Existence of Solutions for the Impulsive Semilinear Fuzzy Integrodifferential Equations with Nonlocal Conditions and Forcing Term with Memory in $n$-dimensional Fuzzy Vector Space $(E^m_N, d_\varepsilon)$

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Abstract

In this paper, we study the existence and uniqueness of solutions for the impulsive semilinear fuzzy integrodifferential equations with nonlocal conditions and forcing term with memory in $n$-dimensional fuzzy vector space $(E^m_N, d_\varepsilon)$ by using Banach fixed point theorem. That is an extension of the result of Kwun et al. [9] to impulsive system.

Key Words: Existence and uniqueness of solutions, impulsive, semilinear, integrodifferential equations, nonlocal, memory

1. Introduction

Many authors have studied several concepts of fuzzy systems. Diamond and Kloeden [4] proved the fuzzy optimal control for the following system:

$$\dot{x}(t) = a(t)x(t) + u(t), \quad x(0) = x_0,$$

where $x(\cdot)$ and $u(\cdot)$ are nonempty compact interval-valued functions on $E^1$. Kwun and Park [6] proved the existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition in $E^1_N$ by using Kuhn-Tucker theorems. The theory of differential equations with discontinuous trajectories during the last twenty years has been to a great extent stimulated by their numerous applications to problem arising in mechanics, electrical engineerin, the theory of automatic control, medicine and biology. For the monographs of the theory of impulsive differential equations, see Bainov and Simenov [1], Lakshmikantham et al. [10], Samoileuko and Perestyuk [15], where numerous properties of their solutions are studied and detailed bibliographies are given. Rogovchenko [14] follows the ideas of the theory of impulsive differential equations which treats the changes of the state of the evolution process due to a short-term perturbations whose duration can be negligible in comparison with the duration of the process as an instant impulses. In 2007, Park et al. [12] studied the existence and uniqueness of fuzzy solutions and controllability for the impulsive semilinear fuzzy integrodifferential equations in one-dimensional fuzzy vector space $E^1_N$. R. Rodríguez-López [13] studied periodic boundary value problems for impulsive fuzzy differential equations. Fuzzy integrodifferential equations are a field of interest, due to their applicability to the analysis of phenomena with memory where imprecision is inherent. Balasubramaniam and Muralisankar [2] proved the existence and uniqueness of fuzzy solutions for the semilinear fuzzy integrodifferential equation with nonlocal initial condition. They considered the semilinear one-dimensional heat equation on a connected domain $(0, 1)$ for material with memory. In one-dimensional fuzzy vector space $E^1_N$, Park et al. [11] proved the existence and uniqueness of fuzzy solutions and presented the sufficient condition of nonlocal controllability for the following semilinear fuzzy integrodifferential equation with nonlocal initial condition. In [7], Kwun et al. proved the existence and uniqueness of fuzzy solutions for the semilinear fuzzy integrodifferential equations by using successive iteration. In [8], Kwun et al. investigated the continuously initial observability for the semilinear fuzzy integrodifferential equations. Bed and Gal [3] studied almost periodic fuzzy-number-valued functions. Gal and N’Guerekata [5] studied almost automorphic fuzzy-number-valued functions. Recently, Kwun et al.[9] study the the existence and uniqueness of solutions and nonlocal controllability for the semilinear fuzzy integrodifferential equations in $n$-dimensional fuzzy vector space.

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In this paper, we study the existence and uniqueness of solutions for the following impulsive nonlinear fuzzy integrodifferential equations with nonlocal conditions and forcing term with memory in $n$-dimensional fuzzy vector space:

$$\frac{dx_i(t)}{dt} = A_i \left[ x_i(t) + \int_0^t G(t-s)x_i(s)ds \right] + f_i(t, x_i(t), \int_0^t q_i(t, s, x_i(s))ds) \text{ on } E_N^i, \quad x_i(0) = x_{il} \in E_N^i, \quad \Delta x_i(t_k) = I_k(x_i(t_k^+)), \quad t \neq t_k, \quad k = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n, \tag{1}$$

where $A_i : [0, T] \to E_N^i$ is fuzzy coefficient, $E_N^i$ is the set of all upper semi-continuously convex fuzzy numbers on $R$ with $E_N^i \neq R^i$ ($i \neq j$), $f_i : [0, T] \times E_N^i \times E_N^i \to E_N^i$ and $q_i : [0, T] \times [0, T] \times E_N^i \to E_N^i$ are nonlinear fuzzy functions, $g_i : E_N^i \to E_N^i$ is a nonlinear continuous function, $G(t)$ is a $n \times n$ continuous matrix such that $\frac{dG(t)}{dt}$ is continuous for $x_i \in E_N^i$ and $t \in [0, T]$ with $\|G(t)\| \leq h_k$, $k = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n$ such that

$$d_H \left( [f_i(s, \xi(s), \eta_i(s))]^\alpha \right) \leq d_H \left( [f_i(s, \xi(s), \eta_i(s))]^\alpha \right) \leq k_1 d_H \left( [\xi(s)]^\alpha , [\eta_i(s)]^\alpha \right) + k_2 d_H \left( [\xi(s)]^\alpha , [\eta_i(s)]^\alpha \right),$$

for all $\xi(s), \eta_i(s) \in (E_N^i)^n (j = 1, 2)$. The nonlinear function $g_i$ is a continuous function and satisfies the Lipschitz condition

$$d_H \left( [g_i(x_i(\cdot))]^\alpha \right) \leq h_i d_H \left( [x_i(\cdot)]^\alpha \right),$$

for all $x_i(\cdot) \in (E_N^i)^n, h_i$ is a finite positive constant.

We call $u \in E^n$ a $n$-dimension fuzzy number.

Wang, Li and Wen [16] defined $n$-dimension fuzzy vector space and investigated its properties.

For any $u_i \in E, \quad i = 1, 2, \ldots, n$, we call the ordered one-dimension fuzzy number class $(u_1, u_2, \ldots, u_n)$ (i.e., the Cartesian product of one-dimension fuzzy numbers $u_1, u_2, \ldots, u_n$) a $n$-dimension fuzzy vector, denote it as $(U_1, U_2, \ldots, U_n)$, and call the collection of all $n$-dimension fuzzy vectors (i.e., the Cartesian product $E \times E \times \cdots \times E$) $n$-dimensional fuzzy vector space, and denote it as $(E)$.

**Definition 2.1.** [16] If $u \in E^n$ and $[u]^\alpha$ is a hyperrectangle, i.e., $[u]^\alpha$ can be represented by $\prod_{i=1}^n[u_i^\alpha, u_i^\alpha]$, i.e., $[u_1^\alpha, u_1^\alpha] \times [u_2^\alpha, u_2^\alpha] \times \cdots \times [u_n^\alpha, u_n^\alpha]$ for every $\alpha \in [0, 1]$.

$\text{Conversely, for any } (u_1, u_2, \ldots, u_n) \in (E)$ with $[u_1^\alpha, u_1^\alpha] \times [u_2^\alpha, u_2^\alpha] \times \cdots \times [u_n^\alpha, u_n^\alpha]$ and $\alpha \in [0, 1]$, there exists a unique $(u_1, u_2, \ldots, u_n) \in (E)$ such that $[u]^\alpha = \prod_{i=1}^n[u_i^\alpha, u_i^\alpha] (i = 1, 2, \ldots, n, \alpha \in [0, 1]).$

**Theorem 2.1.** [16] For any $u \in L(E^n)$ with $[u]^\alpha = \prod_{i=1}^n[u_i^\alpha, u_i^\alpha]$ (i.e., the unique $n$-dimension fuzzy vector determined by $u \in L(E^n)$), we denote the collection of all $n$-dimensional fuzzy numbers by $I(E^n)$.

**Note 2.1.** [16] Theorem 2.1 indicates that fuzzy $n$-cell numbers and $n$-dimensional fuzzy vectors can represent each other, so $L(E^n)$ and $(E)$ may be regarded as identity. If $(u_1, u_2, \ldots, u_n) \in (E)$ is the unique $n$-dimension fuzzy vector determined by $u \in L(E^n)$, then we denote $u = (u_1, u_2, \ldots, u_n)$.

Let $(E_N^i)^n = E_1^i \times E_2^i \times \cdots \times E_n^i, \quad (i = 1, 2, \ldots, n)$ is fuzzy subset of $R$. Then $(E_N^i)^n \subseteq (E^n)$.

**Definition 2.2.** Let $u, v \in (E_N^i)^n \quad (i = 1, 2, \ldots, n)$

$$d_\alpha ([u]^\alpha, [v]^\alpha) = d_H \left( \prod_{i=1}^n[u_i]^\alpha \prod_{i=1}^n[v_i]^\alpha \right) = \left( \sum_{i=1}^n(d_H([u_i]^\alpha, [v_i]^\alpha))^2 \right)^{1/2}$$

where $d_H$ is the Hausdorff distance.

**Definition 2.3.** The complete metric $d_{\infty}$ on $(E_N^i)^n$ is defined by

$$d_{\infty}(u(t), v(t)) = \sup_{0 < \alpha < 1} \left( \int_{\alpha}(...) \right)$$

for any $u, v \in (E_N^i)^n$.

**Definition 2.4.** Let $u, v \in C([0, T] : (E_N^i)^n)$

$$H_1(u, v) = \sup_{0 \leq t \leq T} d_{\infty}(u(t), v(t))$$
Definition 2.5. [16] The derivative $x'(t)$ of a fuzzy process $x \in (E_N^i)^n$ is defined by

$$[x'(t)]^\alpha = \sum_{i=1}^{n} [(x_{it}^i)'(t), (x_{it}^i)'(t)]$$

provided that equation defines a fuzzy $x'(t) \in (E_N^i)^n$.

Definition 2.6. [16] The fuzzy integral $\int_b^a x(t)dt, a, b \in [0, T]$ is defined by

$$[\int_b^a x(t)dt]^\alpha = \sum_{i=1}^{n} \left[ \int_b^a x_{ir}^i(t)dt, \int_b^a x_{ir}^i(t)dt \right]$$

provided that the Lebesgue integrals on the right hand side exist.

3. Existence and Uniqueness

In this section we consider the existence and uniqueness of the fuzzy solution for the equations (1)-(3).

We define

$$A = (A_1, A_2, \ldots, A_n),$$

$$x = (x_1, x_2, \ldots, x_n),$$

$$f = (f_1, f_2, \ldots, f_n),$$

$$g = (g_1, g_2, \ldots, g_n),$$

and

$$x_0 = (x_{01}, x_{02}, \ldots, x_{0n}).$$

Then

$$A, x, f, g, x_0 \in (E_N^i)^n.$$

Instead of the equations (1)-(3), we consider the following fuzzy integro-differential equations in $(E_N^i)^n$.

$$\frac{dx(t)}{dt} = A \left[ x(t) + \int_0^t G(t-s)x(s)ds \right] + f(t, x(t), \int_0^t g(t, s, x(s))ds) \text{ on } (E_N^i)^n$$

(7)

$$x(0) + g(x) = x_0 \in (E_N^i)^n$$

(8)

$$\Delta x(t_k) = I_k(x(t_k)), \; t \neq t_k,$$

(9)

$$k = 1, 2, \ldots, m, i = 1, 2, \ldots, n,$$

with fuzzy coefficient $A : [0, T] \rightarrow (E_N^i)^n$, initial value $x_0 \in (E_N^i)^n$.

Definition 3.1. The fuzzy process $x : I = [0, T] \rightarrow (E_N^i)^n$ with $\alpha$-level set $[x(t)]^\alpha = \Pi_{i=1}^{m}[x_i]^\alpha = \Pi_{i=1}^{m}[x_{ir}^i]^\alpha$ is a fuzzy solution of the equations (7)-(9) without nonhomogeneous term if and only if

$$[x_{ir}^i]'(t) = \min \left\{ A_{ij}^i(t) \left[ x_{ik}^i(t) \right]_N, T \right\}$$

$$+ \int_0^t G(t-s)x_{ir}^i(s)ds : j, k = l, r \},$$

$$[x_{ir}^i]'(t) = \max \left\{ A_{ij}^i(t) \left[ x_{ik}^i(t) \right]_N, T \right\}$$

$$+ \int_0^t G(t-s)x_{ir}^i(s)ds : j, k = l, r \},$$

$x_{ir}^i(0) = x_{ir}^i(0) = x_{ir}^i, i = 1, 2, \ldots, n.$

For the sequel, we need the following assumption:

(H1) $S_i(t)$ is a fuzzy number satisfying, for $y_i \in (E_N^i)^n$,

$$\frac{d}{dt} S_i(t)y_i \in C^2(I \times (E_N^i)^n) \cap C(I \times (E_N^i)^n),$ the equation

$$\frac{d}{dt} S_i(t)y_i = A \left[ S_i(t)y_i + \int_0^t G(t-s)S_i(s)ds \right],$$

where

$$[S(t)]^\alpha = \prod_{i=1}^{n} [S_i(t)]^\alpha = \prod_{i=1}^{n} \left[ (S_i(t))^\alpha, (S_i(t))^\alpha \right],$$

and $\left( S_i(t) \right)^\alpha$ is continuous with $\left| (S_i(t))^\alpha_k \right| \leq c_i, c_i > 0, k = l, r$ for all $t \in I = [0, T]$.

In order to define the solution of (7)-(9), we shall consider the space

$$\Omega_i = \left\{ x_i : J \rightarrow E_N^i \; x_i \in C(J_k, E_N^i), J_k = (t_k, t_{k+1}), k = 0, 1, \ldots, m \right\},$$

and there exist $x_i(t_k), (x_i(t_k))(k = 1, 2, \ldots, m)$, with $x_i(t_k), (x_i(t_k))(k = 1, 2, \ldots, m)$.

Let $\Omega' = \Pi_{i=1}^{m} \Omega_i', \Omega' = \Omega \cap C([0, T] \rightarrow E_N^i), i = 1, 2, \ldots, n$.

Lemma 3.1. If $x$ is an integral solution of (7)-(9), then $x$ is given by

$$x(t) = S(t)(x_0 - g(x))$$

(10)

$$+ \int_0^t S(t-s)f(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau)ds$$

$$+ \sum_{0 < t_k < t} S(t - t_k)I_k(x(t_k^-)), \; \text{for } t \in J.$$

Proof. Let $x$ be a solution of (4)-(6). Define $\omega(s) = S(t-s)x(s)$. Then we have that

$$\frac{d\omega(s)}{ds} = - \frac{dS(t-s)}{ds}x(s) + S(t-s) \frac{dx(s)}{ds}$$
\[
\begin{align*}
&= -A \left[ S(t-s)x(s) + \int_0^t G(t-s)S(s)x(s)\,ds \right. \\
&\quad + S(t-s) \frac{dx(s)}{ds} \left. \right] \\
&\quad + S(t-s) f(s, x(s), \int_0^s q(s, \tau, x(\tau))\,d\tau). \\
\end{align*}
\]
which proves the lemma.

Assume the following:

(H2) There exists \( d_{ik} \), \( k = 1, 2, \ldots, m \), \( i = 1, 2, \ldots, n \), such that

\[
\begin{align*}
&d_H \left( \left[ I_{k_1}(x_i(t_{k_1}^i) - y_i(t_{k_1}^i)) \right]^\alpha, \left[ I_{k_2}(y_i(t_{k_2}^i)) \right]^\alpha \right) \\
&\quad \leq d_H \left( \left[ I_{k_1}(x_i(t_{k_1}^i)) \right]^\alpha, \left[ I_{k_1}(y_i(t_{k_1}^i)) \right]^\alpha \right),
\end{align*}
\]

where \( \sum_{k=1}^m d_{ik} = d_i \).

(H3) \( K_1 = \max \{k_1\}, K_2 = \max \{k_2\}, H = \max \{h_i\}, M = \max \{m_i\}, C = \max \{c_i\}, D = \max \{d_i\} \).

(H4) \( C (H + D + (K_1 + K_2 M^\frac{\alpha}{2})) T < 1. \)

**Theorem 3.1.** Let \( T > 0 \). If hypotheses (H2)-(H4) are hold. Then, for every \( x \in (E_N)^\alpha \), equation (10) have a unique fuzzy solution \( x \in \Omega'_1 \).

**Proof.** For each \( x(t) \in \Omega' \) and \( t \in [0, T] \), define \((Gx)(t) \in \Omega'_1 \) by

\[
\begin{align*}
(Gx)(t) &= S(t)(x_0 - g(x)) \\
&\quad + \int_0^t S(t-s) f(s, x(s), \int_0^s q(s, \tau, x(\tau))\,d\tau)\,ds. \\
&\quad + \sum_{0 < t_k < t} S(t-t_k) I_k(x(t_k^-)),
\end{align*}
\]

Thus, \( G : [0, T] \to \Omega' \) is continuous, so \( G \) is a mapping from \( \Omega'_1 \) into itself. There exist \( G_i \), \( i = 1, 2, \ldots, n \) which is a continuous function from \( \Omega'_1 \). By Definition 2.2, Definition 2.3, some properties of \( d_H \) and inequalities (4), (5) and (6), we have following inequalities. For \( x, y \in \Omega'_1 \),

\[
\begin{align*}
d_H \left( \left[ G_i(x_i(t)) \right]^\alpha, \left[ G_i(y_i(t)) \right]^\alpha \right) \\
&\quad \leq d_H \left( \left[ S_i(t)(x_0 - g(x_i)) \right]^\alpha, \left[ S_i(t)(y_0 - g(y_i)) \right]^\alpha \right) \\
&\quad + \int_0^t S_i(t-s) f_i(s, x_i(s), \int_0^s q_i(s, \tau, x_i(\tau))\,d\tau)\,ds. \\
&\quad + \sum_{0 < t_k < t} S_i(t-t_k) I_k(x(t_k^-)),
\end{align*}
\]
\[
\begin{aligned}
&\leq \left( \sum_{i=1}^{n} \left( \sup_{0<\alpha\leq1} CH \sup_{0<\alpha\leq1} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right) \\
&\quad + C \sup_{0<\alpha\leq1} \int_{0}^{t} \left( K_1 d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right) \\
&\quad + K_2 M \int_{0}^{\tau} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) d\tau \, ds \\
&\quad + CD \sup_{0<\alpha\leq1} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right)^{2/2} \\
\end{aligned}
\]

Hence

\[
H_1(Gx, Gy) = \sup_{0\leq t \leq T} d_\infty \left( Gx(t), Gy(t) \right) \\
\leq \left( \sum_{i=1}^{n} \left( \sup_{0<\alpha\leq1} CH \sup_{0<\alpha\leq1} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right) \\
\quad + C \sup_{0<\alpha\leq1} \int_{0}^{t} \left( K_1 d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right) \\
\quad + K_2 M \int_{0}^{\tau} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) d\tau \, ds \\
\quad + CD \sup_{0<\alpha\leq1} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right)^{2/2} \\
\]

Therefore

\[
\begin{aligned}
&d_\infty \left( Gx(t), Gy(t) \right) \\
&= \sup_{0<\alpha\leq1} d_\infty \left( [G(x(t))]^\alpha, [G(y(t))]^\alpha \right) \\
&\leq \left( \sum_{i=1}^{n} \left( \sup_{0<\alpha\leq1} CH \sup_{0<\alpha\leq1} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right) \\
&\quad + C \sup_{0<\alpha\leq1} \int_{0}^{t} \left( K_1 d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right) \\
&\quad + K_2 M \int_{0}^{\tau} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) d\tau \, ds \\
&\quad + CD \sup_{0<\alpha\leq1} d_H([x_i(\cdot)]^\alpha, [y_i(\cdot)]^\alpha) \right)^{2/2} \\
\end{aligned}
\]
Consider the two semilinear one-dimensional heat equations on a connected domain (0,1) for a material with memory on $E_N(i = 1, 2)$, boundary condition $x_1(t, 0) = x_1(t, 1) = 0 (i = 1, 2)$ and initial conditions $x_0(z_i) + \sum_{k=1}^{p} c_k x_1(t_k, z_i) = x_{0i}(z_i)$, where $x_{0i}(z_i) \in E_N(i = 1, 2)$, $\sum_{k=1}^{p} c_k x_1(t_k, z_i) = g_i(x_i), i = 1, 2$. Let $x_1(t, z_i)(i = 1, 2)$ be the internal energy and $x_0 = (x_0, x_0) = (\bar{0}, \bar{0})$, 

$$\Delta x(t_k) = \left(\Delta x_1(t_k), \Delta x_2(t_k)\right),$$ 

$t \neq t_k, k = 1, 2, \cdots, m$,

$$I_k(x(t_k)) = (I_k(x_1(t_k)), I_k(x_2(t_k)))$$ 

$$= (x_1(t_k), x_2(t_k))$$ 

$$= (x_1(t_k^+, z_1) - x_1(t_k^-, z_1),$$ 

$$x_2(t_k^+, z_2) - x_2(t_k^-, z_2)),$$

and $G(t - s) = (e^{-(t-s)}, e^{-(t-s)})$, then the balance equation becomes 

$$\frac{dx(t)}{dt} = A \begin{pmatrix} x(t) + \int_0^t G(t - s) x(s)ds \\ f(t, x(t), \int_0^t q(t, s, x(s))ds) \end{pmatrix}$$

(11)

$$+ f(t, x(t), \int_0^t q(t, s, x(s))ds) \text{ on } (E_N^\alpha),$$

$$x(0) + g(x) = x_0 \in (E_N^\alpha),$$

$$\Delta x(t_k) = I_k(x(t_k)), t \neq t_k,$$

(12)

(13)

$k = 1, 2, \cdots, m, i = 1, 2, \cdots, n.$

The $\alpha$-level set of fuzzy numbers are the following:

$$\bar{0}^\alpha = [\alpha - 1, 1 - \alpha], [\bar{2}]^\alpha = [\alpha + 1, 3 - \alpha] \text{ for all } \alpha \in [0, 1].$$

Then $\alpha$-level sets of $f(t, x(t), \int_0^t q(t, s, x(s))ds)$ is

$$\begin{aligned}
&\left[ f(t, x(t), \int_0^t q(t, s, x(s))ds) \right]^\alpha \\
&= \left[ 2tx_1(t)^2 + \int_0^t (t - s)x_1(s)ds \right]^\alpha \\
&\times \left[ 2tx_2(t)^2 + \int_0^t (t - s)x_2(s)ds \right]^\alpha \\
&= \left[ \bar{2}^\alpha \cdot \left[ 2tx_1(t)^2 \right]^\alpha + \int_0^t (t - s)x_1(s)ds \right]^\alpha \\
&\times \left[ \bar{2}^\alpha \cdot \left[ 2tx_2(t)^2 \right]^\alpha + \int_0^t (t - s)x_2(s)ds \right]^\alpha \\
&= \left[ (\alpha + 1)t(x_1^\alpha(t))^2 + \int_0^t (t - s)x_1^\alpha(s)ds, \right. \\
&\left. (3 - \alpha)t(x_2^\alpha(t))^2 + \int_0^t (t - s)x_2^\alpha(s)ds \right] \\
&\times \left[ (\alpha + 1)t(x_1^\alpha(t))^2 + \int_0^t (t - s)x_1^\alpha(s)ds, \right. \\
&\left. (3 - \alpha)t(x_2^\alpha(t))^2 + \int_0^t (t - s)x_2^\alpha(s)ds \right].
\end{aligned}$$

Further, we have

$$d_H\left[ f(t, x(t), \int_0^t q(t, s, x(s))ds) \right]^\alpha,$$
\[
\left[ f_t \left(t, y(t), \int_0^t q_i(t, s, y_i(s))ds \right) \right]^{\alpha} = t^{\alpha + 1, 3 - \alpha} \left( \frac{d_H([x(t)]^\alpha, [y(t)]^\alpha)}{\sqrt{2}} \right) + T^2 \frac{d_H([x(t)]^\alpha, [y(t)]^\alpha)}{\sqrt{2}} \\
\leq t^{\alpha + 1, 3 - \alpha} \left( \frac{d_H([x(t)]^\alpha, [y(t)]^\alpha)}{\sqrt{2}} \right) + k_1 d_H([x(t)]^\alpha, [y(t)]^\alpha) + k_2 d_H([x(t)]^\alpha, [y(t)]^\alpha),
\]

where \(k_1, k_2\) and \(b_i\) satisfy the inequality (4), (5) and (6) respectively. Then all the conditions stated in Theorem 3.1 are satisfied, so the problem (11)-(13) has a unique fuzzy solution.

References


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