Fuzzy $\beta$-$(r, s)$-Open Sets in Smooth Bitopological Spaces

Seung On Lee$^1$ and Eun Pyo Lee$^2$

1 Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea
2 Department of Mathematics, Seonam University, Namwon 590-711, Korea

Abstract

We introduce and investigate the concepts of $(T_i, T_j)$-fuzzy $\beta$-$(r, s)$-open sets, $(T_i, T_j)$-fuzzy $\beta$-$(r, s)$-closed sets and fuzzy pairwise $\beta$-$(r, s)$-continuous mappings in smooth bitopological spaces.

Key Words: fuzzy $\beta$-$(r, s)$-open sets, fuzzy $\beta$-$(r, s)$-closed sets, fuzzy pairwise $\beta$-$(r, s)$-continuous mappings

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12] in his classical paper. Using the concept of fuzzy sets, Chang [2] was the first to introduce the concept of a fuzzy topology on a set $X$ by axiomatizing a collection $T$ of fuzzy subsets of $X$, where he referred to each member of $T$ as an open set. In his definition of fuzzy topology, fuzziness in the concept of openness of a fuzzy subset was absent. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [11], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [9]. Kandil [4] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Lee [5] introduced the concept of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil’s fuzzy bitopological spaces.

In this paper, we introduce the concepts of $(T_i, T_j)$-fuzzy $\beta$-$(r, s)$-open sets and fuzzy pairwise $\beta$-$(r, s)$-continuous mappings in smooth bitopological spaces and then investigate some of their characteristic properties.

2. Preliminaries

Let $I$ be the closed unit interval $[0, 1]$ of the real line and let $I_0$ be the half open interval $(0, 1]$ of the real line. For a set $X$, $I^X$ denotes the collection of all mapping from $X$ to $I$. A member $\mu$ of $I^X$ is called a fuzzy set of $X$. By $\hat{0}$ and $\check{1}$ we denote constant mappings on $X$ with value $0$ and $1$, respectively. For any $\mu \in I^X$, $\mu^c$ denotes the complement $\check{1} - \mu$. All other notations are the standard notations of fuzzy set theory.

A Chang’s fuzzy topology on $X$ [2] is a family $T$ of fuzzy sets in $X$ which satisfies the following properties:

1. $\hat{0}, \check{1} \in T$.
2. If $\mu_1, \mu_2 \in T$ then $\mu_1 \land \mu_2 \in T$.
3. If $\mu_k \in T$ for all $k$, then $\bigvee \mu_k \in T$.

The pair $(X, T)$ be called a Chang’s fuzzy topological space. Members of $T$ are called $T$-fuzzy open sets of $X$ and their complements $T$-fuzzy closed sets of $X$.

A system $(X, T_1, T_2)$ consisting of a set $X$ with two Chang’s fuzzy topologies $T_1$ and $T_2$ on $X$ is called a Kandil’s fuzzy bitopological space.

A smooth topology on $X$ is a mapping $T : I^X \to I$ which satisfies the following properties:

1. $T(\hat{0}) = T(\check{1}) = 1$.
2. $T(\mu_1 \land \mu_2) \geq T(\mu_1) \land T(\mu_2)$.
3. $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair $(X, T)$ be called a smooth topological space. For $r \in I_0$, we call $\mu$ a $T$-fuzzy $r$-open set of $X$ if $T(\mu) \geq r$ and $\mu$ a $T$-fuzzy $r$-closed set of $X$ if $T(\mu^c) \geq r$.

A system $(X, T_1, T_2)$ consisting of a set $X$ with two smooth topologies $T_1$ and $T_2$ on $X$ is called a smooth bitopological space. Throughout this paper the indices $i, j$ take values in $\{1, 2\}$ and $i \neq j$.

Let $(X, T)$ be a smooth topological space. Then it is easy to see that for each $r \in I_0$, an $r$-cut

$$T_r = \{\mu \in I^X \mid T(\mu) \geq r\}
$$

is a Chang’s fuzzy topology on $X$. 

Manuscript received Jan. 4, 2010; revised Mar. 11, 2010; Accepted Mar. 15, 2010.

$^1$This work was supported by the research grant of the Chungbuk National University in 2009.

$^2$Corresponding author: Eun Pyo Lee
Let \((X, T)\) be a Chang’s fuzzy topological space and \(r \in I_0\). Then the mapping \(T^r : I^X \to I\) is defined by

\[
T^r(\mu) = \begin{cases} 
1 & \text{if } \mu = 0, 1, \\
r & \text{if } \mu \in T - \{0, 1\}, \\
0 & \text{otherwise}
\end{cases}
\]

becomes a smooth topology.

Hence, we obtain that if \((X, (T_1, T_2))\) is a smooth bitopological space and \(r, s \in I_0\), then \((X, (T_1)_r, (T_2)_s)\) is a Kandil’s fuzzy bitopological space. Also, if \((X, (T_1, T_2))\) is a Kandil’s fuzzy bitopological space and \(r, s \in I_0\), then \((X, (T_1)_r, (T_2)_s)\) is a smooth bitopological space.

**Definition 2.1.** [5] Let \((X, T)\) be a smooth topological space. For each \(r \in I_0\) and for each \(\mu \in I^X\), the \(T\)-fuzzy \(r\)-closure is defined by

\[
T_{-Cl}(\mu, r) = \{\rho \in I^X | \mu \leq \rho, T(\rho) \geq r\}
\]

and the \(T\)-fuzzy \(r\)-interior is defined by

\[
T_{-Int}(\mu, r) = \{\rho \in I^X | \mu \geq \rho, T(\rho) \geq r\}.
\]

**Lemma 2.2.** [5] Let \(\mu\) be a fuzzy set of a smooth topological space \((X, T)\) and let \(r \in I_0\). Then we have:

1. \(T_{-Cl}(\mu, r)^c = T_{-Int}(\mu^c, r)\).
2. \(T_{-Int}(\mu, r)^c = T_{-Cl}(\mu^c, r)\).

**Definition 2.3.** [5, 6] Let \(\mu\) be a fuzzy set of a smooth bitopological space \((X, T_1, T_2)\) and \(r, s \in I_0\). Then \(\mu\) is said to be

1. a \((T_1, T_2)\)-fuzzy \((r, s)\)-semiopen set if \(\mu \leq T_2_{-Cl}(T_1_{-Int}(\mu, r), s)\),
2. a \((T_1, T_2)\)-fuzzy \((r, s)\)-semiclosed set if \(T_2_{-Int}(T_1_{-Cl}(\mu, r), s) \leq \mu\),
3. a \((T_1, T_2)\)-fuzzy \((r, s)\)-preopen set if \(\mu \leq T_2_{-Int}(T_1_{-Cl}(\mu, s), r) \leq \mu\),
4. a \((T_1, T_2)\)-fuzzy \((r, s)\)-precilosed set if \(T_1_{-Cl}(T_2_{-Int}(\mu, r), s) \leq \mu\).

**Definition 2.4.** [5, 6] Let \(f : (X, T_1, T_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)\) be a mapping from a smooth bitopological space \(X\) to a smooth bitopological space \(Y\) and \(r, s \in I_0\). Then \(f\) is called

1. a **fuzzy pairwise** \((r, s)\)-continuous mapping if the induced mapping \(f : (X, T_1) \to (Y, \mathcal{U}_1)\) is fuzzy \(r\)-continuous and the induced mapping \(f : (X, T_2) \to (Y, T_2)\) is fuzzy \(s\)-continuous,
2. a **fuzzy pairwise** \((r, s)\)-semicontinuous mapping if \(f^{-1}(\mu) = (T_1, T_2)\)-fuzzy \((r, s)\)-semiopen set of \(X\) for each \(\mathcal{U}_1\)-fuzzy \(r\)-open set \(\mu\) of \(Y\) and \(f^{-1}(\nu) \) is a \((T_2, T_1)\)-fuzzy \((s, r)\)-semiopen set of \(X\) for each \(\mathcal{U}_2\)-fuzzy \(s\)-open set \(\nu\) of \(Y\),
3. a **fuzzy pairwise** \((r, s)\)-precontinuous mapping if \(f^{-1}(\mu) = (T_1, T_2)\)-fuzzy \((r, s)\)-preopen set of \(X\) for each \(\mathcal{U}_1\)-fuzzy \(r\)-open set \(\mu\) of \(Y\) and \(f^{-1}(\nu) \) is a \((T_2, T_1)\)-fuzzy \((s, r)\)-preopen set of \(X\) for each \(\mathcal{U}_2\)-fuzzy \(s\)-open set \(\nu\) of \(Y\).

3. \((T_1, T_2)\)-fuzzy \(\beta\)-(\(r, s\))-open sets

**Definition 3.1.** Let \(\mu\) be a fuzzy set of a smooth bitopological space \((X, T_1, T_2)\) and \(r, s \in I_0\). Then \(\mu\) is said to be

1. a \((T_1, T_2)\)-fuzzy \(\beta\)-(\(r, s\))-open set if \(\mu \leq T_2_{-Cl}(T_1_{-Int}(\mu, r), s)\),
2. a \((T_1, T_2)\)-fuzzy \(\beta\)-(\(r, s\))-closed set if \(T_2_{-Int}(T_1_{-Cl}(\mu, r), s) \leq \mu\).

**Theorem 3.2.** Let \(\mu\) be a fuzzy set of a smooth bitopological space \((X, T_1, T_2)\) and \(r, s \in I_0\). Then the following statements are equivalent:

1. \(\mu\) is a \((T_1, T_2)\)-fuzzy \(\beta\)-(\(r, s\))-open set.
2. \(\mu^c\) is a \((T_1, T_2)\)-fuzzy \(\beta\)-(\(r, s\))-closed set.

**Proof.** It follows from Lemma 2.2.

**Remark 3.3.** It is clear that every \((T_1, T_2)\)-fuzzy \((r, s)\)-semiopen set is a \((T_1, T_2)\)-fuzzy \(\beta\)-(\(r, s\))-open set and every \((T_1, T_2)\)-fuzzy \((r, s)\)-preopen set is a \((T_1, T_2)\)-fuzzy \(\beta\)-(\(r, s\))-open set. However, the following example show that all the converses need not be true.

**Example 3.4.** Let \(X = \{x, y\}\) and \(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\) and \(\mu_6\) be fuzzy sets of \(X\) defined as

\[
\begin{align*}
\mu_1(x) &= 0.4, & \mu_1(y) &= 0.7; \\
\mu_2(x) &= 0.1, & \mu_2(y) &= 0.2; \\
\mu_3(x) &= 0.8, & \mu_3(y) &= 0.5; \\
\mu_4(x) &= 0.8, & \mu_4(y) &= 0.1; \\
\mu_5(x) &= 0.7, & \mu_5(y) &= 0.6; \\
\mu_6(x) &= 0.5, & \mu_6(y) &= 0.2.
\end{align*}
\]

and

\[
\begin{align*}
\mu_1(x) &= 0.4, & \mu_1(y) &= 0.7; \\
\mu_2(x) &= 0.1, & \mu_2(y) &= 0.2; \\
\mu_3(x) &= 0.8, & \mu_3(y) &= 0.5; \\
\mu_4(x) &= 0.8, & \mu_4(y) &= 0.1; \\
\mu_5(x) &= 0.7, & \mu_5(y) &= 0.6; \\
\mu_6(x) &= 0.5, & \mu_6(y) &= 0.2.
\end{align*}
\]

Define \(T_1 : I^X \to I\) and \(T_2 : I^X \to I\) by

\[
T_1(\mu) = \begin{cases} 
1 & \text{if } \mu = 0, 1, \\
\frac{1}{2} & \text{if } \mu = \mu_1, \\
0 & \text{otherwise};
\end{cases}
\]

\[
T_2(\mu) = \begin{cases} 
1 & \text{if } \mu = 0, 1, \\
\frac{1}{2} & \text{if } \mu = \mu_2, \\
0 & \text{otherwise};
\end{cases}
\]
and
\[ T_2(\mu) = \begin{cases} 1 & \text{if } \mu = 0, 1, \\ \frac{1}{3} & \text{if } \mu = \mu_2, \\ 0 & \text{otherwise.} \end{cases} \]

Then clearly \((T_1, T_2)\) is a smooth bitopology on \(X\). The fuzzy set \(\mu_2 = (T_1, T_2)\)-fuzzy \(\beta\)-\((\frac{1}{3}, \frac{1}{3})\)-open which is not \((T_1, T_2)\)-fuzzy \((\frac{1}{3}, \frac{1}{3})\)-semiopen and \(\mu_4 = (T_2, T_1)\)-fuzzy \(\beta\)-\((\frac{1}{3}, \frac{1}{3})\)-open set which is not a \((T_2, T_1)\)-fuzzy \((\frac{1}{3}, \frac{1}{3})\)-semiopen set. Also, \(\mu_5 = (T_1, T_2)\)-fuzzy \(\beta\)-\((\frac{1}{3}, \frac{1}{3})\)-open set which is not a \((T_1, T_2)\)-fuzzy \((\frac{1}{3}, \frac{1}{3})\)-preopen set and \(\mu_6 = (T_2, T_1)\)-fuzzy \(\beta\)-\((\frac{1}{3}, \frac{1}{3})\)-open set which is not a \((T_2, T_1)\)-fuzzy \((\frac{1}{3}, \frac{1}{3})\)-preopen set.

**Theorem 3.5.** (1) Any union of \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-open sets is a \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-open set.

(2) Any intersection of \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-closed sets is a \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-closed set.

**Proof.** (1) Let \(\{\mu_k\}\) be a collection of \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-open sets. Then for each \(k\), \(\mu_k \leq T_j \cdot \text{Cl}(T_j \cdot \text{Int}(T_j \cdot \text{Cl}(\mu_k, s), r), s)\).

Thus \(\bigvee \mu_k \leq \bigvee T_j \cdot \text{Cl}(T_j \cdot \text{Int}(T_j \cdot \text{Cl}(\mu_k, s), r), s)\).

(2) It follows from (1) using Theorem 3.2.

**Theorem 3.6.** Let \(\mu\) be a \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-open and \((T_j, T_i)\)-fuzzy \((s, r)\)-semiopen set. Then \(\mu\) is a \((T_i, T_j)\)-fuzzy \((r, s)\)-semiopen set.

**Proof.** Let \(\mu\) be a \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-open and \((T_j, T_i)\)-fuzzy \((s, r)\)-semiopen set. Since \(\mu\) is \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-open, \(\mu \leq T_j \cdot \text{Cl}(T_j \cdot \text{Int}(T_j \cdot \text{Cl}(\mu, s), r), s)\).

Also since \(\mu\) is \((T_j, T_i)\)-fuzzy \((s, r)\)-semiopen, \(\mu \geq T_j \cdot \text{Int}(T_j \cdot \text{Int}(T_j \cdot \text{Cl}(\mu, r), s), r)\).

Thus \(\mu \leq T_j \cdot \text{Cl}(T_j \cdot \text{Cl}(\mu, s), r)\).

Hence \(\mu\) is a \((T_i, T_j)\)-fuzzy \((r, s)\)-semiopen set.

**Theorem 3.7.** Let \(\mu\) be a \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-closed and \((T_j, T_i)\)-fuzzy \((s, r)\)-semiopen set. Then \(\mu\) is a \((T_i, T_j)\)-fuzzy \((r, s)\)-semiopen set.

**Proof.** Let \(\mu\) be a \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-closed and \((T_j, T_i)\)-fuzzy \((s, r)\)-semiopen set. Since \(\mu\) is \((T_i, T_j)\)-fuzzy \(\beta\)-(r, s)-closed, \(\mu \geq T_j \cdot \text{Int}(T_j \cdot \text{Cl}(\mu, s), r)\).

Also since \(\mu\) is \((T_j, T_i)\)-fuzzy \((s, r)\)-semiopen, \(\mu \leq T_j \cdot \text{Int}(T_j \cdot \text{Cl}(\mu, r), s)\).

Thus \(\mu \geq T_j \cdot \text{Int}(T_j \cdot \text{Cl}(\mu, s), r)\).

Hence \(\mu\) is a \((T_i, T_j)\)-fuzzy \((r, s)\)-semiopen set.

**4. Fuzzy pairwise \(\beta\)-continuous mappings**

**Definition 4.1.** Let \(f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)\) be a mapping from a smooth bitopological space \(X\) to a smooth bitopological space \(Y\) and \(r, s \in I_0\). Then \(f\) is called

(1) a fuzzy pairwise \(\beta\)-(r, s)-continuous mapping if \(f^{-1}(\mu)\) is a \((T_1, T_2)\)-fuzzy \(\beta\)-(r, s)-open set of \(X\) for each \(U_1\)-fuzzy \(r\)-open set \(\mu\) of \(Y\) and \(f^{-1}(\nu)\) is a \((T_2, T_1)\)-fuzzy \(\beta\)-(r, s)-open set of \(X\) for each \(U_2\)-fuzzy \(s\)-open set \(\nu\) of \(Y\).

(2) a fuzzy pairwise \(\beta\)-(r, s)-open mapping if \(f(\rho)\) is a \((U_1, U_2)\)-fuzzy \(\beta\)-(r, s)-open set of \(Y\) for each \(T_1\)-fuzzy \(r\)-open set \(\rho\) of \(X\) and \(f(\lambda)\) is a \((U_2, U_1)\)-fuzzy \(\beta\)-(s, r)-open set of \(Y\) for each \(T_2\)-fuzzy \(s\)-open set \(\lambda\) of \(X\).

(3) a fuzzy pairwise \(\beta\)-(r, s)-closed mapping if \(f(\rho)\) is a \((U_1, U_2)\)-fuzzy \(\beta\)-(r, s)-closed set of \(Y\) for each \(T_1\)-fuzzy \(r\)-closed set \(\rho\) of \(X\) and \(f(\lambda)\) is a \((U_2, U_1)\)-fuzzy \(\beta\)-(s, r)-closed set of \(Y\) for each \(T_2\)-fuzzy \(s\)-closed set \(\lambda\) of \(X\).

**Theorem 4.2.** Let \((X, T_1, T_2), (Y, U_1, U_2)\) and \((Z, V_1, V_2)\) be smooth bitopological spaces and let \(f : X \rightarrow Y\) and \(g : Y \rightarrow Z\) be mappings and \(r, s \in I_0\). Then the following statements are true.

(1) If \(f\) is fuzzy pairwise \(\beta\)-(r, s)-continuous and \(g\) is fuzzy pairwise \((r, s)\)-continuous then \(g \circ f\) is fuzzy pairwise \(\beta\)-(r, s)-continuous.

(2) If \(f\) is fuzzy pairwise \((r, s)\)-open and \(g\) is fuzzy pairwise \(\beta\)-(r, s)-open then \(g \circ f\) is fuzzy pairwise \(\beta\)-(r, s)-open.

(3) If \(f\) is fuzzy pairwise \((r, s)\)-closed and \(g\) is fuzzy pairwise \(\beta\)-(r, s)-closed then \(g \circ f\) is fuzzy pairwise \(\beta\)-(r, s)-closed.

**Proof.** Straightforward.

**Theorem 4.3.** Let \((X, T_1, T_2)\) and \((Y, U_1, U_2)\) be smooth bitopological spaces and let \(r, s \in I_0\). Then \(f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)\) is a fuzzy pairwise \(\beta\)-(r, s)-continuous mapping if and only if \(f : (X, T_1, T_2) \rightarrow (Y, (U_1)_r, (U_2)_s)\) is a fuzzy pairwise \(\beta\)-continuous mapping.
Proof. Let \( \mu \in (U_1)_r \) and \( \nu \in (U_2)_s \). Then \((U_1)_r \) and \((U_2)_s \) are fuzzy r-open and \( \nu \)-open sets of \( Y \). Since \( \mu \) is a \((U_1)_r \)-fuzzy \( r \)-open set and \( \nu \) is a \((U_2)_s \)-fuzzy \( s \)-open set of \( Y \). Therefore, \( f^{-1}((U_1)_r \times (U_2)_s) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

Conversely, let \( \mu \) be any \((U_1)_r \)-fuzzy \( r \)-open set and \( \nu \) be any \((U_2)_s \)-fuzzy \( s \)-open set of \( Y \). Then \((U_1)_r \) and \((U_2)_s \) are \( r \) and \( s \) fuzzy \( \beta \)-open sets. Since \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) is a fuzzy pairwise \( \beta \)-continuous mapping if and only if \( f : (X, T_1, T_2) \to (Y, (U_1)_r \times (U_2)_s) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

\[ f^{-1}(U_1 \times U_2) \]

Theorem 4.4. Let \( (X, T_1, T_2) \) and \((Y, U_1, U_2)\) be Kandi’s fuzzy bitopological spaces and let \( r, s \in I_0 \). Then \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) is a fuzzy pairwise \( \beta \)-continuous mapping if and only if \( f : (X, T_1, T_2) \to (Y, (U_1)_r \times (U_2)_s) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

Proof. Let \( \mu \) be a \((U_1)_r \)-fuzzy \( r \)-open set and \( \nu \) be a \((U_2)_s \)-fuzzy \( s \)-open set. Then \((U_1)_r \) and \((U_2)_s \) are \( r \) and \( s \) fuzzy \( \beta \)-open sets. Since \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) is a fuzzy pairwise \( \beta \)-continuous mapping, \( f^{-1}(U_1) \) is a \( \beta \)-open set of \( X \). Therefore, \( f^{-1}(U_1) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

Conversely, let \( \mu \) be any \((U_1)_r \)-fuzzy \( r \)-open set and \( \nu \) be any \((U_2)_s \)-fuzzy \( s \)-open set of \( Y \). Then \((U_1)_r \) and \((U_2)_s \) are \( r \) and \( s \) fuzzy \( \beta \)-open sets. Since \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) is a fuzzy pairwise \( \beta \)-continuous mapping if and only if \( f : (X, T_1, T_2) \to (Y, (U_1)_r \times (U_2)_s) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

Theorem 4.5. Let \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) be a mapping and \( r, s \in I_0 \). Then the following statements are equivalent:

1. \( f \) is a fuzzy pairwise \( \beta \)-continuous mapping.
2. \( f^{-1}(U_1) \times (U_2) \) is a \((T_1, T_2)\)-fuzzy \( \beta \)-continuous mapping.

Proof. (1) \( \iff \) (2) It follows from Theorem 3.2. (3) \( \Rightarrow \) (4). Let \( \rho \) be any fuzzy set of \( X \). Then \( f(U_1 \times U_2) \) and \( f^{-1}(U_1) \) are \( \rho \)-open sets. Hence \( f^{-1}(U_1) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

Theorem 4.6. Let \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) be a mapping and \( r, s \in I_0 \). Then the following statements are equivalent:

1. \( f \) is a fuzzy pairwise \( \beta \)-continuous mapping.
2. \( f^{-1}(U_1 \times U_2) \) is a \((T_1, T_2)\)-fuzzy \( \beta \)-continuous mapping.

Proof. Let \( \mu \in (U_1)_r \) and \( \nu \in (U_2)_s \). Then \((U_1)_r \) and \((U_2)_s \) are fuzzy r-open and \( \nu \)-open sets of \( Y \). Therefore, \( f^{-1}(U_1 \times U_2) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

Conversely, let \( \mu \) be any \((U_1)_r \)-fuzzy \( r \)-open set and \( \nu \) be any \((U_2)_s \)-fuzzy \( s \)-open set of \( Y \). Then \((U_1)_r \) and \((U_2)_s \) are \( r \) and \( s \) fuzzy \( \beta \)-open sets. Since \( f : (X, T_1, T_2) \to (Y, U_1, U_2) \) is a fuzzy pairwise \( \beta \)-continuous mapping if and only if \( f : (X, T_1, T_2) \to (Y, (U_1)_r \times (U_2)_s) \) is a fuzzy pairwise \( \beta \)-continuous mapping.

\[ f^{-1}(U_1 \times U_2) \]

Hence \( U_1 \times U_2 \) is a fuzzy pairwise \( \beta \)-continuous mapping.
and

\[ U_2-\text{Cl}(f(\mu), s) \geq f^{-1}(U_2-\text{Cl}(f(\mu), s)) \]
\[ \geq f(T_1-\text{Int}(T_2-\text{Cl}(T_1-\text{Int}(\mu, r), s), r)). \]

(4) \Rightarrow (2) Let \( \mu \) be any \( U_1 \)-fuzzy \( r \)-closed set and \( \nu \) any \( U_2 \)-fuzzy \( s \)-closed set of \( Y \). Then \( f^{-1}(\mu) \) and \( f^{-1}(\nu) \) are fuzzy sets of \( X \). By (4),

\[
\mu = U_1-\text{Cl}(\mu, r) \\
\geq U_1-\text{Cl}(f^{-1}(\mu), r) \\
\geq f(T_1-\text{Int}(T_2-\text{Cl}(f^{-1}(\mu), s), r)), \] 

and

\[
\nu = U_2-\text{Cl}(\nu, s) \\
\geq U_2-\text{Cl}(f^{-1}(\nu), s) \\
\geq f(T_1-\text{Int}(T_2-\text{Cl}(f^{-1}(\mu), r), s), r)). \]

Thus

\[
f^{-1}(\mu) \geq f^{-1}(T_2-\text{Int}(T_1-\text{Int}(f^{-1}(\mu), s), r)) \\
\geq T_2-\text{Int}(T_1-\text{Int}(f^{-1}(\mu), s), r) \]

and

\[
f^{-1}(\nu) \geq f^{-1}(T_1-\text{Int}(T_2-\text{Cl}(T_1-\text{Int}(f^{-1}(\nu), r), s), r)) \\
\geq T_1-\text{Int}(T_2-\text{Cl}(T_1-\text{Int}(f^{-1}(\nu), r), s), r). \]

Therefore \( f^{-1}(\mu) \) is a \((T_1, T_2)\)-fuzzy \( \beta-(r, s) \)-closed set and \( f^{-1}(\nu) \) is a \((T_2, T_1)\)-fuzzy \( \beta-(s, r) \)-closed set of \( X \).

\section*{Acknowledgements}

This work was supported by the research grant of the Chungbuk National University in 2009.

\section*{References}


[7] S. O. Lee and E. P. Lee, "Fuzzy strongly \((r, s)\)\n-semiopen sets," \textit{International J. Fuzzy Logic and In-


\underline{Seung On Lee}
Professor of Chungbuk National University
E-mail : solee@chungbuk.ac.kr

\underline{Eun Pyo Lee}
Professor of Seonam University
E-mail : eplee55@paran.com
Corresponding author