The Properties of Implications and Conjunctions

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Abstract

We investigate the properties of (forcing)-implications, conjunctions and adjointness in a sense Morsi et.al [1,5].

Key words: (forcing)-implications, conjunctions, adjointness

1. Introduction and Preliminaries

Recently, Morsi et.al [1,5] introduced the theory of implications and conjunctions (generalized by t-norm) related by adjointness in many valued logics.

In this paper, we introduce characterizations of (forcing)-implications, conjunctions and adjointness. We investigate the relations of them. In particular, we study the (forcing)-implications, conjunctions and adjointness induced by functions. Let $L$ be a completely distributive lattice with a top 1 and a bottom 0.

Definition 1.1. ([1,5]) A binary operation $A : L \times L \rightarrow L$ is called an implication if it satisfies:

(A1) if $x \leq y$, then $A(x, z) \leq A(y, z)$.

(A2) if $y \leq z$, then $A(x, y) \leq A(x, z)$.

(A3) $A(1, z) = z$.

A binary operation $A : L \times L \rightarrow L$ is called a forcing-implication if it satisfies (A1), (A2) and (H) $y \leq z$ iff $H(y, z) = 1$.

Definition 1.2. ([1,5]) A binary operation $K : L \times L \rightarrow L$ is called a conjunction if it satisfies:

(K1) if $x \leq y$, then $K(x, z) \leq K(y, z)$.

(K2) if $y \leq z$, then $K(x, y) \leq K(x, z)$.

(K3) $K(1, z) = z$.

Definition 1.3. ([1,5]) (1) A binary operation $K$ is called a left adjoint of $A$, denoted by $K \vdash A$, if it satisfies: for all $x, y, z \in L$,

(adjointness) $K(x, y) \leq z$ iff $y \leq A(x, z)$.

(2) A binary operation $H$ is called a left adjoint of $A$, denoted by $H \vdash \text{op} A$, if it satisfies: for all $x, y, z \in L$,

(adjointness) $H(y, z) \leq \text{op} x$ iff $y \leq A(x, z)$

where $\leq \text{op} = \geq$.

Definition 1.4. ([1,5]) A function $N : L \rightarrow L$ is called a negation if it satisfies:

(N1) $N(0) = 1$ and $N(1) = 0$.

(N2) if $x \leq y$, then $N(x) \geq N(y)$.

(N3) $N(N(x)) = x$.

2. Implications and Conjunctions

Theorem 2.1. Let $f : L \rightarrow L$ be an order-isomorphic function ($f$ is bijective and $x \leq y$ iff $f(x) \leq f(y)$) with $f(1) = 1$. Define a binary operation $A : L \rightarrow L$ by

$A(x, y) = f^{-1}(N(f(x)) \lor f(y))$.

Then $A$ is an implication. Moreover, if $L$ is a Boolean algebra, then $A$ is an implication and a forcing-implication.

Proof. It is easily proved

$A(1, z) = f^{-1}(N(f(1)) \lor f(z)) = f^{-1}(f(z)) = z$.

If $L$ is a Boolean algebra, then $1 = N(a) \lor b$ iff $a \leq b$. Thus

$1 = A(x, y) = f^{-1}(N(f(x)) \lor f(y))$

iff $1 = N(f(x)) \lor f(y)$

iff $f(x) \leq f(y)$ iff $x \leq y$.

Hence $A$ is a forcing-implication.

Example 2.2. Let $(P(U), \subseteq, \emptyset, U)$ be a completely distributive lattice.

(1) We define an operator $A : P(U) \rightarrow P(U)$ as follows:

$A(X, Y) = Y$.

Then $A$ is an implication operator.

(2) We define an operator $H : P(U) \rightarrow P(U)$ as follows:

$H(X, Y) = \begin{cases} U & \text{if } X \subseteq Y, \\ \emptyset & \text{if } X \nsubseteq Y. \end{cases}$
Then $H$ is a forcing-implication.

(3) We define an operator $A : P(U) \to P(U)$ as follows

$$A(X, Y) = X^c \cup Y.$$ 

Then $A$ is an implication and forcing implication operator.

**Theorem 2.3.** Let $f : [0, 1] \to [f(0), 1]$ be a bijective strictly-increasing function and $p > 0$. Define binary operations $A_1, A_2 : [0, 1] \times [0, 1] \to [0, 1]$ by

$$A_1(x, y) = f^{-1}\left(\frac{f(y)}{f(x)^p} \wedge 1\right), \quad f(0) \neq 0$$

$$A_2(x, y) = f^{-1}\left(1 - f(x)^p + f(y)\right) \wedge 1), \quad f(0) = 0$$

Then we have the following properties:

(1) $A_1$ and $A_2$ are implications.

(2) If $p = 1$, then $A_1$ and $A_2$ are implications and forcing-implications.

**Proof.** (1) Since $f(1) = 1$, we have:

$$A_1(1, y) = f^{-1}\left(\frac{f(y)}{f(1)^p} \wedge 1\right) = y,$$

$$A_2(1, y) = f^{-1}\left(1 - f(1)^p + f(y)\right) \wedge 1) = y.$$

(2) If $p = 1$, then

$$A_1(x, y) = f^{-1}\left(\frac{f(y)}{f(x)} \wedge 1\right) = 1$$

$\iff \frac{f(y)}{f(x)} \geq 1 \iff x \leq y$

$$A_2(x, y) = f^{-1}\left(1 - f(x) + f(y)\right) \wedge 1) = 1$$

$\iff 1 - f(x) + f(y) \geq 1 \iff x \leq y.$

\[\square\]

**Example 2.4.** (1) Let $f : [0, 1] \to [f(0), 1]$ be a bijective strictly-increasing function as $f(x) = \frac{1}{2}x^2 + \frac{1}{2}$. From Theorem 2.3(1), we define an operator

$$A_1(x, y) = f^{-1}\left(\frac{f(y)}{f(x)} \wedge 1\right)$$

$$= \sqrt{2 - \frac{y^2}{x^2 + 1}} \wedge 1.$$ 

If $p = 1$, then $A_1$ is an implication and forcing-implication.

(2) Let $f : [0, 1] \to [0, 1]$ be a bijective strictly-increasing function as $f(x) = x^2$. From Theorem 2.3(1), we define an operator

$$A_2(x, y) = f^{-1}\left(1 - f(x)^p + f(y)\right) \wedge 1)$$

$$= \sqrt{1 - x^{2p} + y^2} \wedge 1.$$ 

If $p = 1$, then $A_2$ is implications and forcing-implications.

**Theorem 2.5.** Let $f : L \to L$ be an order-isomorphic function with $f(1) = 1$. Define a binary operation $K : L \to L$ by

$$K(x, y) = f^{-1}(f(x) \wedge f(y)).$$ 

Then $K$ is a conjunction.

**Proof.** It is easily proved from

$$K(1, y) = f^{-1}(f(1) \wedge f(y)) = y.$$ 

\[\square\]

**Example 2.6.** Let $(P(U), \subset, \emptyset, U)$ be a completely distributive lattice. We define an operator $K : P(U) \to P(U)$ as follows:

$$K(X, Y) = X \cap Y.$$ 

Then $K$ is a conjunction.

**Theorem 2.7.** Let $f : [0, 1] \to [f(0), 1]$ be a bijective strictly-increasing function and $p > 0$. Define binary operations $K_1, K_2 : [0, 1] \times [0, 1] \to [0, 1]$ by

$$K_1(x, y) = f^{-1}\left(f(x)^p f(y) \vee f(0)\right), \quad f(0) \neq 0$$

$$K_2(x, y) = f^{-1}\left(f(x)^p + f(y) - 1\right) \vee 0), \quad f(0) = 0.$$ 

Then $K_1$ and $K_2$ are conjunctions.

**Proof.** Since $f(1) = 1$, we have:

$$K_1(1, y) = f^{-1}(f(1)^p f(y) \vee f(0)) = y,$$

$$K_2(1, y) = f^{-1}\left((f(1)^p + f(y) - 1) \vee 0\right) = y.$$ 

\[\square\]

**Example 2.8.** (1) Let $f : [0, 1] \to [f(0), 1]$ be a bijective strictly-increasing function as $f(x) = \frac{1}{2}x^2 + \frac{1}{2}$. From Theorem 2.7, we define an operator

$$K_1(x, y) = f^{-1}\left(f(x)^p f(y) \vee f(0)\right)$$

$$= \sqrt{2 - \frac{y^2}{x^2 + 1}} \wedge 1.$$ 

(2) Let $f : [0, 1] \to [0, 1]$ be a bijective strictly-increasing function as $f(x) = x^2$. From Theorem 2.7, we define an operator

$$K_2(x, y) = f^{-1}\left(f(x)^p + f(y) - 1\right) \vee 0)$$

$$= \sqrt{x^{2p} + y^2 - 1} \vee 0.$$ 

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3. The Adjointness for Fuzzy Logics

**Theorem 3.1.** (1) A binary operation $K$ is a left adjoint of $A$ iff for all $x, y, z \in L$,
\[
y \leq A(x, K(x, y)), \quad K(x, A(x, z)) \leq z.
\]
(2) A binary operation $H$ is a left adjoint of $A, H \uparrow_{\text{op}} A$ iff for all $x, y, z \in L$,
\[
y \leq A(H(y, z), z), \quad H(A(x, z), z) \geq x.
\]

**Proof.** (1) Since $K(x, y) \leq K(x, y)$ and $A(x, z) \leq A(x, z)$, by adjointness, we have
\[
y \leq A(x, K(x, y)), \quad K(x, A(x, z)) \leq z.
\]
Conversely, let $K(x, y) \leq z$. By (A2), we have
\[
A(x, z) \geq A(x, K(x, y)) \geq y.
\]
Let $A(x, z) \geq y$. By (K2), we have
\[
K(x, y) \leq K(x, A(x, z)) \leq z.
\]
(2) Since $H(x, y) \uparrow_{\text{op}} H(x, y)$ and $A(x, z) \leq A(x, z)$, by adjointness, we have
\[
y \leq A(H(y, z), y), \quad H(A(x, z), z) \geq x.
\]
Conversely, let $H(y, z) \leq x$. By (A1), we have
\[
A(x, z) \geq A(H(y, z), z) \geq y.
\]
Let $A(x, z) \geq y$. By (A1), we have
\[
H(y, z) \geq H(A(x, z), z) \geq x.
\]
Hence $H(y, z) \uparrow_{\text{op}} x$. □

**Theorem 3.2.** Let $(L, \leq)$ be a distributive complete lattice.
(1) An implication $A$ satisfies $A(x, \Lambda z_i) = \Lambda A(x, z_i)$ iff there exists a conjunction $K$ with $K \vdash A$ defined by
\[
K(x, y) = \Lambda \{z \in L \mid y \leq A(x, z)\}.
\]
(2) A conjunction $K$ satisfies $K(x, \Lambda z_i) = \Lambda K(x, z_i)$ iff there exists an implication $A$ with $K \vdash A$ defined by
\[
A(x, y) = \Lambda \{z \in L \mid K(x, z) \leq y\}.
\]
(3) An implication $A$ satisfies $A(\Lambda x_i, z) = \Lambda A(x_i, z)$ iff there exists a forcing-implication $H$ with $H \uparrow_{\text{op}} A$ defined by
\[
H(x, y) = \Lambda \{z \in L \mid x \leq A(z, y)\}.
\]

**Proof.** (1) ($\Rightarrow$) (K1) If $x_1 \leq x_2$, then $A(x_1, z) \geq A(x_2, z) \geq y$ implies $K(x_1, y) \leq K(x_2, y)$.
(K2) If $y_1 \leq y_2$, then $y_1 \leq y_2 \leq A(x, z)$ implies $K(x, y_1) \leq K(x, y_2)$.
(K3) $K(1, y) = \Lambda \{z \in L \mid y \leq A(1, z) = z\} = y$.
Hence $K$ is a conjunction. Let $y \leq A(x, z)$. Then $K(x, y) \leq z$. Let $K(x, y) \leq z$. Then $A(x, K(x, y)) \leq A(x, z)$ and
\[
A(x, K(x, y)) = A(x, \Lambda \{z \in L \mid y \leq A(x, z)\})
\]
\[
= \Lambda \{A(x, z) \mid y \leq A(x, z)\}
\]
\[
= y.
\]
So, $A(x, z) \geq y$. Hence $K \vdash A$.

($\Leftarrow$) Enough to $\Lambda A(x, z_i) \leq A(x, \Lambda z_i)$. It follows from:
\[
K(x, \Lambda A(x, z_i)) \leq K(x, A(x, z_i)) \leq z_i
\]
\[
\Rightarrow K(x, \Lambda A(x, z_i)) \leq \Lambda z_i
\]
\[
\Rightarrow A(x, z_i) \leq A(x, \Lambda z_i).
\]
(2) ($\Rightarrow$) (A1) If $x_1 \leq x_2$, then $K(x_1, z) \leq K(x_2, z)$.
So, $A(x_1, y) \geq A(x_2, y)$.
(A2) If $y_1 \leq y_2$, then $K(x, z) \leq y_1 \leq y_2$ implies $A(x, y_1) \leq A(x, y_2)$.
(A3) $A(1, y) = \Lambda \{z \in L \mid K(1, z) = z \leq y\} = y$.
Hence $A$ is an implication. Let $K(x, y) \leq z$. By the definition of $A, y \leq A(x, z)$. Let $z \leq A(x, y)$. Then $K(x, A(x, y)) \geq K(x, z)$ and
\[
K(x, A(x, y)) = K(x, \Lambda \{z \in L \mid K(x, z) \leq y\})
\]
\[
= \Lambda \{K(x, z) \mid K(x, z) \leq y\}
\]
\[
\leq y.
\]
So, $K(x, z) \leq y$. Hence $K \vdash A$.

($\Leftarrow$) Enough to $\Lambda K(x, z_i) \geq K(x, \Lambda z_i)$. It follows from:
\[
A(x, \Lambda K(x, z_i)) \geq A(x, K(x, z_i)) \geq z_i
\]
\[
\Rightarrow A(x, \Lambda K(x, z_i)) \geq \Lambda z_i
\]
\[
\Rightarrow K(x, z_i) \geq K(x, \Lambda z_i).
\]
(3) ($\Rightarrow$) Let $H(y, z) = 1$ be given. Then $y \leq z$ from:
\[
z = A(1, z) = A(H(y, z), z)
\]
\[
= A(\Lambda \{x_i \mid y \leq A(x_i, z)\}, z)
\]
\[
\geq \Lambda \{A(x_i, z) \mid y \leq A(x_i, z)\}
\]
\[
\geq y.
\]
Let $y \leq z$. Since $y \leq z = A(1, z)$, we have
\[
H(y, z) = \Lambda \{x_i \mid y \leq A(x_i, z)\} = 1.
\]
Hence $H$ is a forcing-implication. Let $y \leq A(x, z)$. By the definition of $H, x \leq H(y, z)$. Let $x \leq H(y, z)$. Then $A(H(y, z), z) \leq A(x, z)$ and
\[
A(H(y, z), z) = A(\Lambda \{x_i \in L \mid y \leq A(x_i, z)\}, z)
\]
\[
= \Lambda \{A(x_i, z) \mid y \leq A(x_i, z)\}
\]
\[
\geq y.
\]
So, \( A(x, z) \geq y \). Hence \( H \supseteq A \).

(\( \iff \)) Enough to \( A(\bigwedge x_i, z) \leq A(\bigvee x_i, z) \). It follows from:

\[
H\left( \bigwedge A(x_i, z) \right) \geq H(\bigvee (A(x_i, z))) \geq x_i \\
\Rightarrow H\left( \bigwedge A(x_i, z) \right) \geq \bigvee x_i \\
\Rightarrow A(x_i, z) \leq A(\bigvee x_i, z).
\]

\( \square \)

**Theorem 3.3.** Let \( f : [0, 1] \rightarrow [f(0), 1] \) be a bijective strictly-increasing continuous function. Define an implication \( A : [0, 1] \times [0, 1] \rightarrow [0, 1] \) by

\[
A(x, y) = f^{-1}\left( \frac{f(y)}{f(x)} \land 1 \right), \quad f(0) \neq 0.
\]

Then there exists a forcing-implication \( H \) such that \( A = H \) and conjunction \( K \) such that

\[
K(x, y) = f^{-1}\left( f(x)f(y) \lor f(0) \right).
\]

**Proof.** Since \( A \) satisfies \( A(\bigvee x_i, z) = \bigwedge A(x_i, z) \), by Theorem 3.2(3), there exists a forcing-implication \( H \) defined by

\[
H(x, y) = \bigvee \{ z \in L \mid x \leq A(z, y) \}.
\]

Since \( x \leq A(z, y) = f^{-1}\left( \frac{f(y)}{f(z)} \land 1 \right) \), we have \( z \leq f^{-1}\left( \frac{f(y)}{f(z)} \land 1 \right) \). Since \( A \) is continuous from pasting lemma, we have

\[
H(x, y) = f^{-1}\left( \frac{f(y)}{f(x)} \land 1 \right).
\]

Hence \( A = H \). Since \( A \) is continuous, we have \( A(x \land z_i) = \bigwedge A(x, z_i) \). By Theorem 3.2(1), there exists a conjunction \( K \) defined by \( K(x, y) = \bigwedge \{ z \in L \mid y \leq A(z, x) \} \). Since \( y \leq f^{-1}\left( \frac{f(y)}{f(x)} \land 1 \right) \), we have

\[
z \geq f^{-1}(f(x)f(y) \lor f(0)).
\]

Hence \( K(x, y) = f^{-1}(f(x)f(y) \lor f(0)) \). \( \square \)

**Example 3.4.** Let \( f : [0, 1] \rightarrow [f(0), 1] \) be a bijective strictly-increasing function as \( f(x) = \frac{1}{2}x + \frac{1}{2} \). From Theorem 3.3, we define an operator

\[
A(x, y) = f^{-1}\left( \frac{f(y)}{f(x)} \land 1 \right) = \left( \frac{2y-x+1}{x+1} \right) \land 1.
\]

Equivalently,

\[
A(x, y) = \begin{cases} 
1 & \text{if } x \leq y, \\
\frac{2y-x+1}{x+1} & \text{if } x > y.
\end{cases}
\]

(1) \( A \) is an implication satisfying \( A(x \land z_i) = \bigwedge A(x, z_i) \). Hence we can obtain a conjunction \( K \) as follows

\[
K(x, y) = \bigwedge \{ z \in L \mid y \leq A(x, z) = \left( \frac{2z-x+1}{x+1} \right) \land 1 \} = (\frac{2z-x+1}{x+1}) \lor 0.
\]

Furthermore, \( A(x, K(x, y)) = y \lor \frac{1-x}{1+y} \) and \( K(x, A(x, z)) \leq z \) from:

Since \( x \geq K(x, y) \),

\[
A(x, K(x, y)) = \frac{2K(x, y) - x + 1}{x + 1} = y \lor \frac{1-x}{1+y}.
\]

If \( x > z \),

\[
K(x, A(x, z)) = \frac{zA(x, z)+x+1-A(x, z)-1}{x+1} \lor 0 = \frac{x+1}{x+1} = z.
\]

If \( x \leq z \), then \( K(x, A(x, z)) = x \leq z \).

(2) \( A \) is an implication satisfying \( A(\bigvee x_i, z) = \bigwedge A(x_i, z) \). Hence we can obtain a forcing implication \( H \) as follows

\[
H(x, y) = \bigvee \{ z \in L \mid x \leq A(z, y) = \left( \frac{2y-x+1}{x+1} \right) \land 1 \} = (\frac{2y-x+1}{x+1}) \land 1.
\]

Furthermore, \( A(H(y, z), z) = y \lor \frac{z(y+1)}{z+1} \land 1 \) and \( H(A(x, z), z) \geq x \).

**Theorem 3.5.** Let \( f : [0, 1] \rightarrow [0, 1] \) be a bijective strictly-increasing continuous function. Define an implication \( A : [0, 1] \times [0, 1] \rightarrow [0, 1] \) by

\[
A(x, y) = f^{-1}\left( (1 - f(x) + f(y)) \land 1 \right).
\]

Then there exists a forcing-implication \( H \) such that \( A = H \) and conjunction \( K \) such that

\[
K(x, y) = f^{-1}\left( (f(x) + f(y) - 1) \lor f(0) \right).
\]

**Proof.** Since \( A \) satisfies \( A(\bigvee x_i, z) = \bigwedge A(x_i, z) \), by Theorem 3.2(3), there exists a forcing-implication \( H \) defined by

\[
H(x, y) = \bigvee \{ z \in L \mid x \leq A(z, y) \}.
\]

Since \( x \leq A(z, y) = f^{-1}\left( (1 - f(z) + f(y)) \land 1 \right) \), we have \( z \leq f^{-1}\left( (1 - f(x) + f(y)) \land 1 \right) \). Since \( A \) is continuous from pasting lemma, we have

\[
H(x, y) = f^{-1}\left( (1 - f(x) + f(y)) \land 1 \right).
\]

Hence \( A = H \). Since \( A \) is continuous, we have \( A(x \land z_i) = \bigwedge A(x, z_i) \). By Theorem 3.2(2), there exists a conjunction \( K \) defined by \( K(x, y) = \bigwedge \{ z \in L \mid y \leq A(x, z) \} \). Since \( y \leq f^{-1}\left( (1 - f(x) + f(z)) \land 1 \right) \), we have

\[
z \geq f^{-1}\left( (f(x) + f(y) - 1) \lor f(0) \right).
\]

Hence \( K(x, y) = f^{-1}\left( (f(x) + f(y) - 1) \lor f(0) \right) \). \( \square \)

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Example 3.6. Let $f : [0, 1] \to [0, 1]$ be a bijective strictly-increasing function as $f(x) = x^p (p > 0)$. From Theorem 3.5, we define an implication

$$A(x, y) = \left( (1 - x^p + y^p) \land 1 \right)^{\frac{1}{p}}.$$

Since $A$ is an implication satisfying $A(x, \land z_i) = \land A(x, z_i)$ and $A(\lor x_i, z) = \lor A(x_i, z)$, hence we can obtain a forcing implication $H = A$ and a conjunction $K$ as follows

$$K(x, y) = \left( (x^p + y^p - 1) \lor 0 \right)^{\frac{1}{p}}.$$

Theorem 3.7. Let $f : L \to L$ be an order-isomorphic function with $f(1) = 1$. Define a conjunction $K : L \times L \to L$ with $K(x, \lor z_i) = \lor K(x, z_i)$ and $K(x, y) = f^{-1}(f(x) \land f(y))$. Then there exists a forcing-implication $H$ such that $A = H$ with $K \vdash A$ defined as

$$H(x, y) = \begin{cases} 1 & \text{if } x \leq y, \\ y & \text{if } x \geq y. \end{cases}$$

Proof. It is easily proved from Theorem 3.2. \qed

Example 3.8. Let $(P(U), \subseteq, \emptyset, U)$ be a completely distributive lattice. We define an operator $K : P(U) \to P(U)$ as follows:

$$K(X, Y) = X \cap Y.$$

Then $K$ is a conjunction with $K(X, \cup Y) = \cup K(X, Y)$. We obtain an implication operator $A = H$ as follows:

$$A(X, Y) = \bigcup \{ Z \in P(U) \mid X \cap Z \subseteq Y \}$$

$$= \bigcup \{ Z \in P(U) \mid Z \subseteq X^c \cup Y \}$$

$$= X^c \cup Y.$$

Furthermore, $X \cap Z \subseteq Y$ iff $Z \subseteq X^c \cup Y$.

Example 3.9. We define an operator $A : [0, 1] \times [0, 1] \to [0, 1]$ as follows:

$$A(x, y) = \begin{cases} 1 & \text{if } y > 2x - 1, \\ (1 - x) \lor y & \text{if } y \leq 2x - 1. \end{cases}$$

Then $A$ is an implication operator which does not satisfy $A(x, \land z_i) \neq \land A(x, z_i)$ and $A(\lor x_i, z) \neq \lor A(x_i, z)$ because

$$1 = \land_{n \in N} A\left(\frac{3}{4}, \frac{1}{n+1}, \frac{1}{2}\right)$$

$$\neq A(\lor \frac{3}{4}, \frac{1}{n+1}, \frac{1}{2}) = A(\frac{3}{4}, \frac{1}{2}) = \frac{1}{2}$$

$$1 = \land_{n \in N} A\left(\frac{3}{4}, \frac{1}{2} + \frac{1}{n+1}\right)$$

$$\neq A(\frac{3}{4}, \frac{1}{2} + \frac{1}{n+1}, \frac{1}{2}) = A(\frac{3}{4}, \frac{1}{2} + \frac{1}{2}) = \frac{1}{2}.$$

References


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