A Fixed Point for Pair of Maps in Intuitionistic Fuzzy Mtric Space

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Abstract

Park, Park and Kwon[6] is defined the intuitionistic fuzzy metric space in which it is a little revised from Park[5]. According to this paper, Park, Kwon and Park[11] Park and Kwon[10], Park, Park and Kwon[7] are established some fixed point theorems in the intuitionistic fuzzy metric space. Furthermore, Park, Park and Kwon[6] obtained common fixed point theorem in the intuitionistic fuzzy metric space, and also, Park, Park and Kwon[8] proved common fixed points of maps on intuitionistic fuzzy metric spaces. We prove a fixed point for pair of maps with another method from Park, Park and Kwon[7] in intuitionistic fuzzy metric space defined by Park, Park and Kwon[6]. Our research are an extension of Vijayaraju and Marudai’s result[14] and generalization of Park, Park and Kwon[7], Park and Kwon[10].

Key words: t-norm, t-conorm, Intuitionistic Fuzzy Metric Space, Fixed Point.

1. Introduction

Grabiec [1], Park and Kim[9] are studied a fixed point theorem in a fuzzy metric space. Also, Mishra, Shrama and Singh[4], Subremanym[13] are proved a common fixed point theorem in fuzzy metric spaces. Vijayaraju and Marudai[14] obtained fixed point for pair of maps in fuzzy metric spaces.

Recently, Park[5] is defined the intuitionistic fuzzy metric space, and Park, Park and Kwon[6] is defined the intuitionistic fuzzy metric space in which it is a little revised from Park[5]. According to this paper, Park, Kwon and Park[11] Park and Kwon[10], Park, Park and Kwon[7] are established some fixed point theorems in the intuitionistic fuzzy metric space. Furthermore, Park, Park and Kwon[6] obtained common fixed point theorem in the intuitionistic fuzzy metric space, and also, Park, Park and Kwon[8] proved common fixed points of maps on intuitionistic fuzzy metric spaces.

In this paper, we prove a fixed point for pair of maps in intuitionistic fuzzy metric spaces. Our research are an extension of Vijayaraju and Marudai’s result[14] and generalization of Park, Park and Kwon[7], Park and Kwon[10].

2. Preliminaries

We will give some definitions, properties and notation of the intuitionistic fuzzy metric space following by Schweizer and Sklar[12], Grabie[1] and Park, Park and Kwon[6].

Definition 2.1. ([12]) A operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous $t-$norm if $*$ is satisfying the following conditions:

(a) $*$ is commutative and associative,
(b) $*$ is continuous,
(c) $a * 1 = a$ for all $a \in [0,1]$,
(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Definition 2.2. ([12]) A operation $\circ : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous $t-$conorm if $\circ$ is satisfying the following conditions:

(a) $\circ$ is commutative and associative,
(b) $\circ$ is continuous,
(c) $a \circ 1 = a$ for all $a \in [0,1]$,
(d) $a \circ b \geq c \circ d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Remark 2.3. ([5]) The following conditions are satisfied:

(a) For any $r_1, r_2 \in (0,1)$ with $r_1 > r_2$, there exist $r_3, r_4 \in (0,1)$ such that $r_1 * r_3 \geq r_2$ and $r_4 * r_2 \leq r_1$.
(b) For any $r_5 \in (0,1)$, there exist $r_6, r_7 \in (0,1)$ such that $r_6 * r_7 \geq r_5$ and $r_7 * r_7 \leq r_5$.

Definition 2.4. ([6]) The 5--tuple $(X, M, N, *, \circ)$ is said to be an intuitionistic fuzzy metric space if $X$ is an arbitrary set, $*$ is a continuous $t-$norm, $\circ$ is a continuous $t-$conorm and $M, N$ are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that
(a) \( M(x, y, t) > 0 \),
(b) \( M(x, y, t) = 1 \iff x = y \),
(c) \( M(x, y, t) = M(y, x, t) \),
(d) \( M(x, y, t) \ast M(y, z, s) \leq M(x, z, t + s) \),
(e) \( M(x, y, \cdot) : (0, \infty) \to (0, 1] \) is continuous,
(f) \( N(x, y, t) > 0 \),
(g) \( N(x, y, t) = 0 \iff x = y \),
(h) \( N(x, y, t) = N(y, x, t) \),
(i) \( N(x, y, t) \ast N(y, z, s) \geq N(x, z, t + s) \),
(j) \( N(x, y, \cdot) : (0, \infty) \to [0, 1] \) is continuous.

Then \( (M, N) \) is called an intuitionistic fuzzy metric on \( X \). The functions \( M(x, y, t) \) and \( N(x, y, t) \) denote the degree of nearness and the degree of non-nearness between \( x \) and \( y \) with respect to \( t \), respectively.

**Remark 2.5.** (111) In an intuitionistic fuzzy metric space \( (X, M, N, *, \circ) \), \( M(x, y, \cdot) \) is nondecreasing and \( N(x, y, \cdot) \) is nonincreasing for all \( x, y \in X \).

Throughout the paper, we shall use \( N \) to denote the set of natural numbers and \( X \) to denote an intuitionistic fuzzy metric space \( (X, M, N, *, \circ) \) with the following properties:

\[
\lim_{t \to \infty} M(x, y, t) = 1, \quad \lim_{t \to \infty} N(x, y, t) = 0 \quad \text{for all } x, y \in X.
\]

**Definition 2.6.** (110) Let \( X \) be an intuitionistic fuzzy metric space.

(a) A sequence \( \{x_n\} \) in \( X \) is called a Cauchy sequence iff

\[
\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1, \quad \lim_{n \to \infty} N(x_{n+p}, x_n, t) = 0 \quad \text{for each } p \in \mathbb{N}, \ t > 0.
\]

(b) A sequence \( \{x_n\} \) in \( X \) is convergent to \( x \) in \( X \) iff

\[
\lim_{n \to \infty} M(x_n, x, t) = 1, \quad \lim_{n \to \infty} N(x_n, x, t) = 0 \quad \text{for each } t > 0.
\]

(c) \( X \) is said to be complete if every Cauchy sequence in \( X \) is convergent in \( X \).

**Lemma 2.7.** (110) Let \( \{x_n\} \) be a sequence in an intuitionistic fuzzy metric space \( X \). If there exists a positive number \( k \), \( 0 < k < 1 \) such that

\[
M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t), \quad N(x_{n+2}, x_{n+1}, kt) \leq N(x_{n+1}, x_n, t), \quad t > 0, \ n \in \mathbb{N}.
\]

Then \( \{x_n\} \) is a Cauchy sequence.

**Lemma 2.8.** (110) If \( x, y \) are any two points in an intuitionistic fuzzy metric space \( X \) and \( k \) is a positive number with \( k < 1 \), and

\[
M(x, y, kt) \geq M(x, y, t), \quad N(x, y, kt) \leq N(x, y, t),
\]

then \( x = y \).

**Lemma 2.9.** (110) Let \( X \) be a complete intuitionistic fuzzy metric space and \( T \) be a self-map of \( X \) satisfying

\[
M(Tx, Ty, kt) \geq M(x, y, t), \quad N(Tx, Ty, kt) \leq N(x, y, t)
\]

for all \( x, y \in X \) and \( 0 < k < 1 \). Then \( T \) has a unique fixed point in \( X \).

**3. Main Results**

In this section, we prove a fixed point for pair of maps with another method from Park, Park, Kwun[7] in intuitionistic fuzzy metric space defined by Park, Park, Kwun[6]. Our research are an extension of Vijayaraju, Marudai’s result[14] and generalization of Park, Park, Kwun[7], Park, Kwun[10].

**Lemma 3.1.** (110) Let \( \{x_n\} \) is a sequence in an intuitionistic fuzzy metric space \( X \). If

\[
M(x_n, x_{n+1}, t) \geq M(x_0, x_1, \frac{t}{\alpha^n}), \quad N(x_n, x_{n+1}, t) \leq N(x_0, x_1, \frac{t}{\alpha^n}),
\]

where \( \alpha \) is a positive number with \( 0 < \alpha < 1 \) and \( n \in \mathbb{N} \), then \( \{x_n\} \) is a Cauchy sequence.

**Lemma 3.2.** (110) If \( X \) is an intuitionistic fuzzy metric space and \( \{x_n\} \) is a sequence in \( X \) such that

\[
M(x_{i+1}, x_{i+2}, kt) \geq M(x_i, x_{i+1}, t) \ast M(x_{i+1}, x_{i+2}, t),
\]

\[
N(x_{i+1}, x_{i+2}, kt) \leq N(x_i, x_{i+1}, t) \ast N(x_{i+1}, x_{i+2}, t),
\]

where \( 0 < k < 1 \), \( i = 0, 1, 2, \ldots \) and \( t > 0 \), then

\[
M(x_{i+1}, x_{i+2}, kt) \geq M(x_i, x_{i+1}, t), \quad N(x_{i+1}, x_{i+2}, kt) \leq N(x_i, x_{i+1}, t).
\]

**Theorem 3.3.** Let \( X \) be a complete intuitionistic fuzzy metric space. If \( T, S \) are self maps on \( X \) such that

\[
M(Tx, Sy, \beta t) \geq M(x, Tx, t) \ast M(y, Sy, t), \quad N(Tx, Sy, \beta t) \leq N(x, Tx, t) \ast N(y, Sy, t)
\]

for all \( x, y \in X \) and \( 0 < \beta < \frac{1}{2} \), then \( T \) and \( S \) have a unique common fixed point in \( X \).

**Proof.** Let \( x_0 \in X \) be fixed. We define a sequence \( \{x_n\} \subset X \) by

\[
x_{n+1} = Tx_n \text{ if } n \text{ is even, } \quad Sx_n \text{ if } n \text{ is odd}.
\]

Now, we will prove that

\[
M(x_{n+1}, x_n, lt) \geq M(x_0, x_1, t) \ast M(x_n, x_{n+1}, \frac{t}{1-\beta}) \ast M(x_0, x_1, t),
\]

\[
N(x_{n+1}, x_n, lt) \leq N(x_0, x_1, t) \ast N(x_n, x_{n+1}, \frac{t}{1-\beta}) \ast N(x_0, x_1, t).
\]
\[
M(x_1, x_2, \left(\frac{\beta}{1 - \beta} t\right)) \\
= M\left(Tx_0, Sx_1, \beta \cdot \frac{t}{1 - \beta}\right) \\
\geq M(x_0, x_1, \frac{t}{1 - \beta}) \ast M(x_1, x_2, \frac{t}{1 - \beta}) \\
\geq M(x_0, x_1, \frac{t}{1 - \beta}), \quad \text{(by Lemma 3.2)} \\
\geq M(x_0, x_1, t), \quad \text{(because of } \frac{t}{1 - \beta} > t) \\
N(x_1, x_2, \left(\frac{\beta}{1 - \beta} t\right)) \\
= N\left(Tx_0, Sx_1, \beta \cdot \frac{t}{1 - \beta}\right) \\
\leq N(x_0, x_1, \frac{t}{1 - \beta}) \circ N(x_1, x_2, \frac{t}{1 - \beta}) \\
\leq N(x_0, x_1, \frac{t}{1 - \beta}) \\
\leq N(x_0, x_1, t), \quad \text{(because of } \frac{t}{1 - \beta} > t).
\]

Thus the result is true for \( n = 1 \).

Suppose that the result is true for \( n = k \), that is,

\[
M(x_k, x_{k+1}, \left(\frac{\beta}{1 - \beta} k \cdot t\right)) \geq M(x_0, x_1, t) \\
N(x_k, x_{k+1}, \left(\frac{\beta}{1 - \beta} k \cdot t\right)) \leq N(x_0, x_1, t).
\]

Without loss of generality, let us assume that \( k \) is even,

\[
M(x_{k+1}, x_{k+2}, \left(\frac{\beta}{1 - \beta} \cdot k+1 \cdot t\right)) \\
= M\left(Tx_k, Sx_{k+1}, \beta \cdot \frac{k+1}{1 - \beta} \cdot t\right) \\
\geq M(x_k, x_{k+1}, \frac{t}{1 - \beta}) \ast M(x_{k+1}, x_{k+2}, \frac{t}{1 - \beta}) \\
= M(x_k, x_{k+1}, \frac{t}{1 - \beta} \cdot k) \ast M(x_{k+1}, x_{k+2}, \frac{t}{1 - \beta}) \\
\geq M(x_0, x_1, \frac{t}{1 - \beta}) \circ M(x_{k+1}, x_{k+2}, \frac{t}{1 - \beta}) \\
\leq N(x_0, x_1, \frac{t}{1 - \beta}) \circ N(x_{k+1}, x_{k+2}, \frac{t}{1 - \beta}) \\
\leq N(x_0, x_1, t).
\]

Then by Lemma 3.2, we have

\[
M(x_{k+1}, x_{k+2}, \left(\frac{\beta}{1 - \beta} \cdot k+1 \cdot t\right)) \\
\geq M(x_0, x_1, \frac{t}{1 - \beta}) \circ N(x_{k+1}, x_{k+2}, \frac{t}{1 - \beta}) \\
\leq N(x_0, x_1, t).
\]

Hence the result is true for all \( n \). Therefore

\[
M(x_n, x_{n+1}, \left(\frac{\beta}{1 - \beta} \cdot n\right) \geq M(x_0, x_1, t), \\
N(x_n, x_{n+1}, \left(\frac{\beta}{1 - \beta} \cdot n\right) \leq N(x_0, x_1, t),
\]

which can be written as

\[
M(x_n, x_{n+1}, \frac{1}{1 - \beta} \cdot n) \geq M(x_0, x_1, \frac{1}{1 - \beta} \cdot n), \\
N(x_n, x_{n+1}, \frac{1}{1 - \beta} \cdot n) \leq N(x_0, x_1, \frac{1}{1 - \beta} \cdot n).
\]

By Lemma 3.1, \( \{x_n\} \) is a Cauchy sequence in \( X \). Since \( X \) is complete, \( \{x_n\} \) converges to a point \( x \) in \( X \). That is,

\[
\lim_{n \to \infty} M(x_n, x, t) = 1, \quad \lim_{n \to \infty} N(x_n, x, t) = 0.
\]

Now, by Definition 2.3 and assumption of this theorem

\[
M(x, Tx, t) \\
\geq M(x, x_n, \frac{t}{2}) \ast M(x_n, Tx, \frac{t}{2}) \\
= M(x, x_n, \frac{t}{2}) \ast M(Sx_{n-1}, Tx, \frac{t}{2}) \\
\geq M(x, x_n, \frac{t}{2}) \ast M(x, Tx, \frac{t}{2 \beta}) \ast M(x_{n-1}, x_n, \frac{t}{2 \beta}), \\
N(x, Tx, t) \\
\leq N(x, x_n, \frac{t}{2}) \circ N(x_n, Tx, \frac{t}{2}) \\
= N(x, x_n, \frac{t}{2}) \circ N(Sx_{n-1}, Tx, \frac{t}{2}) \\
\leq N(x, x_n, \frac{t}{2}) \circ N(x, Tx, \frac{t}{2 \beta}) \circ N(x_{n-1}, x_n, \frac{t}{2 \beta}).
\]

Taking limit as \( n \to \infty \), we get

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\[ M(x, Tx, t) \geq 1 \cdot M(x, Tx, \frac{t}{2\beta}) \cdot 1 = M(x, Tx, \frac{t}{2\beta}), \]
\[ N(x, Tx, t) \leq 0 \circ N(x, Tx, \frac{t}{2\beta}) \circ 0 = N(x, Tx, \frac{t}{2\beta}). \]

By lemma 2.9, \( Tx = x \).

Similarly, \( Sx = x \).

Now, we will show that \( x \) is a unique common fixed point of \( T \) and \( S \) in \( X \).

Assume that there exist another fixed point \( y \) in \( X(Ty = Sy = y) \). Then

\[ M(x, y, t) = M(Tx, Sy, t) \]
\[ \geq M(x, Tx, \frac{t}{\beta}) \cdot M(y, Sy, \frac{t}{\beta}) = 1, \]
\[ N(x, y, t) = N(Tx, Sy, t) \]
\[ \leq N(x, Tx, \frac{t}{\beta}) \circ N(y, Sy, \frac{t}{\beta}) = 0. \]

Therefore \( M(x, y, t) = 1 \) and \( N(x, y, t) = 0 \). Hence \( x = y \). Thus \( x \) is a unique common fixed point of \( T \) and \( S \) in \( X \).

**Corollary 3.4.** (10) If \( T \) is a self map on a complete intuitionistic fuzzy metric space \( X \) and if there exists a positive number \( \beta \) with \( 0 < \beta < \frac{1}{2} \) such that

\[ M(Tx, Ty, \beta t) \geq M(x, Ty, t) \cdot M(y, Ty, t), \]
\[ N(Tx, Ty, \beta t) \leq N(x, Ty, t) \circ N(y, Ty, t) \]

for all \( x, y \in X \) and \( t \geq 0 \), then \( T \) has a unique fixed point in \( X \).

**Proof.** The proof follows immediately from Theorem 3.3 by putting \( T = S \).

**Theorem 3.5.** Let \( X \) be a complete intuitionistic fuzzy metric space. Also, let \( T \) and \( S \) be two self maps on \( X \) such that

(a) \( M(Tx, Sy, \alpha t) \geq M(x, y, t) \cdot N(Tx, Sy, \alpha t) \leq N(x, y, t) \), where \( 0 < \alpha < 1 \), \( x, y \in X \), \( x \neq y \).

(b) \( S \) is a contraction on \( X \). That is, there exists \( \beta \) with \( 0 < \beta < 1 \) such that \( M(Sx, Sy, \beta t) \geq M(x, y, t) \), \( N(Sx, Sy, \beta t) \leq N(x, y, t) \) for all \( x, y \in X \), and

(c) there exists \( x_0 \in X \) such that

\[ x_{n+1} = \begin{cases} Tx_n & \text{if } n \text{ is even} \\ Sx_n & \text{if } n \text{ is odd} \end{cases} \]

with \( x_m \neq x_l \) if \( m \neq l \).

Then \( T \) and \( S \) have a unique common fixed point in \( X \).

**Proof.** If \( x_1, x_2 \) are two distinct points in \( X \), then it is impossible that \( Tx_1 = x_1 \) and \( Sx_2 = x_2 \). For if \( Tx_1 = x_1 \) and \( Sx_2 = x_2 \), then by (a),

\[ M(x_1, x_2, \alpha t) = M(Tx_1, Sx_2, \alpha t) \geq M(x_1, x_2, t), \]
\[ N(x_1, x_2, \alpha t) = N(Tx_1, Sx_2, \alpha t) \leq N(x_1, x_2, t). \]

This is a contradiction from Remark 2.5. Since \( S \) is contraction, \( S \) has a unique fixed point say \( x \) from Lemma 2.9. Therefore if \( T \) has a fixed point, it is unique and must coincide with \( x \). If \( x_0 = x_1 \), since \( x_1 = Tx_0 = x_0 = Sx_0 \), assume that \( x_0 \neq x_1 \). Let \( x_1, x_2 \) be any two members of \( \{x_n\} \) defined by (c). Then from (a),

\[ M(x_1, x_2, t) \geq M(x_0, x_1, \frac{t}{\alpha}), \]
\[ N(x_1, x_2, t) \leq N(x_0, x_1, \frac{t}{\alpha}). \]

Similarly, from

\[ M(x_2, x_3, \alpha t) = M(Sx_1, Tx_2, \alpha t) \geq M(x_1, x_2, t), \]
\[ N(x_2, x_3, \alpha t) = N(Sx_1, Tx_2, \alpha t) \leq N(x_1, x_2, t), \]

we have

\[ M(x_2, x_3, t) = M(Sx_1, Tx_2, t) \geq M(x_0, x_1, \frac{t}{\alpha^2}), \]
\[ N(x_2, x_3, t) = N(Sx_1, Tx_2, t) \leq N(x_0, x_1, \frac{t}{\alpha^2}). \]

\[ \cdots \cdots \cdots \cdots \]
\[ M(x_n, x_{n+1}, t) \geq M(x_0, x_1, \frac{t}{\alpha^n}), \]
\[ N(x_n, x_{n+1}, t) \leq N(x_0, x_1, \frac{t}{\alpha^n}). \]

Hence by Lemma 3.1 and Lemma 2.7, \( \{x_n\} \) is a Cauchy sequence. Since \( X \) is complete, it converges to \( y_0 \) in \( X \). Therefore it satisfied the Definition 2.6(b).

Suppose that \( n \) is even integer. Then

\[ M(y_0, Ty_0, t) \geq M(y_0, x_n, \frac{t}{2}) \cdot M(x_n, Ty_0, \frac{t}{2}) \]
\[ = M(y_0, x_n, \frac{t}{2}) \cdot M(Sx_{n-1}, Ty_0, \frac{t}{2}) \]
\[ \geq M(y_0, x_n, \frac{t}{2}) \cdot M(x_{n-1}, y_0, \frac{t}{2\alpha}), \]
\[ N(y_0, Ty_0, t) \leq N(y_0, x_n, \frac{t}{2}) \circ N(x_n, Ty_0, \frac{t}{2}) \]
\[ = N(y_0, x_n, \frac{t}{2}) \circ N(Sx_{n-1}, Ty_0, \frac{t}{2}) \]
\[ \leq N(y_0, x_n, \frac{t}{2}) \circ N(x_{n-1}, y_0, \frac{t}{2\alpha}). \]

Taking limit as \( n \to \infty \), we get

\[ M(y_0, Ty_0, t) \geq 1 \cdot 1 = 1, \quad N(y_0, Ty_0, t) \leq 0 \cdot 0 = 0. \]

Thus \( y_0 = Ty_0 \). We know that \( y_0 \) is a fixed point of \( T \). Therefore \( y_0 = x \). Hence \( T \) and \( S \) have a unique common fixed point in \( X \).
Corollary 3.6. ([7]) (Intuitionistic fuzzy Banach contraction theorem) Let $X$ be a complete intuitionistic fuzzy metric space and $T : X \to X$ be a mapping satisfying

$$M(Tx, Ty, \alpha t) \geq M(x, y, t), \quad N(Tx, Ty, \alpha t) \leq N(x, y, t)$$

where $0 < \alpha < 1$, $x, y \in X$ and all $t > 0$. Then $T$ has a unique fixed point in $X$.

Proof. We proved this corollary from Theorem 3.5 by putting $T = S$. Also, in this proof, we used another method with respect to [7].

4. Example

Example 4.1. Let $(X, d)$ be a metric space in $X = [0, 1]$. Denote $x \ast y = \min\{x, y\}$, $x \circ y = \max\{x, y\}$ for all $x, y \in X$ and let $M_d, N_d$ be fuzzy sets on $X^2 \times (0, \infty)$ as follows:

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

where for any $x, y \in X$, $t > 0$, $d(x, y) = |x - y|$.

Define maps $T, S : X \to X$ by $Tx = 1 - x$, $Sx = \frac{3}{4} - \frac{x}{2}$ for all $x \in X$. Then $(X, M_d, N_d, \ast, \circ)$ is an intuitionistic fuzzy metric space.

Also,

$$M_d(Sx, Sy, t) = \frac{t}{t + d(Sx, Sy)} = \frac{t}{t + \frac{3}{2}|y - x|} \geq \frac{t}{t + \frac{3}{2}|y - x|} = M_d(x, y, t)$$

$$N_d(Sx, Sy, t) = \frac{d(Sx, Sy)}{t + d(Sx, Sy)} = \frac{\frac{3}{2}|y - x|}{t + \frac{3}{2}|y - x|} \leq \frac{\frac{3}{2}|y - x|}{t + \frac{3}{2}|y - x|} = N_d(x, y, t)$$

Clearly, $T(\frac{1}{2}) = \frac{1}{2} = S(\frac{1}{2})$ and $\frac{1}{2}$ is the only fixed point of both $T$ and $S$ in $X$.

But since

$$M_d(Tx, Sy, t) = \frac{t}{t + |\frac{1}{2} - x + \frac{1}{2}|}$$

$$N_d(Tx, Sy, t) = \frac{1}{t + |\frac{1}{2} - x + \frac{1}{2}|}$$

If $x = \frac{3}{4}, y = \frac{1}{2}$, then

$$M_d(Tx, Sy, t) = \frac{t}{t + \frac{1}{2}}, \quad N_d(Tx, Sy, t) = \frac{\frac{3}{2}}{t + \frac{1}{2}} = N(x, y, t)$$

Hence we can know that Theorem 3.5 gives only some sufficient conditions for which $T$ and $S$ have a common unique fixed point in $X$.

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