Fuzzy Semi-Weakly r-Semicontinuous 함수에 관한 연구
Fuzzy Semi-Weakly r-Semicontinuous Mappings

민원근
Won Keun Min
강원대학교 수학과

요 약
Fuzzy semi-weakly r-semicontinuous 함수의 개념을 소개하며 특성을 조사한다. 본 논문에서 소개된 함수와 fuzzy r-semicontinuity와 fuzzy weakly r-semicontinuity의 관계를 밝힌다.

Abstract
In this paper, we introduce the concept of fuzzy semi-weakly r-semicontinuous mappings on a fuzzy topological space and study characterizations for such mappings. And we investigate the relationships among fuzzy r-semicontinuity, fuzzy semi-weakly r-semicontinuity and fuzzy weakly r-semicontinuity.

Key Words: fuzzy semi-weakly r-semicontinuous, fuzzy weakly r-semicontinuous, fuzzy S-weakly r-continuous, fuzzy r-irresolute.

1. 서 론

Chang [1] defined fuzzy topological spaces using fuzzy sets introduced by Zadeh [10]. In [3, 4], Chattopadhyay, Hazra and Samanta introduced the concept of smooth fuzzy topological spaces which are a generalization of fuzzy topological spaces.

Lee and Kim [8] introduced and studied the concept of fuzzy weakly \( r \)-semicontinuous mappings in fuzzy topological spaces defined by Chattopadhyay, which is a generalized concept of fuzzy weakly semicontinuous mappings defined in the Chang’s fuzzy topological spaces.

In this paper, we introduce and study the concept of fuzzy semi-weakly \( r \)-semicontinuous mappings on the fuzzy topological space which is a generalization of fuzzy \( r \)-irresolute mappings. In particular, we investigate the relationships among fuzzy \( r \)-semicontinuity, fuzzy weakly \( r \)-semicontinuity and fuzzy semi-weakly \( r \)-semicontinuity.

2. Preliminaries

Let I be the unit interval \([0,1]\) of the real line. A member \( \mu \) of \( I^X \) is called a fuzzy set of \( X \). By 0 and 1, we denote constant maps on \( X \) with value 0 and 1, respectively. For any \( \mu \in I^X, \mu^c \) denotes the complement \( 1 - \mu \). All other notations are standard notations of fuzzy set theory.

A fuzzy point \( x_\alpha \) in \( X \) is a fuzzy set \( x_\alpha \) is defined as follows

\[
x_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}
\]

A fuzzy point \( x_\alpha \) is said to belong to a fuzzy set \( A \) in \( X \), denoted by \( x_\alpha \in A \), if \( \alpha \leq A(x) \) for \( x \in X \).

A fuzzy set \( A \) in \( X \) is the union of all fuzzy points which belong to \( A \).

Let \( f : X \to Y \) be a mapping and \( \alpha \in I^X \) and \( \beta \in I^Y \). Then \( f(\alpha) \) is a fuzzy set in \( Y \), defined by

\[
f(\alpha)(y) = \sup_{z \in f^{-1}(y)} \alpha(z), \quad \text{if } f^{-1}(y) \neq \emptyset, \]

\[
0, \quad \text{otherwise},
\]

for \( y \in Y \) and \( f^{-1}(\beta) \) is a fuzzy set in \( X \), defined by \( f^{-1}(\beta)(x) = \beta(f(x)) \), \( x \in X \).

A fuzzy topology [3, 4] on \( X \) is a map \( T : I^X \to I \) which satisfies the following properties:

1. \( T(0) = T(1) = 1 \).
2. \( T(\mu_1 \land \mu_2) \geq T(\mu_1) \land T(\mu_2) \) for \( \mu_1, \mu_2 \in I^X \).
3. \( T(\mu_1 \lor \mu_2) \geq T(\mu_1) \lor T(\mu_2) \) for \( \mu_1 \in I^X \).

The pair \((X,T)\) is called a fuzzy topological space. And \( \mu \in I^X \) is said to be fuzzy \( r \)-open (resp., fuzzy \( r \)-closed) if \( T(\mu) \geq r \) (resp., \( T(\mu^c) \geq r \)).

The \( r \)-closure and the \( r \)-interior of \( A \), denoted by...
cl(A, r) and int(A, r), respectively, are defined as
\[ \text{cl}(A, r) = \{ x \in X : \text{scl}(B, r) \subseteq A \} \]
\[ \text{int}(A, r) = \{ x \in X : B \subseteq A \} \]
and
\[ \text{cl}(A, r) = \cap \{ B \subseteq X : B \subseteq A \} \]
\[ \text{int}(A, r) = \cup \{ B \subseteq X : B \subseteq A \} \]
Define 2.1 [6]. Let A be a fuzzy set in an FTS \((X, T)\) and \(r \in (0, 1] = \mathbb{L}_r\). Then A is said to be fuzzy \(r\)-semiopen if there is a fuzzy \(r\)-open set \(B \subseteq X\) such that \(B \subseteq A \subseteq \text{cl}(B, r)\).

Let \(A \subseteq X\) in an FTS \((X, T)\) and \(r \in (0, 1] = \mathbb{L}_r\). The fuzzy \(r\)-semi-closure and the fuzzy \(r\)-semi-interior of A, denoted by \(\text{scl}(A, r)\) and \(\text{int}(A, r)\), respectively, are defined as:
\[ \text{scl}(A, r) = \cap \{ B \subseteq X : B \subseteq A \} \]
\[ \text{int}(A, r) = \cup \{ B \subseteq X : B \subseteq A \} \]

Define 2.2 [6, 7, 8, 9]. Let \(f : X \rightarrow Y\) be a mapping from FTS’s \(X\) and \(Y\). Then \(f\) is said to be
1. fuzzy \(r\)-irresolute [7] if for each fuzzy \(r\)-semiopen set \(B \subseteq Y\), \(f^{-1}(B)\) is a fuzzy \(r\)-semiopen set in \(X\).
2. fuzzy \(r\)-semicontinuous [6] if for each fuzzy \(r\)-semiopen set \(B \subseteq Y\), \(f^{-1}(B)\) is a fuzzy \(r\)-semiopen set in \(X\).
3. fuzzy weakly \(r\)-semicontinuous [8] if for each fuzzy \(r\)-open set \(B \subseteq Y\), \(f^{-1}(B) \subseteq \text{int}(\text{scl}(B, r), r)\).
4. fuzzy \(S\)-weakly \(r\)-continuous [9] if \(f^{-1}(B) \subseteq \text{int}(f^{-1}(\text{cl}(B, r)), r)\) for each fuzzy \(r\)-open set \(B \subseteq Y\).

3. Main Results

Definition 3.1. Let \(f : X \rightarrow Y\) be a mapping from FTS’s \(X\) and \(Y\) and \(r \in (0, 1] = \mathbb{L}_r\). Then \(f\) is said to be fuzzy \(S\)-weakly \(r\)-semicontinuous if \(f^{-1}(A) \subseteq \text{int}(f^{-1}(\text{scl}(A, r)), r)\) for each fuzzy \(r\)-semiopen set \(A\) of \(Y\).

Remark 3.2. Every fuzzy \(r\)-semiopen set of \(X\) is fuzzy \(r\)-semiopen but the converse is not always true.

Example 3.3. Let \(X = I\) and let \(A_1\) and \(A_2\) be fuzzy sets of \(X\) defined as:
\[ A_1(x) = \begin{cases} \frac{1}{4} x + 1, & \text{for } x \in I, \\ 1, & \text{for } x \in I \end{cases} \]
\[ A_2(x) = \begin{cases} \frac{1}{2} x + 1, & \text{for } x \in I, \\ 1, & \text{for } x \in I \end{cases} \]
Define a fuzzy topology \(T : I^X \rightarrow I\) by
\[ T(\sigma) = \begin{cases} 1, & \text{if } \sigma = 0, \\ \frac{1}{2}, & \text{if } \sigma = A_1, \\ 0, & \text{otherwise.} \end{cases} \]

Consider the identity mapping \(f : (X, T) \rightarrow (X, U)\). We know that every fuzzy set \(B\) containing \(A_1\) is fuzzy \(\frac{1}{2}\)-semiopen in the FTS \((X, U)\) and \(\text{scl}(B, \frac{1}{2}) = 1\).

Hence the mapping \(f\) is a fuzzy semi-weakly \(\frac{1}{2}\)-semicontinuous mapping but it is not fuzzy \(\frac{1}{2}\)-semicontinuous.

Remark 3.4. Every fuzzy semi-weakly \(r\)-semi continuous mapping is fuzzy weakly \(r\)-semicontinuous but the converse is not always true.

Example 3.5. Let \(X = I\) and let \(A_1\), \(A_2\) and \(A_3\) be fuzzy sets of \(X\) defined as:
\[ A_1(x) = \begin{cases} \frac{1}{10}, & \text{for } x \in I, \\ 0, & \text{otherwise.} \end{cases} \]
\[ A_2(x) = \begin{cases} \frac{3}{10}, & \text{for } x \in I, \\ 1, & \text{for } x \in I \end{cases} \]
\[ A_3(x) = \begin{cases} \frac{8}{10}, & \text{for } x \in I, \\ 0, & \text{otherwise.} \end{cases} \]
 Define a fuzzy topology \(T : I^X \rightarrow I\) by
\[ T(\sigma) = \begin{cases} 1, & \text{if } \sigma = 0, \\ \frac{1}{2}, & \text{if } \sigma = A_1, A_3, \\ 0, & \text{otherwise.} \end{cases} \]

Consider the identity mapping \(f : (X, T) \rightarrow (X, U)\). Then obviously f is fuzzy weakly \(\frac{1}{2}\)-semicontinuous. Consider a fuzzy semiopen set \(B\) in \((X, U)\) defined as \(B(x) = \frac{1}{4}\) for \(x \in I\). Then \(\text{sint}(f^{-1}(\text{scl}(B, \frac{1}{2}))) = \frac{1}{2}\) and \(\text{sint}(A_2, \frac{1}{2}) = \frac{1}{2} - A_3\) in the FTS \((X, T)\) and \(f^{-1}(B) = B \supseteq \frac{1}{2} - A_3\). Hence the mapping \(f\) is not a fuzzy
semi-weakly $\frac{1}{2}$-semicontinuous.

Now the following implications are obtained.

fuzzy r-irresolute $\Rightarrow$ fuzzy weakly r-semicont.
$\Rightarrow$ fuzzy semi-weakly r-semicont. $\Rightarrow$ fuzzy weakly r-semicont. $\Rightarrow$ fuzzy S-weakly r-cont.

**Theorem 3.6.** Let $f: (X, T) \to (X, U)$ be a mapping on FTS’s $(X, T)$ and $(Y, U)$ $(r \in \mathcal{L})$. Then $f$ is a fuzzy semi-weakly r-semicontinuous if and only if for every fuzzy point $x_0$ and each fuzzy r-semiopen set $V$ containing $f(x_0)$, there exists a fuzzy r-semiopen set $U$ containing $x_0$ such that $f(U) \subseteq \text{scl}(V, r)$.

**Proof.** Suppose $f$ is a fuzzy semi-weakly r-semi-continuous mapping. Let $x_0$ be a fuzzy point in $X$ and $V$ a fuzzy r-semiopen set containing $f(x_0)$. Then there exists a fuzzy r-semiopen set $B$ such that $f(x_0) \subseteq B \subseteq V$. From the hypothesis, it follows

$$f^{-1}(B) \subseteq \text{sint}(f^{-1}(\text{scl}(B, r)), r) \subseteq \text{sint}(f^{-1}(\text{scl}(V, r)), r).$$

Set $U=\text{sint}(f^{-1}(\text{scl}(B, r)), r)$. Since $U$ is a fuzzy r-semiopen set such that $f^{-1}(B) \subseteq U \subseteq \text{sint}(f^{-1}(\text{scl}(V, r)), r) \subseteq f^{-1}(\text{scl}(V, r))$, we have $f(U) \subseteq \text{scl}(V, r)$.

For the converse, let $V$ be a fuzzy r-semiopen set in $Y$. For each $x_0 \in f^{-1}(V)$, there exists a fuzzy r-semiopen set $U_{x_0}$ containing $x_0$ such that $f(U_{x_0}) \subseteq \text{scl}(V, r)$. This implies

$$f^{-1}(V) \subseteq \bigcup\{U_{x_0} : x_0 \in f^{-1}(V) \subseteq f^{-1}(\text{scl}(V, r)).$$

Since $\bigcup\{U_{x_0} : x_0 \in f^{-1}(V)\}$ is a fuzzy r-semiopen set containing $f^{-1}(V)$, we have $f^{-1}(V) \subseteq \text{sint}(f^{-1}(\text{scl}(V, r)), r)$. Hence $f$ is a fuzzy semi-weakly r-semi-continuous function.

**Theorem 3.7.** ([7]) Let $A$ be a fuzzy set in an FTS $(X, T)$ and $r \in \mathcal{L}$. Then we have

1. $\text{i-scl}(A, r)=\text{sint}([1-A], r)$,
2. $\text{i-sint}(A, r)=\text{scl}([1-A], r)$.

**Theorem 3.8.** Let $f: (X, T) \to (X, U)$ be a mapping on FTS’s $(X, T)$ and $(Y, U)$ $(r \in \mathcal{L})$. Then the following statements are equivalent:

1. $f$ is fuzzy semi-weakly r-semicontinuous.
2. $\text{scl}(f^{-1}(\text{sint}(F, r)), r) \subseteq f^{-1}(F)$ for each fuzzy r-semiclosed set $F$ in $Y$.
3. $\text{scl}(f^{-1}(\text{sint}(B, r)), r) \subseteq f^{-1}(\text{scl}(B, r))$ for each fuzzy set $B$ in $Y$.
4. $f^{-1}(\text{sint}(B, r)) \subseteq \text{sint}(f^{-1}(\text{scl}(B, r)), r)$ for each fuzzy set $B$ in $Y$.
5. $\text{scl}(f^{-1}(V, r)) \subseteq f^{-1}(\text{scl}(V, r))$ for a fuzzy r-semiopen set $V$ in $Y$.

**References**


