
\[
\begin{bmatrix}
0 & 0 & 0 \\
1 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix} w(t) + \begin{bmatrix}
1 & -1 \\
h_1 & -h_1 \\
1 & 0 \\
0 & 0
\end{bmatrix} u_a(t) \\
\begin{bmatrix}
0 & h_1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x(t) \\
x(t) + \begin{bmatrix}
0 & 0 & 0 \\
0 & -j_1 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix} \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix} z(t) \\
\begin{bmatrix}
0.058 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix} [x(t) + \begin{bmatrix}
0 & 0 & 1
\end{bmatrix} w(t)]
\]

where the membership function \( M_i \ (i = 1, 2, 3) \) are depicted in Fig. 1

\[
\begin{bmatrix}
u_a(t) \\
d(t)
\end{bmatrix}, a_1 = -0.222, a_2 = -0.278, a_3 = -0.294, \\
b_1 = 0.4067, b_2 = 0.2917, b_3 = 0.1382, b_4 = 0.0222, \\
h_2 = 0.0278, h_3 = 0.0294, \\
j_1 = 0.0407, j_2 = 0.0292, j_3 = 0.0138.
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{M_i.png}
\caption{Membership function}
\end{figure}

We obtain the left coprime factors in (7) using \( L_i \ (i = 1, 2, 3) \) as follows:

\[
\begin{bmatrix}
-217 & -226 \\
-7.84 & -8.33 \\
0 & 0
\end{bmatrix}, \begin{bmatrix}
-229 \\
0 \\
0
\end{bmatrix}, \begin{bmatrix}
-8.47 \\
0 \\
0
\end{bmatrix}
\]

We discuss a design method such that \( \| W_r(s) - Q(\mu) P_{21}(\mu) \| \leq \gamma \). Unfortunately, the above mentioned method does not give a good result in the steam generator level system since \( P_{21}(\mu) \) has the blocking zero at the origin for each rule. Let \( P_{21}(\mu) = \frac{1}{s} P_{21}(\mu) \). We obtain \( Q(\mu) = \bar{Q}(\mu) \frac{1}{s} \) where \( \bar{Q}(\mu) \) is found such that \( \| W_r(s) - \bar{Q}(\mu) P_{21}(\mu) \| \leq \gamma \) is satisfied. The maximum singular value plots of \( W_r(s) \) is shown in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig_2.png}
\caption{The maximum singular value}
\end{figure}

A simulation of the steam generator control system has been done with the following two cases of scenario:

- **case 1:** 100% power level, a 5% step disturbance on \( \alpha(t) \) at 500sec, and a drift (5%/min ramp) on the water level sensor at 1000sec.
- **case 2:** 30% power level, a 5% step disturbance on \( \alpha(t) \) at 500sec, and a -50% error on the water level sensor at 1000sec.

In order to compute \( J \) in (17), we use \( J = \sqrt{\int_{t=0}^{T} \alpha(t)^2 dt} \) for a finite time period \( T \). The filtered residual signal is depicted in Fig. 3. The solid line shows the simulation result of case 2, and the dotted line shows the simulation result of case 1. In both cases, the effects of the step disturbance \( \alpha(t) \) on the filtered residual signal are very small while the effects of the sensor faults are remarkable.

Fig. 4 shows the unfiltered residual signals (i.e. \( Q(\mu) = I \)) of case 1. In Fig. 4, we can observe that the effect of the disturbance \( \alpha(t) \) at 500sec is big and the effect of the drift on the water level sensor is very small.

5. Conclusion

In this paper, we have described a fault detection scheme for T-S fuzzy systems with the IQC's. We design the residual generator from the left coprime factor model and the fuzzy filter minimizing the effect of disturbances. The norm of the residual signal is compared with a threshold level to determine whether or not a fault occurs. The developed scheme has been successfully implemented to detect fault occurrence of the nuclear steam generator water level control system.
A Fault Detection System Design for Uncertain Fuzzy Systems

Fig. 3. The filtered residual signal

Fig. 4. The unfiltered residual signal

References


Seog-Hwan Yoo received his B.S. and M.S. degree in electrical engineering from Seoul National University, PhD from the University of Florida. Since 1991 he is working at Daegu University. The areas of his research interests are robust control, model reduction and intelligent control.

Phone : 053-850-6621  
Fax : 053-850-6619  
E-mail : shryu@daegu.ac.kr

Byung-Jae Choi received his B.S. degree in electronic engineering at Kyungpook National University, and M.S. degree and Ph.D. from KAIST. Since 1999 he is working at Daegu University. The areas of his research interests are intelligent control and systems.

Phone : 053-850-6633  
Fax : 053-850-6619  
E-mail : bjchoi@daegu.ac.kr