Fuzzy \((r, s)\)-semi-preopen sets and fuzzy \((r, s)\)-semi-precontinuous maps

Seok Jong Lee and Jin Tae Kim

Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea

Abstract

In this paper, we introduce the concepts of fuzzy \((r, s)\)-semi-preopen sets and fuzzy \((r, s)\)-semi-precontinuous mappings on intuitionistic fuzzy topological spaces in Šostak’s sense. The relations among fuzzy \((r, s)\)-semitopological, fuzzy \((r, s)\)-precontinuous, and fuzzy \((r, s)\)-semi-precontinuous mappings are discussed. The concepts of fuzzy \((r, s)\)-semi-preinterior, fuzzy \((r, s)\)-semi-preclosure, fuzzy \((r, s)\)-semi-preneighborhood, and fuzzy \((r, s)\)-quasi-semi-preneighborhood are given. Using these concepts, the characterization for the fuzzy \((r, s)\)-semi-precontinuous mapping is obtained. Also, we introduce the notions of fuzzy \((r, s)\)-semi-preopen and fuzzy \((r, s)\)-semi-preclosed mappings on intuitionistic fuzzy topological spaces in Šostak’s sense, and then we investigate some of their characteristic properties.

Key words: fuzzy \((r, s)\)-semi-preopen set, fuzzy \((r, s)\)-semi-precontinuous mapping, fuzzy \((r, s)\)-semi-preopen mapping, fuzzy \((r, s)\)-semi-preclosed mapping

1. Introduction

The concept of fuzzy topological spaces was introduced by Chang [2]. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [14], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [3], and by Ramadan [13].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [4, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak’s sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. Thakur and Singh [15] introduced the concepts of fuzzy semi-preopen sets and fuzzy semi-precontinuous mappings on Chang’s fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy \((r, s)\)-semi-preopen sets and fuzzy \((r, s)\)-semi-precontinuous mappings on intuitionistic fuzzy topological spaces in Šostak’s sense. The relations among fuzzy \((r, s)\)-semitopological, fuzzy \((r, s)\)-precontinuous, and fuzzy \((r, s)\)-semi-precontinuous mappings are discussed. The concepts of fuzzy \((r, s)\)-semi-preinterior, fuzzy \((r, s)\)-semi-preclosure, fuzzy \((r, s)\)-semi-preneighborhood, and fuzzy \((r, s)\)-quasi-semi-preneighborhood are given. Using these concepts, the characterization for the fuzzy \((r, s)\)-semi-precontinuous mapping is obtained. Also, we introduce the notions of fuzzy \((r, s)\)-semi-preopen and fuzzy \((r, s)\)-semi-preclosed mappings on intuitionistic fuzzy topological spaces in Šostak’s sense, and then we investigate some of their characteristic properties.

2. Preliminaries

For the nonstandard definitions and notations we refer to [9, 10].

Definition 2.1. ([6]) Let \(X\) be a nonempty set. An intuitionistic fuzzy topology in Šostak’s sense(SoIFT for short) \(T = (T_1, T_2)\) on \(X\) is a mapping \(T : I(X) \to I \otimes I\) which satisfies the following properties:

1. \(T_1(\emptyset) = T_1(\{1\}) = 1\) and \(T_2(\emptyset) = T_2(\{0\}) = 0\).

2. \(T_1(A \cap B) \geq T_1(A) \wedge T_1(B)\) and \(T_2(A \cap B) \leq T_2(A) \vee T_2(B)\).

3. \(T_1(\bigcup A_i) \geq \bigwedge T_1(A_i)\) and \(T_2(\bigcup A_i) \leq \bigvee T_2(A_i)\).

The \((X, T) = (X, T_1, T_2)\) is said to be an intuitionistic fuzzy topological space in Šostak’s sense(SoIFTS for short). Also, we call \(T_1(A)\) a gradation of openness of \(A\) and \(T_2(A)\) a gradation of nonopenness of \(A\).
Definition 2.2. ([5, 8]) Let \((X, T_1, T_2)\) be a SoFTS and 
\((r, s) \in I \otimes I\). Then

1. an intuitionistic fuzzy point \(x_{(\alpha, \beta)}\) in \(X\) is said to be 
   quasi-coincident with the intuitionistic fuzzy set \(A\) in 
   \(X\), denoted by \(x_{(\alpha, \beta)} \triangleq A\), if and only if \(\mu_A(x) > \beta\) 
   or \(\gamma_A(x) < \alpha\).

2. two intuitionistic fuzzy sets \(A\) and \(B\) in \(X\) are said 
   to be quasi-coincident, denoted by \(A \triangleq B\), if and only 
   if there exists an element \(x \in X\) such that \(\mu_A(x) > \gamma_B(x)\) or \(\gamma_A(x) < \mu_B(x)\).

The word ’not quasi-coincident’ will be abbreviated as \(\triangleq\).

Definition 2.3. ([12]) Let \(A\) be an intuitionistic fuzzy set 
in a SoFTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then \(A\) is said 
 to be

1. fuzzy \((r, s)\)-preopen if \(A \subseteq \text{int}(\text{cl}(A, r, s), r, s)\).

2. fuzzy \((r, s)\)-preclosed if \(\text{cl}(\text{int}(A, r, s), r, s) \subseteq A\).

Definition 2.4. ([12]) Let \((X, T_1, T_2)\) be a SoFTS. For each 
\((r, s) \in I \otimes I\) and for each \(A \in I(X)\), the fuzzy 
\((r, s)\)-preinterior is defined by

\[
\text{pint}(A, r, s) = \bigcup\{B \in I(X) \mid B \subseteq A, \ B \text{ is fuzzy } (r, s)\text{-preopen}\}
\]

and the fuzzy \((r, s)\)-preclosure is defined by

\[
\text{pcl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B, \ B \text{ is fuzzy } (r, s)\text{-preclosed}\}.
\]

Definition 2.5. ([11, 12]) Let \(f : (X, T_1, T_2) \rightarrow 
(Y, U_1, U_2)\) be a mapping from a SoFTS \(X\) to a SoFTS 
\(Y\) and \((r, s) \in I \otimes I\). Then \(f\) is called 

1. a fuzzy \((r, s)\)-semi-closed mapping if \(f(A)\) is a fuzzy 
   \((r, s)\)-semi-closed set in \(Y\) for each fuzzy \((r, s)\)-
   closed set \(A \subseteq X\),

2. a fuzzy \((r, s)\)-precontinuous mapping if \(f^{-1}(B)\) is a 
   fuzzy \((r, s)\)-preopen set in \(X\) for each fuzzy \((r, s)\)-
   open set \(B \subseteq Y\),

3. a fuzzy \((r, s)\)-preopen mapping if \(f(A)\) is a fuzzy 
   \((r, s)\)-preopen set in \(Y\) for each fuzzy \((r, s)\)-open set 
   \(A \subseteq X\),

4. a fuzzy \((r, s)\)-preclosed mapping if \(f(A)\) is a fuzzy 
   \((r, s)\)-preclosed set in \(Y\) for each fuzzy \((r, s)\)-closed set 
   \(A \subseteq X\).

Definition 2.6. ([10]) Let \(x_{(\alpha, \beta)}\) be an intuitionistic fuzzy 
point in a SoFTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then an 
intuitionistic fuzzy set \(A\) in \(X\) is called

1. a fuzzy \((r, s)\)-neighborhood of \(x_{(\alpha, \beta)}\) if there is a 
   fuzzy \((r, s)\)-open set \(B\) in \(X\) such that \(x_{(\alpha, \beta)} \in B \subseteq A\),

2. a fuzzy \((r, s)\)-semineighborhood of \(x_{(\alpha, \beta)}\) if there is a 
   fuzzy \((r, s)\)-semi-open set \(B\) in \(X\) such that \(x_{(\alpha, \beta)} \in B \subseteq A\).

3. Fuzzy \((r, s)\)-semi-preopen sets and fuzzy 
\((r, s)\)-semi-precontinuous mappings

Now, we define the notions of fuzzy \((r, s)\)-semi-preopen sets and fuzzy \((r, s)\)-semi-precontinuous mappings 
on intuitionistic fuzzy topological spaces in Šostak’s sense, and then we investigate some of their properties.

Theorem 3.1. Let \(A\) be an intuitionistic fuzzy set in a 
SoFTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then \(A\) is a fuzzy 
\((r, s)\)-preopen set in \(X\) if and only if there is a fuzzy \((r, s)\)-
open set \(B\) in \(X\) such that \(A \subseteq B \subseteq \text{cl}(A, r, s)\).

Proof. Let \(A\) be a fuzzy \((r, s)\)-preopen set in \(X\). Then \(A \subseteq 
\text{int}(\text{cl}(A, r, s), r, s)\). Put \(B = \text{int}(\text{cl}(A, r, s), r, s)\). Then \(B\) 
is a fuzzy \((r, s)\)-open set in \(X\) and \(A \subseteq B \subseteq \text{cl}(A, r, s)\).

Conversely, let \(B\) be a fuzzy \((r, s)\)-open set in \(X\) such that 
\(A \subseteq B \subseteq \text{cl}(A, r, s)\). Then \(A \subseteq B = \text{int}(B) \subseteq 
\text{int}(\text{cl}(A, r, s), r, s)\). Hence \(A\) is a fuzzy \((r, s)\)-preopen set.

Definition 3.2. Let \(A\) be an intuitionistic fuzzy set in a 
SoFTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then \(A\) is called

1. a fuzzy \((r, s)\)-semi-preopen set if there is a fuzzy 
   \((r, s)\)-semi-preopen set \(B\) in \(X\) such that \(B \subseteq A \subseteq 
   \text{cl}(B, r, s)\).

2. a fuzzy \((r, s)\)-semi-preclosed set if there is a fuzzy 
   \((r, s)\)-semi-preclosed set \(B\) in \(X\) such that \(\text{int}(B, r, s) \subseteq 
   A \subseteq B\).

Theorem 3.3. Let \(A\) be an intuitionistic fuzzy set in a 
SoFTS \((X, T_1, T_2)\) and \((r, s) \in I \otimes I\). Then the following 
statements are equivalent :

1. \(A\) is a fuzzy \((r, s)\)-semi-preopen set.

2. \(A^c\) is a fuzzy \((r, s)\)-semi-preclosed set.

Proof. Straightforward.

Remark 3.4. It is clear that every fuzzy \((r, s)\)-semiopen(resp. 
fuzzy \((r, s)\)-semi-closed) set and every fuzzy \((r, s)\)-preopen(resp. 
fuzzy \((r, s)\)-preclosed) set is fuzzy \((r, s)\)-semi-preopen(resp. fuzzy 
\((r, s)\)-semi-preclosed) for each \((r, s) \in I \otimes I\). However, the following 
example shows that all of the converses need not be true.
Example 3.5. Let $X = \{x, y\}$ and let $A_1, A_2, A_3$, and $A_4$ be intuitionistic fuzzy sets in $X$ defined as

$$
A_1(x) = (0.2, 0.8), \quad A_1(y) = (0.3, 0.5);
$$

$$
A_2(x) = (0.8, 0.1), \quad A_2(y) = (0.8, 0.1);
$$

$$
A_3(x) = (0.5, 0.2), \quad A_3(y) = (0.2, 0.5);
$$

and

$$
A_4(x) = (0.1, 0.9), \quad A_4(y) = (0.2, 0.6).
$$

Define $T : I(X) \rightarrow I \otimes I$ by

$$
T(A) = (T_1(A), T_2(A)) = \begin{cases} 
(1, 0) & \text{if } A = A_1, \\
(\frac{1}{3}, \frac{1}{3}) & \text{if } A = A_2, \\
(0, 1) & \text{otherwise}.
\end{cases}
$$

Then clearly $T$ is a SoIFT on $X$. Since $A_2 \subseteq \frac{1}{3} = \text{int}(\text{cl}(A_2, \frac{1}{3}, \frac{2}{3}, \frac{1}{3}))$, $A_2$ is a fuzzy $(\frac{1}{3}, \frac{1}{3})$-preopen set and hence $A_2$ is fuzzy $(\frac{1}{2}, \frac{1}{2})$-preopen. But $A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen set, because $A_2 \not\subseteq \text{cl}(\text{int}(A_2, \frac{1}{3}, \frac{2}{3}, \frac{1}{3})) = A_1'$. Since $A_4 \subseteq \text{cl}(A_4, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = A_1$, $A_4$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-preopen set. Also, $A_3$ is a fuzzy $(\frac{1}{3}, \frac{1}{3})$-preopen set, because $A_3 \subseteq \text{cl}(\text{int}(A_3, \frac{1}{3}, \frac{2}{3}, \frac{1}{3})) = A_1$.

Theorem 3.6. Let $A$ be an intuitionistic fuzzy set in a SoIFTS $(X, T_1, T_2)$ and $(r, s) \in I \otimes I$. Then the following statements are true:

1. For each fuzzy $(r, s)$-semi-preopen set $B$ in $X$, $B \subseteq A \subseteq \text{cl}(B, r, s)$ implies that $A$ is fuzzy $(r, s)$-semi-preopen in $X$.

2. For each fuzzy $(r, s)$-semi-preclosed set $B$ in $X$, $B \subseteq A \subseteq \text{cl}(B, r, s)$ implies that $A$ is fuzzy $(r, s)$-semi-preclosed in $X$.

Proof. (1) Let $C$ be a fuzzy $(r, s)$-preopen set in $X$ such that $C \subseteq B \subseteq \text{cl}(C, r, s)$. Then clearly $C \subseteq A$ and $B \subseteq \text{cl}(C, r, s)$ implies that $\text{cl}(B, r, s) \subseteq \text{cl}(C, r, s)$. Thus $C \subseteq A \subseteq \text{cl}(B, r, s) \subseteq \text{cl}(C, r, s)$. Hence $A$ is a fuzzy $(r, s)$-semi-preopen set in $X$.

(2) Similar to (1). □

Theorem 3.7. Let $(X, T_1, T_2)$ be a SoIFTS and $(r, s) \in I \otimes I$.

1. If $\{A_i\}$ is a family of fuzzy $(r, s)$-semi-preopen sets in $X$, then $\bigcup A_i$ is fuzzy $(r, s)$-semi-preopen.

2. If $\{A_i\}$ is a family of fuzzy $(r, s)$-semi-preclosed sets in $X$, then $\bigcap A_i$ is fuzzy $(r, s)$-semi-preclosed.

Proof. (1) Let $\{A_i\}$ be a collection of fuzzy $(r, s)$-semi-preopen sets in $X$. Then for each $i$, there is a fuzzy $(r, s)$-preopen set $B_i$ in $X$ such that $B_i \subseteq A_i \subseteq \text{cl}(B_i, r, s)$. So

$$
\bigcup B_i \subseteq \bigcup A_i \subseteq \bigcup \text{cl}(B_i, r, s) \subseteq \text{cl}(\bigcup B_i, r, s)
$$

and $\bigcup B_i$ is fuzzy $(r, s)$-preopen. Hence $\bigcup A_i$ is a fuzzy $(r, s)$-semi-preopen set.

(2) It follows from (1) using Theorem 3.3. □

The following example shows that the intersection(resp. union) of two fuzzy $(r, s)$-semi-preopen(resp. fuzzy $(r, s)$-semi-preclosed) sets need not be a fuzzy $(r, s)$-semi-preopen(resp. fuzzy $(r, s)$-semi-preclosed) set for each $(r, s) \in I \otimes I$.

Example 3.8. Let $X = \{x, y\}$ and let $A_1$ and $A_2$ be intuitionistic fuzzy sets in $X$ defined as

$$
A_1(x) = (0.1, 0.7), \quad A_1(y) = (0.4, 0.3);
$$

and

$$
A_2(x) = (0.8, 0.1), \quad A_2(y) = (0.2, 0.4).
$$

Define $T : I(X) \rightarrow I \otimes I$ by

$$
T(A) = (T_1(A), T_2(A)) = \begin{cases} 
(1, 0) & \text{if } A = A_1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_2, \\
(0, 1) & \text{otherwise}.
\end{cases}
$$

Then clearly $T$ is a SoIFT on $X$. Since $A_1 \subseteq \frac{1}{2} = \text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}))$, $A_1$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-preopen, $A_2$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen set and hence $A_2$ is fuzzy $(\frac{1}{2}, \frac{1}{2})$-semi-preopen. But $A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{2})$-semiopen set, because $A_2 \not\subseteq \text{cl}(\text{int}(A_2, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})) = A_1'$. Since $A_4 \subseteq \text{cl}(A_4, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = A_1$, $A_4$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-preopen set. Also, $A_3$ is a fuzzy $(\frac{1}{2}, \frac{1}{2})$-preopen set, because $A_3 \subseteq \text{cl}(\text{int}(A_3, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})) = A_1$.

Definition 3.9. Let $(X, T_1, T_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy $(r, s)$-preinterior is defined by

$$
\text{spint}(A, r, s) = \{B \in I(X) \mid B \subseteq A, \text{ } B \text{ is fuzzy } (r, s)-\text{preopen}\}
$$

and the fuzzy $(r, s)$-preclosure is defined by

$$
\text{spcl}(A, r, s) = \{B \in I(X) \mid A \subseteq B, \text{ } B \text{ is fuzzy } (r, s)-\text{preclosed}\}.
$$

Obviously spcl$(A, r, s)$ is the smallest fuzzy $(r, s)$-semi-preclosed set which contains $A$, and spint$(A, r, s)$ is the greatest fuzzy $(r, s)$-semi-preopen set which is contained in $A$. Also, spcl$(A, r, s) = A$ for any fuzzy $(r, s)$-semi-preclosed set $A$, and spint$(A, r, s) = A$ for any fuzzy $(r, s)$-semi-preopen set $A$. Moreover, we have

$$
\text{int}(A, r, s) \subseteq \text{pint}(A, r, s) \subseteq \text{spint}(A, r, s) \subseteq A \subseteq \text{spcl}(A, r, s) \subseteq \text{pcl}(A, r, s) \subseteq \text{cl}(A, r, s).
$$
Also, we have the following results:

1. \( \text{spcl}(0, r, s) = 0 \).
2. \( \text{spcl}(A, r, s) \supseteq A \).
3. \( \text{spcl}(A \cup B, r, s) \supseteq \text{spcl}(A, r, s) \cup \text{spcl}(B, r, s) \).
4. \( \text{spcl}(\text{spcl}(A, r, s), r, s) = \text{spcl}(A, r, s) \).
5. \( \text{spint}(0, r, s) = 0 \).
6. \( \text{spint}(A, r, s) \subseteq A \).
7. \( \text{spint}(A \cap B, r, s) \subseteq \text{spint}(A, r, s) \cap \text{spint}(B, r, s) \).
8. \( \text{spint}(\text{spint}(A, r, s), r, s) = \text{spint}(A, r, s) \).

**Definition 3.10.** Let \( A \) be an intuitionistic fuzzy set and \( x_{(\alpha, \beta)} \) an intuitionistic fuzzy point in a SoIFTS \( (X, \mathcal{T}_1, \mathcal{T}_2) \) and \((r, s) \in I \otimes I \). Then \( A \) is called

1. a **fuzzy \((r, s)\)-semi-preneighborhood** of \( x_{(\alpha, \beta)} \) if there is a fuzzy \((r, s)\)-semi-preopen set \( B \) in \( X \) such that \( x_{(\alpha, \beta)} \in B \subseteq A \).

2. a **fuzzy \((r, s)\)-quasi-semi-preneighborhood** of \( x_{(\alpha, \beta)} \) if there is a fuzzy \((r, s)\)-semi-preopen set \( B \) in \( X \) such that \( x_{(\alpha, \beta)}qB \subseteq A \).

**Theorem 3.11.** Let \( A \) be an intuitionistic fuzzy set in a SoIFTS \( (X, \mathcal{T}_1, \mathcal{T}_2) \) and \((r, s) \in I \otimes I \). Then \( A \) is fuzzy \((r, s)\)-semi-preopen if and only if \( A \) is a fuzzy \((r, s)\)-semipreneighborhood of \( x_{(\alpha, \beta)} \) for each intuitionistic fuzzy point \( x_{(\alpha, \beta)} \in A \).

**Proof.** Straightforward. \( \square \)

**Theorem 3.12.** Let \( A \) be an intuitionistic fuzzy set in a SoIFTS \( (X, \mathcal{T}_1, \mathcal{T}_2) \) and \((r, s) \in I \otimes I \). Then an intuitionistic fuzzy point \( x_{(\alpha, \beta)} \) is contained in \( \text{spcl}(A, r, s) \) if and only if every fuzzy \((r, s)\)-quasi-semi-preneighborhood of \( x_{(\alpha, \beta)} \) is quasi-coincidence with \( A \).

**Proof.** Suppose \( x_{(\alpha, \beta)} \in \text{spcl}(A, r, s) \) and there exists a fuzzy \((r, s)\)-quasi-semi-preopen set \( B \) of \( x_{(\alpha, \beta)} \) such that \( AqB \). Then there is a fuzzy \((r, s)\)-semi-preopen set \( C \) in \( X \) such that \( x_{(\alpha, \beta)}qC \subseteq B \), which shows that \( AqC \) and hence \( A \subseteq C^c \). Since \( C^c \) is fuzzy \((r, s)\)-semi-preclosed in \( X \), \( \text{spcl}(A, r, s) \subseteq C^c \). Thus \( x_{(\alpha, \beta)} \in C^c \). But \( x_{(\alpha, \beta)} \notin C^c \), because \( x_{(\alpha, \beta)}qC \). This is a contradiction.

Conversely, suppose every fuzzy \((r, s)\)-quasi-semi-preneighborhood of \( x_{(\alpha, \beta)} \) is quasi-coincidence with \( A \). If \( x_{(\alpha, \beta)} \notin \text{spcl}(A, r, s) \), then there is a fuzzy \((r, s)\)-semi-preclosed set \( B \) in \( X \) such that \( A \subseteq B \) and \( x_{(\alpha, \beta)} \notin B \). So \( B^c \) is a fuzzy \((r, s)\)-semi-preopen set in \( X \) such that \( x_{(\alpha, \beta)}qB \cap B^c = 0 \). This is a contradiction. \( \square \)

**Definition 3.13.** Let \( f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2) \) be a mapping from a SoIFTS \( X \) to a SoIFTS \( Y \) and \((r, s) \in I \otimes I \). Then \( f \) is called a fuzzy \((r, s)\)-semi-precontinuous mapping if \( f^{-1}(B) \) is a fuzzy \((r, s)\)-semi-preopen set in \( X \) for each fuzzy \((r, s)\)-open set \( B \) in \( Y \).

**Remark 3.14.** It is clear that every fuzzy \((r, s)\)-semicontinuous and every fuzzy \((r, s)\)-precontinuous mapping is fuzzy \((r, s)\)-semi-precontinuous for each \((r, s) \in I \otimes I \). However, the following examples show that all of the converses need not be true.

**Example 3.15.** Let \( X = \{x, y\} \) and let \( A_1, A_2 \) and \( B \) be intuitionistic fuzzy sets in \( X \) defined as

\[
A_1(x) = (0.2, 0.7), \quad A_1(y) = (0.3, 0.5);
\]
\[
A_2(x) = (0.7, 0.2), \quad A_2(y) = (0.7, 0.2);
\]
and
\[
B(x) = (0.7, 0.2), \quad B(y) = (0.6, 0.3).
\]

Define \( T : I(X) \rightarrow I \otimes I \) and \( U : I(X) \rightarrow I \otimes I \) by

\[
T(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\
(0, 1) & \text{otherwise};
\end{cases}
\]

and

\[
U(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_2, \\
(0, 1) & \text{otherwise}.
\end{cases}
\]

Then clearly \( T \) and \( U \) are SoIFTS on \( X \). Consider a mapping \( f : (X, T) \rightarrow (X, U) \) defined by \( f(x) = x \) and \( f(y) = y \). Then it is easy to see that \( B \) is a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-preopen set in \( (X, T) \) and \( B \subseteq f^{-1}(A_2) \subseteq \text{cl}(B, \frac{1}{2}, \frac{1}{2}) = 1 \). So \( f^{-1}(A_2) = A_2 \) is a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-semi-preopen set in \( (X, T) \) and hence \( f \) is a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-semi-precontinuous mapping. But \( f^{-1}(A_1) = A_2 \) is not a fuzzy \((\frac{1}{2}, \frac{1}{2})\)-semiopen set in \( (X, T) \), because \( A_2 \subseteq \text{cl}(\text{int}(A_1), \frac{1}{2}, \frac{1}{2}) = A_1^c \) in \( (X, T) \). Hence \( f \) is not fuzzy \((\frac{1}{2}, \frac{1}{2})\)-semicontinuous.

**Example 3.16.** Let \( X = \{x, y\} \) and let \( A_1, A_2 \) and \( B \) be intuitionistic fuzzy sets in \( X \) defined as

\[
A_1(x) = (0.2, 0.5), \quad A_1(y) = (0.3, 0.3); 
\]
\[
A_2(x) = (0.5, 0.3), \quad A_2(y) = (0.3, 0.4); 
\]
and
\[
B(x) = (0.2, 0.6), \quad B(y) = (0.2, 0.4).
\]

Define \( T : I(X) \rightarrow I \otimes I \) and \( U : I(X) \rightarrow I \otimes I \) by

\[
T(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\
(0, 1) & \text{otherwise};
\end{cases}
\]

and

\[
U(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_2, \\
(0, 1) & \text{otherwise}.
\end{cases}
\]
Then clearly $T$ and $U$ are SIOFTs on $X$. Consider a mapping $f : (X, T) \to (X, U)$ defined by $f(x) = x, f(y) = y$. Then it is easy to see that $B$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, T)$ and $B \subseteq f^{-1}(A_2) = A_2 \subseteq \text{cl}(B, \frac{1}{2}, \frac{1}{3}) = A_1'$. So $f^{-1}(A_2) = A_2$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, T)$ and hence $f$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-precontinuous mapping. But $f^{-1}(A_2) = A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, T)$, because $A_2 \not\subseteq \text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1$ in $(X, T)$. Hence $f$ is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-precontinuous.

**Theorem 3.17.** Let $f : (X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping from a SIOFT $X$ to a SIOFT $Y$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

1. $f$ is fuzzy $(r, s)$-semi-precontinuous.
2. For each fuzzy $(r, s)$-closed set $B$ in $Y$, $f^{-1}(B)$ is a fuzzy $(r, s)$-semi-preclosed set in $X$.
3. For each intuitionistic fuzzy point $x_{(a, b)}$ in $X$ and each fuzzy $(r, s)$-open set $B$ in $Y$ such that $f(x_{(a, b)}) \in B$, there is a fuzzy $(r, s)$-semi-preopen set $A$ in $X$ such that $x_{(a, b)} \in A$ and $f(A) \subseteq B$.
4. For each intuitionistic fuzzy point $x_{(a, b)}$ in $X$ and each fuzzy $(r, s)$-neighborhood $B$ of $f(x_{(a, b)})$, $f^{-1}(B)$ is a fuzzy $(r, s)$-semi-preneighborhood of $x_{(a, b)}$.
5. For each intuitionistic fuzzy point $x_{(a, b)}$ in $X$ and each fuzzy $(r, s)$-neighborhood $B$ of $f(x_{(a, b)})$, there is a fuzzy $(r, s)$-semi-preneighborhood $A$ of $x_{(a, b)}$ such that $f(A) \subseteq B$.
6. For each intuitionistic fuzzy set $B$ in $Y$, $\text{spcl}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{cl}(B, r, s))$.
7. For each intuitionistic fuzzy set $A$ in $X$, $f(\text{spcl}(A, r, s)) \subseteq \text{cl}(f(A), r, s)$.
8. For each intuitionistic fuzzy set $B$ in $Y$, $f^{-1}(\text{int}(B, r, s)) \subseteq \text{spint}(f^{-1}(B), r, s)$.

**Proof.** (1) $\Rightarrow$ (2) It is obvious.

(1) $\Rightarrow$ (3) Let $x_{(a, b)}$ be an intuitionistic fuzzy point in $X$ and $B$ a fuzzy $(r, s)$-open set in $Y$ such that $f(x_{(a, b)}) \in B$. Then $x_{(a, b)} \in f^{-1}(B)$. Put $A = f^{-1}(B)$. Then by (1), $A$ is a fuzzy $(r, s)$-semi-preopen set in $X$ such that $x_{(a, b)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(3) $\Rightarrow$ (1) Let $B$ be a fuzzy $(r, s)$-open set in $Y$ and $x_{(a, b)} \in f^{-1}(B)$. Then $f(x_{(a, b)}) \in B$. By (3), there is a fuzzy $(r, s)$-semi-preopen set $A_{x_{(a, b)}}$ in $X$ such that $x_{(a, b)} \in A_{x_{(a, b)}}$ and $f(A_{x_{(a, b)}}) \subseteq B$. Thus $x_{(a, b)} \in A_{x_{(a, b)}} \subseteq f^{-1}(\text{cl}(A_{x_{(a, b)}}, r, s)) \subseteq f^{-1}(B)$. So we have

$$f^{-1}(B) = \bigcup \{x_{(a, b)} \mid x_{(a, b)} \in f^{-1}(B)\} \subseteq \bigcup \{A_{x_{(a, b)}} \mid x_{(a, b)} \in f^{-1}(B)\} \subseteq f^{-1}(B).$$

Thus $f^{-1}(B) = \bigcup \{A_{x_{(a, b)}} \mid x_{(a, b)} \in f^{-1}(B)\}$ and hence $f^{-1}(B)$ is fuzzy $(r, s)$-semi-preopen in $X$. Therefore $f$ is a fuzzy $(r, s)$-semi-precontinuous mapping.

(1) $\Rightarrow$ (4) Let $x_{(a, b)}$ be an intuitionistic fuzzy point in $X$ and $B$ a fuzzy $(r, s)$-neighborhood of $f(x_{(a, b)})$. Then there is a fuzzy $(r, s)$-open set $C$ in $Y$ such that $f(x_{(a, b)}) \in C \subseteq B$ and hence $x_{(a, b)} \in f^{-1}(C) \subseteq f^{-1}(B)$. Since $f$ is fuzzy $(r, s)$-semi-precontinuous, $f^{-1}(C)$ is a fuzzy $(r, s)$-semi-preopen set in $X$. Thus $f^{-1}(B)$ is a fuzzy $(r, s)$-semi-preneighborhood of $x_{(a, b)}$.

(4) $\Rightarrow$ (5) Let $x_{(a, b)}$ be an intuitionistic fuzzy point in $X$ and $B$ a fuzzy $(r, s)$-neighborhood of $f(x_{(a, b)})$. By (4), $A = f^{-1}(B)$ is a fuzzy $(r, s)$-semi-preneighborhood of $x_{(a, b)}$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(5) $\Rightarrow$ (3) Let $x_{(a, b)}$ be an intuitionistic fuzzy point in $X$ and $B$ a fuzzy $(r, s)$-open set in $Y$ such that $f(x_{(a, b)}) \in B$. Then $B$ is a fuzzy $(r, s)$-neighborhood of $f(x_{(a, b)})$. By (5), there is a fuzzy $(r, s)$-semi-preneighborhood $A$ of $x_{(a, b)}$ in $X$ such that $x_{(a, b)} \in A$ and $f(A) \subseteq B$. Thus there is a fuzzy $(r, s)$-semi-preopen set $C$ in $X$ such that $x_{(a, b)} \in C \subseteq A$ and hence $f(C) \subseteq f(A) \subseteq B$.

(2) $\Rightarrow$ (6) Let $B$ be an intuitionistic fuzzy set in $Y$. Then $\text{cl}(B, r, s)$ is a fuzzy $(r, s)$-closed set in $Y$ and $f^{-1}(B) \subseteq f^{-1}(\text{cl}(B, r, s))$. By (2), $f^{-1}(\text{cl}(B, r, s))$ is a fuzzy $(r, s)$-semi-preclosed set in $X$. Hence

$$\text{spcl}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{cl}(B, r, s)).$$

(6) $\Rightarrow$ (2) Let $B$ be a fuzzy $(r, s)$-closed set in $Y$. Then by (6),

$$f^{-1}(B) \subseteq \text{spcl}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{cl}(B, r, s)) = f^{-1}(B).$$

Hence $f^{-1}(B) = \text{spcl}(f^{-1}(B), r, s)$. Thus $f^{-1}(B)$ is a fuzzy $(r, s)$-semi-preclosed set in $X$.

(6) $\Rightarrow$ (7) Let $A$ be an intuitionistic fuzzy set in $X$. Then $f(A)$ is an intuitionistic fuzzy set in $Y$. By (6),

$$\text{spcl}(A, r, s) \subseteq \text{spcl}(f^{-1}(f(A)), r, s) \subseteq f^{-1}(\text{cl}(f(A), r, s)).$$

Thus $f(\text{spcl}(A, r, s)) \subseteq \text{cl}(f(A), r, s)$.

(7) $\Rightarrow$ (6) Let $B$ be an intuitionistic fuzzy set in $Y$. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in $X$. By (7),

$$f(\text{spcl}(f^{-1}(B), r, s)) \subseteq \text{cl}(f(f^{-1}(B)), r, s) \subseteq \text{cl}(B, r, s).$$
4. Fuzzy $(r,s)$-semi-preopen and fuzzy $(r,s)$-semi-preclosed mappings

We define the notions of fuzzy $(r,s)$-semi-preopen and fuzzy $(r,s)$-semi-preclosed mappings on intuitionistic fuzzy topological spaces in Šostak’s sense, and then we investigate some of their properties.

**Definition 4.1.** Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from a SoIFTs $X$ to a SoIFTs $Y$ and $(r,s) \in I \otimes I$. Then $f$ is called

1. a fuzzy $(r,s)$-semi-preopen mapping if $f(A)$ is a fuzzy $(r,s)$-semi-preopen set in $Y$ for each fuzzy $(r,s)$-open set $A$ in $X$,

2. a fuzzy $(r,s)$-semi-preclosed mapping if $f(A)$ is a fuzzy $(r,s)$-semi-preclosed set in $Y$ for each fuzzy $(r,s)$-closed set $A$ in $X$.

**Remark 4.2.** It is obvious that every fuzzy $(r,s)$-semi-preopen(resp. fuzzy $(r,s)$-semi-closed) and every fuzzy $(r,s)$-preopen(resp. fuzzy $(r,s)$-preclosed) mapping is fuzzy $(r,s)$-semi-preopen(resp. fuzzy $(r,s)$-semi-preclosed). However, the following examples show that all of the converses need not be true.

**Example 4.3.** Let $X = \{x, y\}$ and let $A_1, A_2$ and $B$ be intuitionistic fuzzy sets in $X$ defined as

$$A_1(x) = (0.9, 0.1), \quad A_1(y) = (0.5, 0.4);$$

$$A_2(x) = (0.4, 0.5), \quad A_2(y) = (0.4, 0.2);$$

and

$$B(x) = (0.5, 0.4), \quad B(y) = (0.3, 0.4).$$

Define $T : I(X) \rightarrow I \otimes I$ and $U : I(X) \rightarrow I \otimes I$ by

$$T(A) = (T_1(A), T_2(A)) =

\begin{cases}
(1,0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\
(0,1) & \text{otherwise};
\end{cases}
$$

and

$$U(A) = (U_1(A), U_2(A)) =

\begin{cases}
(1,0) & \text{if } A = 0, 1, \\
(\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\
(0,1) & \text{otherwise}.
\end{cases}
$$

Then clearly $T$ and $U$ are SoIFTs on $X$. Consider a mapping $f : (X, T) \rightarrow (X, U)$ defined by $f(x) = x$ and $f(y) = y$. Since $B \subseteq \text{int}(cl[B, \frac{1}{2}, \frac{1}{3}], \frac{1}{2}, \frac{1}{3}) = 1$ in $(X, U)$, $B$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, U)$. Also, $f(A_1)$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set, because $B \subseteq cl[B, \frac{1}{2}, \frac{1}{3}] = 1$ in $(X, U)$. Thus $f$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen mapping. But $f(A_1) = A_1$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-closed set in $(X, U)$, because $A_1 \not\subseteq cl[\text{int}(A_1, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}] = 0$ in $(X, U)$. Thus $f$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-closed mapping.
Theorem 4.5. Let \( f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2) \) be a mapping from a SoFTS \( X \) to a SoFTS \( Y \) and \( (r, s) \in I \otimes I \). Then \( f \) is a fuzzy \((r, s)\)-semi-preopen mapping if and only if \( f(\text{int}(A, r, s)) \subseteq \text{spint}(f(A), r, s) \) for each intuitionistic fuzzy set \( A \) in \( X \).

Proof. Let \( f \) be a fuzzy \((r, s)\)-semi-preopen mapping. Since \( \text{int}(A, r, s) \) is fuzzy \((r, s)\)-open in \( X \), \( f(\text{int}(A, r, s)) \) is a fuzzy \((r, s)\)-semi-preopen set in \( Y \). Hence

\[
f(\text{int}(A, r, s)) = \text{spint}(f(\text{int}(A, r, s), r, s) \subseteq \text{spint}(f(A), r, s).
\]

Conversely, let \( A \) be a fuzzy \((r, s)\)-open set in \( X \). By hypothesis, \( f(A) = f(\text{int}(A, r, s)) \subseteq \text{spint}(f(A), r, s) \subseteq f(A) \). So \( f(A) = \text{spint}(f(A), r, s) \). Thus \( f(A) \) is a fuzzy \((r, s)\)-semi-preopen set in \( Y \). Hence \( f \) is a fuzzy \((r, s)\)-semi-preopen mapping.

\( \square \)

Theorem 4.6. Let \( f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2) \) be a mapping from a SoFTS \( X \) to a SoFTS \( Y \) and \( (r, s) \in I \otimes I \). Then \( f \) is a fuzzy \((r, s)\)-semi-preclosed mapping if and only if \( \text{spcl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s)) \) for each intuitionistic fuzzy set \( A \) in \( X \).

Proof. Let \( f \) be a fuzzy \((r, s)\)-semi-preclosed mapping. Since \( f(\text{cl}(A, r, s)) \) is a fuzzy \((r, s)\)-closed set in \( X \), \( f(\text{cl}(A, r, s)) \) is a fuzzy \((r, s)\)-semi-preclosed set in \( Y \). Since \( f(A) \subseteq f(\text{cl}(A, r, s)) \), we have \( \text{spcl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s)) \). Conversely, let \( A \) be a fuzzy \((r, s)\)-closed set in \( X \). By hypothesis, \( f(A) \subseteq f(\text{cl}(A, r, s)) = f(A) \). So \( f(A) = f(\text{cl}(A, r, s)) \). Thus \( f(A) \) is a fuzzy \((r, s)\)-semi-preclosed set in \( Y \). Hence \( f \) is a fuzzy \((r, s)\)-semi-preclosed mapping.

\( \square \)

References


제자소개

Seok Jong Lee

한국 편지 및 지능시스템학회 부회장
현재 충북대학교 수학과 교수
제 15권 1호 (2005년 2월호) 참조
E-mail : sjl@cbnu.ac.kr

Jin Tae Kim

현재 충북대학교 수학과 박사과정
E-mail : kjmath@hanmail.net