Fuzzy \((r, s)\)-irresolute maps

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Abstract

Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined intuitionistic fuzzy topological spaces in Sostak’s sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. M. N. Mukherjee and S. P. Sinha [10] introduced the concept of fuzzy irresolute maps on Chang’s fuzzy topological spaces. In this paper, we introduce the concepts of fuzzy \((r, s)\)-irresolute, fuzzy \((r, s)\)-presemiopen, fuzzy almost \((r, s)\)-open, and fuzzy weakly \((r, s)\)-continuous maps on intuitionistic fuzzy topological spaces in Sostak’s sense. Using the notions of fuzzy \((r, s)\)-neighborhoods and fuzzy \((r, s)\)-semineighborhoods of a given intuitionistic fuzzy points, characterizations of fuzzy \((r, s)\)-irresolute maps are displayed. The relations among fuzzy \((r, s)\)-irresolute maps, fuzzy \((r, s)\)-continuous maps, fuzzy almost \((r, s)\)-continuous maps, and fuzzy weakly \((r, s)\)-continuous maps are discussed.

Key words: fuzzy \((r, s)\)-continuous, fuzzy almost \((r, s)\)-continuous, fuzzy \((r, s)\)-irresolute, fuzzy \((r, s)\)-presemiopen, fuzzy almost \((r, s)\)-open, fuzzy weakly \((r, s)\)-continuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [14]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Sostak [13], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [3], and by Ramadan [11].


In this paper, we introduce the concepts of fuzzy \((r, s)\)-irresolute, fuzzy \((r, s)\)-presemiopen, fuzzy almost \((r, s)\)-open, and fuzzy weakly \((r, s)\)-continuous maps on intuitionistic fuzzy topological spaces in Sostak’s sense. Using the notions of fuzzy \((r, s)\)-neighborhoods and fuzzy \((r, s)\)-semineighborhoods of a given intuitionistic fuzzy points, characterizations of fuzzy \((r, s)\)-irresolute maps are displayed. The relations among fuzzy \((r, s)\)-irresolute maps, fuzzy \((r, s)\)-continuous maps, fuzzy almost \((r, s)\)-continuous maps, and fuzzy weakly \((r, s)\)-continuous maps are discussed.

2. Preliminaries

We will denote the unit interval \([0, 1]\) of the real line by \(I\). A member \(\mu\) of \(I^X\) is called a fuzzy set in \(X\). For any \(\mu \in I^X\), \(\mu^c\) denotes the complement \(1 - \mu\). By \(\emptyset\) and \(I\) we denote constant maps on \(X\) with value \(\emptyset\) and \(I\), respectively. All other notations are standard notations of fuzzy set theory.

Let \(X\) be a nonempty set. An intuitionistic fuzzy set \(A\) is an ordered pair

\[ A = (\mu_A, \gamma_A) \]

where the functions \(\mu_A : X \rightarrow I\) and \(\gamma_A : X \rightarrow I\) denote the degree of membership and the degree of nonmembership, respectively and \(\mu_A + \gamma_A \leq 1\). Obviously every fuzzy set \(\mu\) in \(X\) is an intuitionistic fuzzy set of the form \((\mu, 1 - \mu)\).

Definition 2.1 ([11]) Let \(A = (\mu_A, \gamma_A)\) and \(B = (\mu_B, \gamma_B)\) be intuitionistic fuzzy sets in \(X\). Then

(1) \(A \subseteq B \text{ iff } \mu_A \leq \mu_B \text{ and } \gamma_A \geq \gamma_B\).
(2) \( A = B \) iff \( A \subseteq B \) and \( B \subseteq A \).

(3) \( A^c = (\gamma_A, \mu_A) \).

(4) \( A \cap B = (\mu_A \wedge \mu_B, \gamma_A \lor \gamma_B) \).

(5) \( A \cup B = (\mu_A \lor \mu_B, \gamma_A \land \gamma_B) \).

(6) \( \emptyset = (\emptyset, \tilde{1}) \) and \( 1 = (\tilde{1}, 0) \).

Let \( f \) be a map from a set \( X \) to a set \( Y \). Let \( A = (\mu_A, \gamma_A) \) be an intuitionistic fuzzy set in \( X \) and \( B = (\mu_B, \gamma_B) \) an intuitionistic fuzzy set in \( Y \). Then

1. The image of \( A \) under \( f \), denoted by \( f(A) \), is an intuitionistic fuzzy set in \( Y \) defined by

\[
f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).
\]

2. The inverse image of \( B \) under \( f \), denoted by \( f^{-1}(B) \), is an intuitionistic fuzzy set in \( X \) defined by

\[
f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).
\]

A smooth fuzzy topology on \( X \) is a map \( T : I^X \to I \) which satisfies the following properties:

1. \( T(\emptyset) = T(\tilde{1}) = 1 \).

2. \( T(\mu_1 \land \mu_2) \geq T(\mu_1) \land T(\mu_2) \).

3. \( T(\nu_1) \geq \bigwedge T(\mu_i) \).

The pair \( (X, T) \) is called a smooth fuzzy topological space.

An intuitionistic fuzzy topology on \( X \) is a family of intuitionistic fuzzy sets in \( X \) which satisfies the following properties:

1. \( \emptyset, 1 \in T \).

2. If \( A_1, A_2 \in T \), then \( A_1 \cap A_2 \in T \).

3. If \( A_i \in T \) for each \( i \), then \( \bigcup A_i \in T \).

The pair \( (X, T) \) is called an intuitionistic fuzzy topological space.

Let \( I(X) \) be a family of all intuitionistic fuzzy sets in \( X \) and let \( I \otimes I \) be the set of the pair \( \langle r, s \rangle \) such that \( r, s \in I \) and \( r + s \leq 1 \).

Definition 2.2 ([5]) Let \( X \) be a nonempty set. An intuitionistic fuzzy topology in Sostak’s sense (SoFTS for short) \( T = (T_1, T_2) \) on \( X \) is a map \( T : I(X) \to I \otimes I \) which satisfies the following properties:

1. \( T_1(\emptyset) = T_1(\tilde{1}) = 1 \) and \( T_2(\emptyset) = T_2(\tilde{1}) = 0 \).

2. \( T_1(A \cap B) \geq T_1(A) \land T_1(B) \) and \( T_2(A \cap B) \leq T_2(A) \lor T_2(B) \).

3. \( T_1(\bigcup A_i) \geq \bigwedge T_1(A_i) \) and \( T_2(\bigcup A_i) \leq \bigvee T_2(A_i) \).

The \( (X, T) = (X, T_1, T_2) \) is said to be an intuitionistic fuzzy topological space in Sostak’s sense (SoFTS for short). Also, we call \( T_1(A) \) a gradation of openness of \( A \) and \( T_2(A) \) a gradation of nonopenness of \( A \).

Definition 2.3 ([8]) Let \( A \) be an intuitionistic fuzzy set in SoFTS \( (X, T_1, T_2) \) and \( \langle r, s \rangle \in I \otimes I \). Then \( A \) is said to be

1. fuzzy \( \langle r, s \rangle \)-open if \( T_1(A) \geq r \) and \( T_2(A) \leq s \),

2. fuzzy \( \langle r, s \rangle \)-closed if \( T_1(A^c) \geq r \) and \( T_2(A^c) \leq s \).

Definition 2.4 ([8]) Let \( (X, T_1, T_2) \) be a SoFTS. For each \( \langle r, s \rangle \in I \otimes I \) and for each \( A \in I(X) \), the fuzzy \( \langle r, s \rangle \)-interior is defined by

\[
\text{int}(A, r, s) = \bigcup \{ B \in I(X) \mid B \subseteq A, \ B \text{ is fuzzy } \langle r, s \rangle \text{-open} \}
\]

and the fuzzy \( \langle r, s \rangle \)-closure is defined by

\[
\text{cl}(A, r, s) = \bigcap \{ B \in I(X) \mid A \subseteq B, \ B \text{ is fuzzy } \langle r, s \rangle \text{-closed} \}.
\]

Lemma 2.5 ([8]) For an intuitionistic fuzzy set \( A \) in a SoFTS \( (X, T_1, T_2) \) and \( \langle r, s \rangle \in I \otimes I \),

1. \( \text{int}(A, r, s)^c = \text{cl}(A^c, r, s) \),

2. \( \text{cl}(A, r, s)^c = \text{int}(A^c, r, s) \).

Let \( (X, T_1, T_2) \) be an intuitionistic fuzzy topological space in Sostak’s sense. Then it is easy to see that for each \( \langle r, s \rangle \in I \otimes I \), the family \( T_{(r,s)} \) defined by

\[
T_{(r,s)} = \{ A \in I(X) \mid T_1(A) \geq r \text{ and } T_2(A) \leq s \}
\]

is an intuitionistic fuzzy topology on \( X \).

Let \( (X, T) \) be an intuitionistic fuzzy topological space and \( \langle r, s \rangle \in I \otimes I \). Then the map \( T_{(r,s)} : I(X) \to I \otimes I \) defined by

\[
T_{(r,s)}(A) = \begin{cases} (1, 0) & \text{if } A = 0, 1, \\ (r, s) & \text{if } A \in T - \{0, 1\}, \\ (0, 1) & \text{otherwise} \end{cases}
\]

becomes an intuitionistic fuzzy topology in Sostak’s sense on \( X \).
Let \( \alpha, \beta \in [0, 1] \) with \( \alpha + \beta \leq 1 \). An intuitionistic fuzzy point \( x_{(\alpha, \beta)} \) in \( X \) is an intuitionistic fuzzy set in \( X \) defined by

\[
x_{(\alpha, \beta)}(y) = \begin{cases} 
(\alpha, \beta) & \text{if } y = x, \\
(0, 1) & \text{if } y \neq x.
\end{cases}
\]

In this case, \( x \) is called the support of \( x_{(\alpha, \beta)} \), \( \alpha \) the value of \( x_{(\alpha, \beta)} \), and \( \beta \) the nonvalue of \( x_{(\alpha, \beta)} \). An intuitionistic fuzzy point \( x_{(\alpha, \beta)} \) is said to belong to an intuitionistic fuzzy set \( A = (\mu_A, \gamma_A) \) in \( X \), denoted by \( x_{(\alpha, \beta)} \in A \), if \( \mu_A(x) \geq \alpha \) and \( \gamma_A(x) \leq \beta \). An intuitionistic fuzzy set \( A \) in \( X \) is the union of all intuitionistic fuzzy points which belong to \( A \).

**Definition 2.6** ([8]) Let \( A \) be an intuitionistic fuzzy set in a SoIPTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then \( A \) is said to be

1. **fuzzy \((r, s)\)-semiopen if** there is a fuzzy \((r, s)\)-open set \( B \) in \( X \) such that \( B \subseteq A \subseteq \text{cl}(B, r, s) \).
2. **fuzzy \((r, s)\)-semiclosed if** there is a fuzzy \((r, s)\)-closed set \( B \) in \( X \) such that \( \text{int}(B, r, s) \subseteq A \subseteq B \).

**Theorem 2.7** ([8]) Let \( A \) be an intuitionistic fuzzy set in a SoIPTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then the following statements are equivalent:

1. \( A \) is a fuzzy \((r, s)\)-semiopen set.
2. \( A^{c} \) is a fuzzy \((r, s)\)-semiclosed set.
3. \( \text{cl}(\text{int}(A, r, s), r, s) \supseteq A \).
4. \( \text{int}(\text{cl}(A^{c}, r, s), r, s) \subseteq A^{c} \).

**Theorem 2.8** ([8]) Let \((X, T_1, T_2)\) be a SoIPTS and \((r, s) \in I \otimes I\).

1. If \( \{A_i\} \) is a family of fuzzy \((r, s)\)-semiopen sets in \( X \), then \( \bigcup A_i \) is fuzzy \((r, s)\)-semiopen.
2. If \( \{A_i\} \) is a family of fuzzy \((r, s)\)-semiclosed sets in \( X \), then \( \bigcap A_i \) is fuzzy \((r, s)\)-semiclosed.

**Definition 2.9** ([8]) Let \((X, T_1, T_2)\) be a SoIPTS. For each \((r, s) \in I \otimes I\) and for each \( A \in I(X) \), the fuzzy \((r, s)\)-semiinterior is defined by

\[
\text{si}(A, r, s) = \bigcup \{ B \in I(X) \mid B \subseteq A, \ B \text{ is fuzzy } (r, s)\text{-semiopen} \},
\]

and the fuzzy \((r, s)\)-semiclosure is defined by

\[
\text{sc}(A, r, s) = \bigcap \{ B \in I(X) \mid A \subseteq B, \ B \text{ is fuzzy } (r, s)\text{-semiclosed} \}.
\]

**Definition 2.10** ([12]) Let \( A \) be an intuitionistic fuzzy set in a SoIPTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then \( A \) is said to be

1. **fuzzy \((r, s)\)-regular open if** \( \text{int}(\text{cl}(A, r, s), r, s) = A \).
2. **fuzzy \((r, s)\)-regular closed if** \( \text{cl}(\text{int}(A, r, s), r, s) = A \).

**Theorem 2.11** ([12]) Let \( A \) be an intuitionistic fuzzy set in a SoIPTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then the following statements are equivalent:

1. \( A \) is fuzzy \((r, s)\)-regular open.
2. \( A^{c} \) is fuzzy \((r, s)\)-regular closed.

**Theorem 2.12** ([12]) (1) The fuzzy \((r, s)\)-closure of a fuzzy \((r, s)\)-open set is fuzzy \((r, s)\)-regular closed for each \((r, s) \in I \otimes I\).

(2) The fuzzy \((r, s)\)-interior of a fuzzy \((r, s)\)-closed set is fuzzy \((r, s)\)-regular open for each \((r, s) \in I \otimes I\).

**Definition 2.13** ([9, 12]) Let \( f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2) \) be a map from a SoIPTS \( X \) to a SoIPTS \( Y \) and \((r, s) \in I \otimes I\). Then \( f \) is called

1. a fuzzy \((r, s)\)-continuous map if \( f^{-1}(B) \) is a fuzzy \((r, s)\)-open set in \( X \) for each fuzzy \((r, s)\)-open set \( B \) in \( Y \).
2. a fuzzy \((r, s)\)-open map if \( f(A) \) is a fuzzy \((r, s)\)-open set in \( Y \) for each fuzzy \((r, s)\)-open set \( A \) in \( X \).
3. a fuzzy \((r, s)\)-semicontinuous map if \( f^{-1}(B) \) is a fuzzy \((r, s)\)-semiopen set in \( X \) for each fuzzy \((r, s)\)-open set \( B \) in \( Y \).
4. a fuzzy \((r, s)\)-semiopen map if \( f(A) \) is a fuzzy \((r, s)\)-semiopen set in \( Y \) for each fuzzy \((r, s)\)-open set \( A \) in \( X \).
5. a fuzzy almost \((r, s)\)-continuous map if \( f^{-1}(B) \) is a fuzzy \((r, s)\)-open set in \( X \) for each fuzzy \((r, s)\)-regular open set \( B \) in \( Y \).

**Theorem 2.14** ([12]) Let \( f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2) \) be a map from a SoIPTS \( X \) to a SoIPTS \( Y \) and \((r, s) \in I \otimes I\). Then the following statements are equivalent:

1. \( f \) is a fuzzy almost \((r, s)\)-continuous map.
(2) \( f^{-1}(B) \subseteq \text{int}(f^{-1}(\text{cl}(B, r, s), r, s)) \) for each fuzzy \((r, s)\)-open set \(B\) in \(Y\).

(3) \( \text{cl}(f^{-1}(\text{cl}(B, r, s), r, s)) \subseteq f^{-1}(B) \) for each fuzzy \((r, s)\)-closed set \(B\) in \(Y\).

**Theorem 2.15** (8) Let \( f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2) \) be a map and \((r, s) \in I \otimes I\). Then the following statements are equivalent:

(1) \( f \) is a fuzzy \((r, s)\)-semicontinuous map.

(2) \( f^{-1}(B) \) is a fuzzy \((r, s)\)-semiclosed set in \(X\) for each fuzzy \((r, s)\)-closed set \(B\) in \(Y\).

(3) \( \text{int}(\text{cl}(f^{-1}(B), r, s), r, s) \subseteq f^{-1}(\text{cl}(B, r, s)) \) for each intuitionistic fuzzy set \(B\) in \(Y\).

(4) \( f(\text{int}(\text{cl}(A, r, s), r, s)) \subseteq \text{cl}(f(A), r, s) \) for each intuitionistic fuzzy set \(A\) in \(X\).

### 3. Fuzzy \((r, s)\)-Irresolute Maps

Now, we define the notions of fuzzy \((r, s)\)-irresolute, fuzzy \((r, s)\)-presemiopen, fuzzy almost \((r, s)\)-open, and fuzzy weakly \((r, s)\)-continuous maps on intuitionistic fuzzy topological spaces in Sostak’s sense, and then we investigate some of their properties.

**Definition 3.1** Let \( f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2) \) be a map from a SoIFTS \(X\) to a SoIFTS \(Y\) and \((r, s) \in I \otimes I\). Then \(f\) is called

(1) a fuzzy \((r, s)\)-irresolute map if \( f^{-1}(B) \) is a fuzzy \((r, s)\)-semiopen set in \(X\) for each fuzzy \((r, s)\)-semiopen set \(B\) in \(Y\),

(2) a fuzzy \((r, s)\)-presemiopen map if \( f(A) \) is a fuzzy \((r, s)\)-semiopen set in \(Y\) for each fuzzy \((r, s)\)-semiopen set \(A\) in \(X\),

(3) a fuzzy almost \((r, s)\)-open map if \( f(A) \) is a fuzzy \((r, s)\)-open set in \(Y\) for each fuzzy \((r, s)\)-regular open set \(A\) in \(X\),

(4) a fuzzy weakly \((r, s)\)-continuous map if for every fuzzy \((r, s)\)-open set \(B\) in \(Y\), \( f^{-1}(B) \subseteq \text{int}(f^{-1}(\text{cl}(B, r, s)), r, s) \).

**Definition 3.2** Let \( x_{(a, b)} \) be an intuitionistic fuzzy point in a SoIFTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then an intuitionistic fuzzy set \(A\) in \(X\) is called

(1) a fuzzy \((r, s)\)-neighborhood of \(x_{(a, b)}\) if there is a fuzzy \((r, s)\)-open set \(B\) in \(X\) such that \(x_{(a, b)} \in B \subseteq A\),

(2) a fuzzy \((r, s)\)-semineighborhood of \(x_{(a, b)}\) if there is a fuzzy \((r, s)\)-semiopen set \(B\) in \(X\) such that \(x_{(a, b)} \in B \subseteq A\).

**Theorem 3.3** Let \( f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2) \) be a map from a SoIFTS \(X\) to a SoIFTS \(Y\) and \((r, s) \in I \otimes I\). Then the following statements are equivalent:

(1) \( f \) is fuzzy \((r, s)\)-irresolute.

(2) \( f^{-1}(B) \) is a fuzzy \((r, s)\)-semiclosed set in \(X\) for each fuzzy \((r, s)\)-semiclosed set \(B\) in \(Y\).

(3) For every intuitionistic fuzzy point \( x_{(a, b)} \) in \(X\) and every fuzzy \((r, s)\)-semiclosed set \(B\) in \(Y\) such that \( f(x_{(a, b)}) \in B \), there is a fuzzy \((r, s)\)-semiclosed set \(A\) in \(X\) such that \( x_{(a, b)} \in A \) and \( f(A) \subseteq B \).

(4) For every intuitionistic fuzzy point \( x_{(a, b)} \) in \(X\) and every fuzzy \((r, s)\)-semineighborhood \(B\) of \( f(x_{(a, b)}) \) in \(Y\), \( f^{-1}(B) \) is a fuzzy \((r, s)\)-semineighborhood of \( x_{(a, b)} \) in \(X\).

(5) For every intuitionistic fuzzy point \( x_{(a, b)} \) in \(X\) and every fuzzy \((r, s)\)-semineighborhood \(B\) of \( f(x_{(a, b)}) \) in \(Y\), there is a fuzzy \((r, s)\)-semineighborhood \(A\) of \( x_{(a, b)} \) in \(X\) such that \( f(A) \subseteq B \).

(6) \( f(\text{sc}(A, r, s)) \subseteq \text{sc}(f(A), r, s) \) for each intuitionistic fuzzy set \(A\) in \(X\).

(7) \( \text{sc}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{sc}(B, r, s)) \) for each intuitionistic fuzzy set \(B\) in \(Y\).

**Proof.** \((1) \Rightarrow (2)\) It is obvious.

(3) \( \Rightarrow (1) \) Let \( B \) be a fuzzy \((r, s)\)-semiopen set in \(Y\) and \( x_{(a, b)} \) an intuitionistic fuzzy point in \(X\) such that \( x_{(a, b)} \in f^{-1}(B) \). Then \( f(x_{(a, b)}) \in B \). Thus there is a fuzzy \((r, s)\)-semiopen set \(A\) in \(X\) such that \( x_{(a, b)} \in A \) and \( f(A) \subseteq B \). Then \( A \subseteq f^{-1}(B) \). Thus

\[
x_{(a, b)} \in A \subseteq \text{cl}(A, r, s, r, s) \subseteq \text{cl}(f^{-1}(B), r, s, r, s).
\]

Hence

\[
f^{-1}(B) = \bigcup \{ x_{(a, b)} \mid x_{(a, b)} \in f^{-1}(B) \} \subseteq \text{cl}(f^{-1}(B), r, s, r, s).
\]

Therefore \( f \) is a fuzzy \((r, s)\)-irresolute map.

\((1) \Rightarrow (4) \) Let \( x_{(a, b)} \) be an intuitionistic fuzzy point in \(X\) and \( B \) a fuzzy \((r, s)\)-semineighborhood of \( f(x_{(a, b)}) \) in \(Y\). Then there is a fuzzy \((r, s)\)-semiopen set \(C\) in \(Y\) such that \( f(x_{(a, b)}) \in C \subseteq B \). Hence \( x_{(a, b)} \in f^{-1}(C) \subseteq \text{int}(f^{-1}(\text{cl}(B, r, s)), r, s) \).
$f^{-1}(B)$. Since $f$ is fuzzy $(r,s)$- irresolute, $f^{-1}(C)$ is a fuzzy $(r,s)$-semiopen set in $X$. Thus $f^{-1}(B)$ is a fuzzy $(r,s)$-semiopenhood of $x_{(0,\theta)}$.

(4) $\Rightarrow$ (5) Let $x_{(0,\theta)}$ be an intuitionistic fuzzy point in $X$ and $B$ a fuzzy $(r,s)$-semiopenhood of $f(x_{(0,\theta)})$ in $Y$. By (4), $f^{-1}(B)$ is a fuzzy $(r,s)$-semiopenhood of $x_{(0,\theta)}$ in $X$. Let $f^{-1}(B) = A$. Then $f(A) = f(f^{-1}(B)) \subseteq B$.

(5) $\Rightarrow$ (3) Let $x_{(0,\theta)}$ be an intuitionistic fuzzy point in $X$ and $B$ a fuzzy $(r,s)$-semiopen set in $Y$ such that $f(x_{(0,\theta)}) \in B$. Then since $B$ is a fuzzy $(r,s)$-semiopenhood of $f(x_{(0,\theta)})$, by (5), there is a fuzzy $(r,s)$-semiopenhood $A$ of $x_{(0,\theta)}$ in $X$ such that $f(A) \subseteq B$. Then there is a fuzzy $(r,s)$-semiopen set $C$ in $X$ such that $x_{(0,\theta)} \in C \subseteq A$ and hence $f(C) \subseteq f(A) \subseteq B$.

(2) $\Rightarrow$ (6) Let $A$ be an intuitionistic fuzzy set in $X$. Since $\operatorname{scl}(f(A), r, s)$ is fuzzy $(r,s)$-semiopen in $Y$, by (2), $f^{-1}(\operatorname{scl}(f(A), r, s))$ is a fuzzy $(r,s)$-semiopen set in $X$. Since $f(A) \subseteq \operatorname{scl}(f(A), r, s)$, we have $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\operatorname{scl}(f(A), r, s))$. Hence

\[
\operatorname{scl}(A, r, s) \subseteq \operatorname{scl}(f^{-1}(\operatorname{scl}(f(A), r, s)), r, s) \subseteq f^{-1}(\operatorname{scl}(f(A), r, s)).
\]

Therefore

\[
f(\operatorname{scl}(A, r, s)) \subseteq f^{-1}(\operatorname{scl}(f(A), r, s)) \subseteq \operatorname{scl}(f(A), r, s).
\]

(6) $\Rightarrow$ (2) Let $B$ be a fuzzy $(r,s)$-semiopen set in $Y$. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in $X$. By (6),

\[
f(\operatorname{scl}(f^{-1}(B), r, s)) \subseteq \operatorname{scl}(f(f^{-1}(B)), r, s) \subseteq \operatorname{scl}(B, r, s) = B.
\]

Thus $\operatorname{scl}(f^{-1}(B), r, s) \subseteq f^{-1}(\operatorname{scl}(f^{-1}(B), r, s)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = \operatorname{scl}(f^{-1}(B), r, s)$. Therefore $f^{-1}(B)$ is a fuzzy $(r,s)$-semiopen set in $X$.

(6) $\Rightarrow$ (7) Let $B$ be an intuitionistic fuzzy set in $Y$. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in $X$. By (6),

\[
f(\operatorname{scl}(f^{-1}(B), r, s)) \subseteq \operatorname{scl}(f(f^{-1}(B)), r, s) \subseteq \operatorname{scl}(B, r, s).
\]

Hence

\[
\operatorname{scl}(f^{-1}(B), r, s) \subseteq f^{-1}(\operatorname{scl}(f^{-1}(B), r, s)) \subseteq f^{-1}(\operatorname{scl}(B, r, s)).
\]

(7) $\Rightarrow$ (6) Let $A$ be an intuitionistic fuzzy set in $X$. Then $f(A)$ is an intuitionistic fuzzy set in $Y$. By (7),

\[
\operatorname{scl}(A, r, s) \subseteq \operatorname{scl}(f^{-1}(f(A)), r, s) \subseteq f^{-1}(\operatorname{scl}(f(A), r, s)).
\]

Hence

\[
f(\operatorname{scl}(A, r, s)) \subseteq f(\operatorname{scl}(f(A), r, s)) \subseteq \operatorname{scl}(f(A), r, s).
\]

**Lemma 3.4** Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a map from a SofITS $X$ to a SofITS $Y$ and let $A$ and $B$ be intuitionistic fuzzy sets in $X$ and $Y$, respectively. Then $f^{-1}(B) \subseteq A$ if and only if $(f(A))^c \subseteq B$.

**Proof.**

\[
f^{-1}(B) \subseteq A \iff f^{-1}(B^c) = f^{-1}(B)^c \subseteq A^c \iff f(A^c) \subseteq f^{-1}(B)^c \subseteq B^c \iff (f(A^c))^c \subseteq B.
\]

**Theorem 3.5** Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a map from a SofITS $X$ to a SofITS $Y$ and $(r, s) \in I \times I$. Then $f$ is fuzzy almost $(r,s)$-open if and only if for each intuitionistic fuzzy set $B$ in $Y$ and each fuzzy $(r,s)$-regular closed set $A$ in $X$ such that $f^{-1}(B) \subseteq A$, there is a fuzzy $(r,s)$-regular closed set $C$ in $Y$ such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.

**Proof.** Let $f$ be a fuzzy almost $(r,s)$-open map, $B$ an intuitionistic fuzzy set in $Y$, and $A$ a fuzzy $(r,s)$-regular closed set in $X$ such that $f^{-1}(B) \subseteq A$. Let $C = (f(A))^c$. Then $C$ is a fuzzy $(r,s)$-closed set in $Y$ and by Lemma 3.4, $B \subseteq C$. Also, we have

\[
f^{-1}(C) = f^{-1}((f(A))^c) = (f^{-1}(f(A))^c)^c \subseteq (A^c)^c = A.
\]

Conversely, let $A$ be a fuzzy $(r,s)$-regular open set in $X$. Let $B = f(A)^c = D = A^c$. Then we have

\[
f^{-1}(B) = f^{-1}(f(A)^c) = (f^{-1}(f(A)))^c \subseteq A^c = D.
\]

By hypothesis, there is a fuzzy $(r,s)$-closed set $C$ in $Y$ such that $f(A)^c = B \subseteq C$ and $f^{-1}(C) \subseteq D = A^c$. Then $A \subseteq f^{-1}(C)^c = f^{-1}(C)^c$. Hence $f(A) = C^c$. Therefore $f(A)$ is a fuzzy $(r,s)$-open set in $Y$ and consequently $f$ is a fuzzy almost $(r,s)$-open map.

**Theorem 3.6** Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a map from a SofITS $X$ to a SofITS $Y$ and $(r, s) \in I \times I$. Then $f$ is a fuzzy almost $(r,s)$-open map if and only if $f(\operatorname{int}(A, r, s)) \subseteq \operatorname{int}(f(A), r, s)$ for each fuzzy $(r,s)$-semiopen set $A$ in $X$.

**Proof.** Let $f$ be a fuzzy almost $(r,s)$-open map and $A$ a fuzzy $(r,s)$-semiopen set in $X$. Then $\operatorname{int}(A, r, s) \subseteq \operatorname{int}(\operatorname{cl}(A, r, s), r, s) \subseteq \operatorname{A}$. Note that $\operatorname{cl}(A, r, s)$ is a fuzzy $(r,s)$-closed set in $X$. By Theorem 2.1.2 (2), $\operatorname{int}(\operatorname{cl}(A, r, s), r, s)$ is a fuzzy $(r,s)$-regular open set in $X$. Since $f$ is a fuzzy almost $(r,s)$-open map,
\[ f(\text{int}(\text{cl}(A, r, s), r, s)) \text{ is a fuzzy } (r, s)-\text{open set in } Y. \text{ Thus we have} \]
\[ f(\text{int}(A, r, s)) \subseteq f(\text{int}(\text{cl}(A, r, s), r, s)) \]
\[ = \text{int}(f(\text{int}(\text{cl}(A, r, s), r, s)), r, s) \]
\[ \subseteq \text{int}(f(A), r, s). \]

Conversely, let \( A \) be a fuzzy \((r, s)\)-regular open set in \( X \). Then \( A \) is fuzzy \((r, s)\)-open and hence \( \text{int}(A, r, s) = A \). Since \( \text{int}(\text{cl}(A, r, s), r, s) = A \), \( A \) is a fuzzy \((r, s)\)-semiclosed set. So
\[ f(A) = f(\text{int}(A, r, s)) \subseteq \text{int}(f(A), r, s) \subseteq f(A). \]

Thus \( f(A) = \text{int}(f(A), r, s) \) and so \( f(A) \) is fuzzy \((r, s)\)-open in \( Y \). Hence \( f \) is a fuzzy almost \((r, s)\)-open map.

**Theorem 3.7** Let \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) be a map from a SoIFTS \( X \) to a SoIFTS \( Y \) and \((r, s) \in I \otimes I \). If \( f \) is a fuzzy almost \((r, s)\)-continuous map if and only if for every intuitionistic fuzzy point \( x_{(a, b)} \) in \( X \) and every fuzzy \((r, s)\)-neighborhood \( B \) of \( f(x_{(a, b)}) \), there is a fuzzy \((r, s)\)-neighborhood \( A_x \) of \( x_{(a, b)} \) such that \( f(A_x) \subseteq \text{int}(B, r, s), r, s) \).

**Proof.** Let \( x_{(a, b)} \) be an intuitionistic fuzzy point in \( X \) and \( B \) a fuzzy \((r, s)\)-neighborhood of \( f(x_{(a, b)}) \). Then there is a fuzzy \((r, s)\)-open set \( C \) in \( Y \) such that \( f(x_{(a, b)}) \in C \subseteq B \). So \( x_{(a, b)} \) in \( f^{-1}(C) \subseteq f^{-1}(B) \). Since \( f \) is a fuzzy almost \((r, s)\)-continuous map, by Theorem 2.14,
\[ f^{-1}(C) \subseteq \text{int}(f^{-1}(\text{cl}(C, r, s), r, s)), r, s) \]
\[ \subseteq \text{int}(f^{-1}(\text{cl}(B, r, s), r, s)), r, s). \]

Put \( A = f^{-1}(\text{cl}(B, r, s), r, s)). \) Then \( x_{(a, b)} \in f^{-1}(A) \subseteq \text{int}(A, r, s) \subseteq A \). By Theorem 2.12 (2), \( \text{int}(B, r, s), r, s) \) is fuzzy \((r, s)\)-regular open. Since \( f \) is fuzzy almost \((r, s)\)-continuous, \( A = f^{-1}(\text{cl}(B, r, s), r, s)) \) is a fuzzy \((r, s)\)-open set. Thus \( A \) is a fuzzy \((r, s)\)-neighborhood of \( x_{(a, b)} \) and
\[ f(A) = f(f^{-1}(\text{int}(\text{cl}(B, r, s), r, s))) \]
\[ \subseteq \text{int}(\text{cl}(B, r, s), r, s). \]

Conversely, let \( B \) be a fuzzy \((r, s)\)-regular open set in \( Y \) and \( x_{(a, b)} \in f^{-1}(B) \). Then \( B \) is fuzzy \((r, s)\)-open and a fuzzy \((r, s)\)-neighborhood of \( f(x_{(a, b)}) \). By hypothesis, there is a fuzzy \((r, s)\)-neighborhood of \( A_{x_{(a, b)}} \) such that \( f(A_{x_{(a, b)}}) \subseteq \text{int}(B, r, s), r, s) \). Since \( A_{x_{(a, b)}} \) is a fuzzy \((r, s)\)-neighborhood of \( x_{(a, b)} \), there is a fuzzy \((r, s)\)-open set \( C_{x_{(a, b)}} \) in \( X \) such that
\[ x_{(a, b)} \in C_{x_{(a, b)}} \subseteq A_{x_{(a, b)}} \subseteq f^{-1}(f(A_{x_{(a, b)}})) \]
\[ \subseteq f^{-1}(B). \]

So we have
\[ f^{-1}(B) = \bigcup \{ x_{(a, b)} \mid x_{(a, b)} \in f^{-1}(B) \} \]
\[ \subseteq \bigcup \{ C_{x_{(a, b)}} \mid x_{(a, b)} \in f^{-1}(B) \} \]
\[ \subseteq f^{-1}(B). \]

Thus \( f^{-1}(B) = \bigcup \{ C_{x_{(a, b)}} \mid x_{(a, b)} \in f^{-1}(B) \} \) and so \( f^{-1}(B) \) is a fuzzy \((r, s)\)-open set in \( X \). Hence \( f \) is a fuzzy almost \((r, s)\)-continuous map.

**Theorem 3.8** Let \( (X, T) \) and \( (Y, U) \) be SoIFTSs and \((r, s) \in I \otimes I \). If \( f : (X, T) \to (Y, U) \) is a fuzzy \((r, s)\)-irresolute map, then \( f \) is a fuzzy \((r, s)\)-semicontinuous map.

**Proof.** Let \( B \) be a fuzzy \((r, s)\)-open set in \( Y \). Then \( B \) is a fuzzy \((r, s)\)-semitopological set in \( Y \). Since \( f \) is a fuzzy \((r, s)\)-irresolute map, \( f^{-1}(B) \) is a fuzzy \((r, s)\)-semitopological set in \( X \). Hence \( f \) is a fuzzy \((r, s)\)-semicontinuous map.

The following example shows that the converse of Theorem 3.8 need not be true.

**Example 3.9** Let \( X = \{x, y, z\} \) and let \( A_1, A_2 \) and \( B \) be intuitionistic fuzzy sets in \( X \) defined as
\[ A_1(x) = (0, 0.9), \quad A_1(y) = (0.3, 0.6), \quad A_1(z) = (0.3, 0.6); \]
\[ A_2(x) = (0.9, 0), \quad A_2(y) = (0.3, 0.6), \quad A_2(z) = (0.3, 0.6); \]

and
\[ B(x) = (0.9, 0), \quad B(y) = (0.7, 0.3), \quad B(z) = (0.7, 0.3). \]

Define \( T : I(X) \to I \otimes I \) and \( U : I(X) \to I \otimes I \) by
\[ T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise}; \end{cases} \]

and
\[ U(A) = (U_1(A), U_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise}. \end{cases} \]

Then clearly \( T \) and \( U \) are SoIFTSs on \( X \). Consider a map \( f : (X, T) \to (X, U) \) defined by \( f(x) = x \), \( f(y) = y \) and \( f(z) = z \). It is easy to see that \( f \) is a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-seminetopological map and \( B \) is a fuzzy \((\frac{1}{3}, \frac{1}{2})\)-semiclosed set in \((X, U)\). But \( f^{-1}(B) = B \) is not a fuzzy \((\frac{1}{3}, \frac{1}{2})\)-semitopological set in \((X, T)\). Hence \( f \) is not a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-irresolute map.

**Theorem 3.10** Let \( f : (X, T) \to (Y, U) \) be a map from a SoIFTS \( X \) to a SoIFTS \( Y \) and \((r, s) \in I \otimes I \). If \( f \) is fuzzy \((r, s)\)-semicontinuous and fuzzy almost \((r, s)\)-open, then \( f \) is a fuzzy \((r, s)\)-irresolute map.
Proof. Let $B$ be a fuzzy $(r, s)$-semiclosed set in $Y$. Then $\text{int}(\text{cl}(B, r, s), r, s) \subseteq B$. Since $f$ is fuzzy $(r, s)$-semicontinuous, by Theorem 2.15,
\[
\text{int}(\text{cl}(f^{-1}(B), r, s), r, s) \subseteq f^{-1}(\text{cl}(B, r, s)).
\]
Thus we have
\[
\text{int}(\text{cl}(f^{-1}(B), r, s), r, s) = \text{int}(\text{int}(\text{cl}(f^{-1}(B), r, s), r, s), r, s) \\ \subseteq \text{int}(f^{-1}(\text{cl}(B, r, s)), r, s).
\]
Since $f$ is fuzzy $(r, s)$-semicontinuous and $\text{cl}(B, r, s)$ is fuzzy $(r, s)$-closed, $f^{-1}(\text{cl}(B, r, s))$ is fuzzy $(r, s)$-semiclosed in $X$. Since $f$ is a fuzzy almost $(r, s)$-open map,
\[
f(\text{int}(f^{-1}(\text{cl}(B, r, s)), r, s)) \\ \subseteq \text{int}(f(f^{-1}(\text{cl}(B, r, s))), r, s) \\ \subseteq \text{int}(\text{cl}(B, r, s), r, s) \subseteq B.
\]
Hence we have
\[
\text{int}(\text{cl}(f^{-1}(B), r, s), r, s) \\ \subseteq f^{-1}(f(\text{int}(\text{cl}(f^{-1}(B), r, s), r, s))) \\ \subseteq f^{-1}(f(\text{int}(f^{-1}(\text{cl}(B, r, s)), r, s))) \\ \subseteq f^{-1}(f(B)).
\]
Thus $f^{-1}(B)$ is a fuzzy $(r, s)$-semiclosed set in $X$. Therefore $f$ is a fuzzy $(r, s)$-irresolute map.

Remark 3.11 Clearly a fuzzy $(r, s)$-continuous map is a fuzzy almost $(r, s)$-continuous map for each $(r, s) \in I \otimes I$. That the converse need not be true is shown by the following example. Also, the example shows that a fuzzy almost $(r, s)$-continuous map need not be a fuzzy $(r, s)$-irresolute map for each $(r, s) \in I \otimes I$.

Example 3.12 Let $X = \{x, y, z\}$ and let $A, A_2$ and $B$ be intuitionistic fuzzy sets in $X$ defined as
\[
A(x) = (0, 0.5), \quad A(y) = (0.3, 0.5), \quad A(z) = (0.3, 0.5);
\]
and
\[
B(x) = (0, 0.7), \quad B(y) = (0.2, 0.7), \quad B(z) = (0.2, 0.7).
\]
Define $T : I(X) \rightarrow I \otimes I$ and $U : I(X) \rightarrow I \otimes I$ by
\[
T(A) = (T_1(A), T_2(A)) = \begin{cases} 
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\
(0, 1) & \text{otherwise};
\end{cases}
\]
and
\[
U(C) = (U_1(C), U_2(C)) = \begin{cases} 
(1, 0) & \text{if } C = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } C = A, \\
(0, 1) & \text{otherwise};
\end{cases}
\]
Then clearly $T$ and $U$ are SolfTs on $X$. Consider a map $f : (X, T) \rightarrow (X, U)$ defined by $f(x) = x$ and $f(y) = y$. Then it is easy to see that $f$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-continuous map and $B$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen set in $(X, T)$. But since $f^{-1}(B) = B$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-irresolute map.

Then clearly $T$ and $U$ are SolfTs on $X$. Consider a map $f : (X, T) \rightarrow (X, U)$ defined by $f(x) = x$ and $f(y) = y$. Then it is easy to see that $f$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-continuous map and $B$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen set in $(X, T)$. But since $f^{-1}(B) = B$ is not a fuzzy $(\frac{1}{2}, \frac{1}{2})$-irresolute map.

Remark 3.14 Clearly a fuzzy $(r, s)$-continuous map is a fuzzy weakly $(r, s)$-continuous map for each $(r, s) \in I \otimes I$. That the converse need not be true is shown by the following example. Also, the following example shows that a fuzzy weakly $(r, s)$-continuous map is neither a fuzzy
(\(r, s\))- irresolute map nor a fuzzy almost \((r, s)\)-continuous map for each \((r, s) \in I \otimes I\).

**Example 3.15** Let \(X = \{x, y, z\}\) and let \(A_1, A_2\) and \(B\) be intuitionistic fuzzy sets in \(X\) defined as

\[
A_1(x) = (0.4, 0.3), \quad A_1(y) = (0.4, 0.4), \quad A_1(z) = (0.1, 0.5);
\]

\[
A_2(x) = (0.0, 0.5), \quad A_2(y) = (0.3, 0.5), \quad A_2(z) = (0.1, 0.6);
\]

and

\[
B(x) = (0.3, 0), \quad B(y) = (0.4, 0.3), \quad B(z) = (0.5, 0.2).
\]

Define \(T : I(X) \to I \otimes I\) and \(U : I(X) \to I \otimes I\) by

\[
T(A) = (T_1(A), T_2(A)) = \begin{cases} 
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\
(0, 1) & \text{otherwise};
\end{cases}
\]

and

\[
U(A) = (U_1(A), U_2(A)) = \begin{cases} 
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_2, \\
(0, 1) & \text{otherwise}.
\end{cases}
\]

Then clearly \(T\) and \(U\) are SolIFTs on \(X\). Consider a map \(f : (X, T) \to (X, U)\) defined by \(f(x) = x, f(y) = y\) and \(f(z) = z\). Note that

\[
f^{-1}(0) = 0 \subseteq \text{int}(f^{-1}(\text{cl}(0, \frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}))) = 0,
\]

\[
f^{-1}(1) = 1 \subseteq \text{int}(f^{-1}(\text{cl}(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}))) = 1,
\]

and

\[
f^{-1}(A_2) = A_2 \subseteq \text{int}(f^{-1}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}))) = A_1.
\]

Hence \(f\) is a fuzzy weakly \((\frac{1}{2}, \frac{1}{2})\)-continuous map. On the other hand, since \(f^{-1}(A_2) = A_2\) is not fuzzy \((\frac{1}{2}, \frac{1}{2})\)-open in \((X, T), f\) is not a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-continuous map.

**Example 3.16** Let \(X = \{x, y, z\}\) and let \(A_1\) and \(A_2\) be intuitionistic fuzzy sets in \(X\) defined as

\[
A_1(x) = (0, 1), \quad A_1(y) = (0.2, 0.7), \quad A_1(z) = (0.1, 0.7);
\]

and

\[
A_2(x) = (0, 1), \quad A_2(y) = (0.3, 0.7), \quad A_2(z) = (0.1, 0.7).
\]

Define \(T : I(X) \to I \otimes I\) and \(U : I(X) \to I \otimes I\) by

\[
T(A) = (T_1(A), T_2(A)) = \begin{cases} 
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\
(0, 1) & \text{otherwise};
\end{cases}
\]

and

\[
U(A) = (U_1(A), U_2(A)) = \begin{cases} 
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_2, \\
(0, 1) & \text{otherwise}.
\end{cases}
\]

Then clearly \(T\) and \(U\) are SolIFTs on \(X\). Consider a map \(f : (X, T) \to (X, U)\) defined by \(f(x) = x, f(y) = y\) and \(f(z) = z\). Then it is easy to see that \(f\) is a fuzzy \((\frac{1}{2}, \frac{1}{2})\)- irresolute map. Since \(f^{-1}(A_2) = A_2\) is not fuzzy \((\frac{1}{2}, \frac{1}{2})\)-open in \((X, T), f\) is not a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-continuous map.

Theorem 3.17

(1) Fuzzy \((r, s)\)- irresolute maps and fuzzy \((r, s)\)-continuous maps are independent notions.

(2) Fuzzy \((r, s)\)- irresolute maps and fuzzy almost \((r, s)\)-continuous maps are independent notions.

(3) Fuzzy \((r, s)\)- irresolute maps and fuzzy weakly \((r, s)\)- continuous maps are independent notions.

**Remark 3.18** It is clear that every fuzzy \((r, s)\)- presemiopen map is a fuzzy \((r, s)\)-semiopen map for each \((r, s) \in I \otimes I\). However, the converse may be false as shown by the following example.

**Example 3.19** Let \(X = \{x, y, z\}\) and let \(A_1, A_2\) and \(B\) be intuitionistic fuzzy sets in \(X\) defined as

\[
A_1(x) = (1, 0), \quad A_1(y) = (0.3, 0.5), \quad A_1(z) = (0.1, 0.5);
\]

\[
A_2(x) = (0, 1), \quad A_2(y) = (0.3, 0.5), \quad A_2(z) = (0.1, 0.5);
\]

and

\[
B(x) = (1, 0), \quad B(y) = (0.4, 0.2), \quad B(z) = (0.1, 0.1).
\]
Define $T : I(X) \to I \otimes I$ and $U : I(X) \to I \otimes I$ by

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise}; \end{cases}$$

and

$$U(A) = (U_1(A), U_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise}. \end{cases}$$

Then clearly $T$ and $U$ are SoIIFTs on $X$. Consider a map $f : (X, T) \to (X, U)$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Then it is easy to see that $f$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen map. Obviously, $B$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen set in $(X, T)$. But since $f(B) = B$ is not a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen set in $(X, U)$, $f$ is not fuzzy $(\frac{1}{2}, \frac{1}{2})$-presemiopen map.

References


