On some properties of vague bi-groups and fuzzy bi-functions

Lee-Chae Jang*, Taekyun Kim**, Byungje Lee*** and Won-Joo Kim****

*Dept. of Mathematics and Computer Science, Konkuk University
**Division of General Education, Kwangwoon University
***Dept. of Wireless Communications Engineering, Kwangwoon University
****The Research Institute of natural Sciences, Konkuk University

Abstract

M. Demirci [Vague groups, J. Math. Anal. Appl. vol.230, pp. 142-156, 1999] studied the vague group operation on a crisp set as a fuzzy function and established the vague group structure on a crisp set. In this paper we consider bi-groups which are studied by A. A. A. Agboola and L. S. Akinola. And we also will define vague bi-groups and fuzzy bi-functions and we investigate some basic operations on the vague bi-group and fuzzy bi-functions.

Key Words: bi-groups, vague groups, fuzzy equality, vague binary operation, vague bi-groups, fuzzy bi-functions.

1. Introduction

Fuzzy sets proposed by Zadeh in 1968 ([8]) and fuzzy settings of various algebraic concepts were studied by several authors. Many authors have worked to present the fuzzy setting of various algebraic concepts based on the papers [1,5,6,7].

To get a more general extension, Demirci [5,7] defined the concept of vague group based on fuzzy equalities and fuzzy functions. He also established the vague group structure on a crisp set.

The concept of fuzzy equality and fuzzy function given in [5,6,7] provides a good tool for fuzzifying the group operation on a crisp set.

Let \( X, Y \) be crisp sets. A mapping \( E_X: X \times X \rightarrow [0,1] \) is called a fuzzy equality on \( X \) if and only if the following conditions are satisfied:

(i) \( E_X(x,y) = 1 \) if and only if \( x = y \), \( \forall x \in X, y \in X \),

(ii) \( E_X(x,y) = E_X(y,x) \), \( \forall x \in X \), and

(iii) \( \min \{E_X(x,y), E_Y(x,z)\} \leq E_X(x,z), \forall x,y,z \in X \).

The real number \( E_X(x,y) \) is called the degree of \( x \) and \( y \) in \( X \).

Let \( E_X \) and \( E_Y \) be two fuzzy equalities on \( X \) and \( Y \), respectively. \( f: X \rightarrow Y \) is called fuzzy function with respect to \( E_X \) and \( E_Y \) if and only if the membership function \( \mu_f: X \times Y \rightarrow [0,1] \) of \( f \) satisfies the following conditions:

(i) \( \forall x \in X, \exists y \in Y \) such that \( \mu_f(x,y) > 0 \), and

(ii) \( \min \{\mu_f(x,z), \mu_f(y,w), E_Y(x,y)\} \leq E_Y(z,w) \), \( \forall x,y \in X \), \( \forall w,z \in Y \).

A fuzzy function \( f \) is called a strong fuzzy function if and only if it satisfies the following additionally condition:

\( \forall x \in X, \exists y \in Y \) such that \( \mu_f(x,y) = 1 \).

In this paper we consider bi-groups which are defined by A.A.A. Agboola and L.S. Akinola [2] and W.B. Vasabtha Kadasamy [3]. And we also will define vague bi-groups and fuzzy bi-functions and we investigate some basic operations on vague bi-groups and fuzzy bi-functions.

2. Vague bigroups.

In this section, we consider vague binary operations, vague closed under the operations, vague semigroups, vague groups, fuzzy functions, etc.

**Definition 2.1** ([6]) (1) A strong fuzzy function \( f: X \times X \rightarrow X \) with respect to a fuzzy equality \( E_{X \times X} \) on \( X \times X \) and fuzzy equality \( E_X \) on \( X \) is said to be a vague binary operation with respect to \( E_{X \times X} \) and \( E_X \).

(2) A vague binary operation \( f \) with respect to \( E_{X \times X} \) and \( E_X \) is said to be transitive of first order if it satisfies the following condition:
A vague binary operation \( f \) on \( X \) with respect to \( E_{X \times X} \) and \( E_X \) is said to be transitive of the second order if it satisfies the following condition:

\[
\min [\mu_f(a,b,c), E_X(c,d)] \leq \mu_f(a,b,d), \quad \forall a,b,c \in X.
\]

(3) A vague binary operation \( f \) on \( X \) with respect to \( E_{X \times X} \) and \( E_X \) is said to be transitive of the third order if it satisfies the following condition:

\[
\min [\mu_f(a,b,c), E_X(a,d)] \leq \mu_f(d,b,c), \quad \forall a,b,c \in X.
\]

(4) A vague binary operation \( f \) on \( X \) with respect to \( E_{X \times X} \) and \( E_X \) is said to be transitive of the third order if it satisfies the following condition:

\[
\min [\mu_f(a,b,c), E_X(a,d)] \leq \mu_f(d,b,c), \quad \forall a,b,c \in X.
\]

(5) Let \( f \) be a vague binary operation on \( X \). A crisp subset \( B \) of \( X \) is said to be vague closed under \( f \) if it satisfies the following condition:

\[
\mu_f(a,b,c) = 1 \Rightarrow c \in B, \quad \forall a,b \in B, \forall c \in X.
\]

\textbf{Definition 2.2} ([(6)]) Let \( \ast \) be a vague binary operation on \( X \) with respect to a fuzzy equality \( E_{X \times X} \) on \( X \times X \) and \( E_X \) on \( X \). Then

(1) \((X, \ast)\) is called a vague semigroup if the membership function \( \mu : X \times X \times X \rightarrow [0,1] \) of \( \ast \) satisfies the following condition:

\[
\min [\mu_{\ast}(b,c,d), \mu_{\ast}(a,d,m), \mu_{\ast}(a,b,q), \mu_{\ast}(q,c,w)] \leq E_X(m,w), \quad \forall a,b,c,d,q,m,w \in X.
\]

(2) A vague semigroup \((X, \ast)\) is called a vague monoid if and only if it satisfies the following condition:

\[
\exists e \in X \text{ such that } \min [\mu_{\ast}(e,a,a), \mu_{\ast}(a,e,a)] = 1, \quad \forall a \in X.
\]

(3) A vague monoid \((X, \ast)\) is called a vague group if it satisfies the following condition:

\[
\forall a \in X, \exists a^{-1} \in X \text{ such that } \min [\mu_{\ast}(a^{-1},a,e), \mu_{\ast}(a,a^{-1},e)] = 1.
\]

(4) A vague group \((X, \ast)\) is said to be abelian (commutative) if \( \ast \) satisfies the following condition:

\[
\min [\mu_{\ast}(a,b,m), \mu_{\ast}(b,a,w)] \leq E_X(m,w), \quad \forall a,b,m,w \in X.
\]

\textbf{Proposition 2.3} ([(6)]) Let \( \ast \) be a \( X \) with respect to \( E_{X \times X} \) and \( E_X \) if \((X, \ast)\) is a semigroup and \( \ast \) is transitive of the second and the third order, then \((X, \ast)\) is a vague group.

Now, we will define vague bi-group and investigate cancellative law of vague bi-group, and some properties of them.

\textbf{Definition 2.4} (1) A set \((X, \oplus, \odot)\) with two vague binary operation \( \oplus \) and \( \odot \) is called a vague bi-group if there exist two proper subsets \( X_1 \) and \( X_2 \) of \( X \) such that

(i) \( X = X_1 \cup X_2 \),

(ii) \((X_1, \oplus)\) is a vague group,

(iii) \((X_2, \odot)\) is a vague group.

(2) A vague bigroup \((X, \oplus, \odot)\) is said to be abelian if \((X_1, \oplus)\) and \((X_2, \odot)\) are vague abelian.

\textbf{Theorem 2.5} (Cancellation law) Let \((X, \oplus, \odot)\) be vague bi-group with respect to \( E_{X \times X} \) on \( X \times X \) and \( E_X \) on \( X \). Then we have

(1) \[
\min [\mu_{\oplus}(a,b,a), \mu_{\oplus}(a,\odot,a)] \leq E_X(b,c),
\]

for \( \forall a,b,c \in X \)

and

(2) \[
\min [\mu_{\odot}(b,\oplus,a_1), \mu_{\odot}(c,\odot,a_1)] \leq E_X(b,c),
\]

for \( \forall a_1,b_1,c_1,a_1 \in X \)

and

(3) \[
\min [\mu_{\odot}(b_2,\oplus,a_2), \mu_{\odot}(c_2,\odot,a_2)] \leq E_X(b,c),
\]

for \( \forall a_2,b_2,c_2,a_2 \in X \).

\textbf{Proposition 2.6} Let \((X, \oplus, \odot)\) be a vague bi-group with respect to \( E_{X \times X} \) on \( X \times X \) and \( E_X \) on \( X \) with \( X = X_1 \cup X_2 \). Then we have

(1) The identity of \((X_1, \oplus)\) and \((X_2, \odot)\) is unique.

(2) Each element of \((X, \oplus, \odot)\) has a unique inverse element in \( X \). That is, each element of \((X_1, \oplus)\) and \((X_2, \odot)\) have a unique inverse element in \( X \).

(3) For all \( a = [a_1] \) if \( a \in X_1 \) and \( [a_2] \) if \( a \in X_2 \), we have

\[
\oplus^{-1}(\oplus^{-1}a_1) = a_1 \quad \text{and} \quad \odot^{-1}(\odot^{-1}a_2) = a_2.
\]

\textbf{Proof.} (1) Suppose that \( e_1 \) and \( f_1 \) are identities in \((X_1, \oplus)\) and that \( e_2 \) and \( f_2 \) are identities in \((X_2, \odot)\).

Then we have

\[
\mu_{\oplus}(e_1,a_1,a_1) = \mu_{\oplus}(a_1,e_1,a_1) = 1
\]

and

\[
\mu_{\odot}(e_2,a_2,a_2) = \mu_{\odot}(a_2,e_2,a_2) = 1.
\]

Also, we have

\[
\mu_{\oplus}(f_1,a_1,a_1) = \mu_{\oplus}(a_1,f_1,a_1) = 1
\]

and

\[
\mu_{\odot}(f_2,a_2,a_2) = \mu_{\odot}(a_2,f_2,a_2) = 1.
\]

Thus, by cancellation law, we see that

\[
\text{...}
\]

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\[
\min \mu_{\oplus}(a_1, e_1, a_1), \mu_{\oplus}(a_1, f_1, a_1) \leq E_X(e_1, f_1)
\]
and
\[
\min \mu_{\ominus}(a_2, e_2, a_2), \mu_{\ominus}(a_2, e_2, a_2) \leq E_X(e_2, f_2).
\]

Then, we obtain that
\[
E_X(e_2, f_2) = 1 \quad \text{and} \quad E_X(e_1, f_1) = 1.
\]
So we have \(e_2 = f_2\) and \(e_1 = f_1\) and hence the identity of \((X_1, \oplus)\) and \((X_2, \ominus)\) is unique.

(2) Let \(e = \{e_1\} \text{ if } e \in X_1 \quad \text{and} \quad e = \{e_2\} \text{ if } e \in X_2\).
Suppose that \(b = \{b_1\} \text{ if } b \in X_1\) and \(c = \{c_1\} \text{ if } c \in X_1\)
are inverse of \(a = \{a_1\} \text{ if } a \in X_1 \quad \text{and} \quad a = \{a_2\} \text{ if } a \in X_2\).
Then we have
\[
\mu_{\oplus}(a_1, b_1, c_1) = \mu_{\oplus}(b_1, a_1, e_1) = 1
\]
and
\[
\mu_{\ominus}(a_2, b_2, c_2) = \mu_{\ominus}(b_2, a_2, e_2) = 1.
\]
Since \(e = \{c_1\} \text{ if } c \in X_1 \quad \text{and} \quad c = \{c_2\} \text{ if } c \in X_2\) is inverse of \(a = \{a_1\} \text{ if } a \in X_1 \quad \text{and} \quad a = \{a_2\} \text{ if } a \in X_2\),
we also see that
\[
\mu_{\oplus}(a_1, c_1, e_1) = \mu_{\oplus}(c_1, a_1, e_1) = 1
\]
and
\[
\mu_{\ominus}(a_2, c_2, e_2) = \mu_{\ominus}(c_2, a_2, e_2) = 1.
\]
Thus, by cancellation law, we see that
\[
\min \mu_{\oplus}(a_1, b_1, c_1), \mu_{\ominus}(a_1, c_1, e_1) \leq E_X(b_1, c_1)
\]
and
\[
\min \mu_{\ominus}(a_2, b_2, c_2), \mu_{\ominus}(a_2, c_2, e_2) \leq E_X(b_2, c_2).
\]
Then, we obtain that
\[
E_X(b_1, c_1) = 1 \quad \text{and} \quad E_X(b_2, c_2) = 1.
\]
So we have \(b = c\) and hence the identity of \((X_1, \oplus, \ominus)\) is unique.

(3) Let \(e = \{e_1\} \text{ if } e \in X_1 \quad \text{and} \quad e = \{e_2\} \text{ if } e \in X_2\).

Suppose that \(a = \{a_1\} \text{ if } a \in X_1 \quad \text{and} \quad a = \{a_2\} \text{ if } a \in X_2\)
has a unique inverse \(a^* = \{a_1^*\} \text{ if } a \in X_1 \quad \text{and} \quad a^* = \{a_2^*\} \text{ if } a \in X_2\).
Then we note that
\[
\mu_{\ominus}(a_1^*, a_2, e_2) = 1 = \mu_{\oplus}(a_1, a_2) \quad \text{and} \quad \mu_{\ominus}(a_2^*, a_2, e_2) = 1 = \mu_{\ominus}(a_2, a_2)\]
we note that
\[
\mu_{\oplus}(a_1^*, a_1, e_1) = 1 = \mu_{\ominus}(a_1, a_1) \quad \text{and} \quad \mu_{\ominus}(a_2^*, a_2, e_2) = 1 = \mu_{\ominus}(a_2, a_2)\]
Thus, by cancellation law, we have
\[
\min \mu_{\ominus}(\ominus^{-1}(a_1), a_1, e_1), \mu_{\ominus}(\ominus^{-1}(a_1), \ominus^{-1} \ominus^{-1}(a_1), e_1) \leq E_X(a_1, \ominus^{-1} \ominus^{-1}(a_1))
\]
and
\[
\min \mu_{\ominus}(\ominus^{-1}(a_2), a_2, e_2), \mu_{\ominus}(\ominus^{-1}(a_2), \ominus^{-1} \ominus^{-1}(a_2), e_2) \leq E_X(a_2, \ominus^{-1} \ominus^{-1}(a_2))
\]
Then, we obtain that
\[
E_X(a_1, \ominus^{-1} \ominus^{-1}(a_1)) = 1 \quad \text{and} \quad E_X(a_2, \ominus^{-1} \ominus^{-1}(a_2)) = 1.
\]
So we have \(a_1 = a_1\) and \(a_2 = a_2\).

From Proposition 2.3, we can obtain the following proposition.

**Proposition 2.7** If \((X_1, \oplus)\) is a semigroup and \(\oplus\) is a transitive of the second and third order, and if \((X_2, \ominus)\) is a semigroup and \(\ominus\) is a transitive of the second and third order, then \((X, \oplus, \ominus)\) is a vague bigroup.

### 3. Fuzzy bi-functions.

In this section, we define fuzzy bi-functions and investigate some characterizations of them.

**Definition 3.1** Let \(X\) be crisp sets. If there exist two proper subsets \(X_1\) and \(X_2\) such that
(i) \(X_1 \cup X_2 = X\),
(ii) \(X_1\) is closed under \(\oplus\),
(iii) \(X_2\) is closed under \(\ominus\),
then \(X\) is called a crisp bi-set.

**Example 3.2** Let \(\mathbb{Q}, \mathbb{Q}^+, \mathbb{Q}^-\) be the set of rational numbers, the set of nonnegative rational integers, the set of negative integers, respectively. Then \((\mathbb{Q}, +, \cdot)\) is a crisp bi-set. In fact, \(\mathbb{Q} = \mathbb{Q}^+ \cup \mathbb{Q}^-\) and \(\mathbb{Q}^+\) is closed under the usual addition \(+\) and \(\mathbb{Q}^-\) is closed under the usual multiplication \(\cdot\).

**Definition 3.3** Let \(X\) be a crisp biset with \(X = X_1 \cup X_2\). A mapping \(E_X : X \times X \to [0, 1]\) is called a fuzzy bi-equality on \(X\) if it satisfies the following conditions:
(i) \(E_X(x_1, y_1) = 1\) and \(E_X(x_2, y_2) = 1\) if and only if
\[
x_1 = y_1 \quad \text{and} \quad x_2 = y_2,
\]
where \(x = \{x_1\} \text{ if } x \in X_1 \quad \text{and} \quad y = \{y_1\} \text{ if } y \in Y_1\).
(ii) \(E_X(x_1, y_1) = E_X(y_1, x_1)\) and \(E_X(x_2, y_2) = E_X(y_2, x_2)\),
(iii) \(\min[E_X(x_1, y_1), E_X(y_1, z_1)] \leq E_X(x_1, z_1)\).
and $\min[E_X(x_2,y_2), E_X(y_2,z_2)] \leq E_X(x_2,z_2)$

where $x = \{x_1 \text{ if } x \in X_1, x_2 \text{ if } x \in X_2\}$, $y = \{y_1 \text{ if } y \in Y_1, y_2 \text{ if } y \in Y_2\}$, and

$z = \{z_1 \text{ if } z \in Z_1, z_2 \text{ if } z \in Z_2\}$.

Now, we consider the following notation:

$X \otimes Y = (X_1 \times Y_1) \cup (X_2 \times Y_2)$

where $X_i \times Y_i$ is the Cartesian product of $X_i$ and $Y_i$ for $i = 1, 2$.

**Definition 3.4** Let $X$ and $Y$ be crisp bi-sets with $X = X_1 \cup X_2$ and $Y = Y_1 \cup Y_2$, respectively and let $E_X$ and $E_Y$ be three fuzzy bi-equalities on $X$ and $Y$, respectively.

1. If $f : X \rightarrow Y$ is called a fuzzy bi-function with respect to $E_X$ and $E_Y$ if it satisfies the following conditions:
   - (I) there exists two fuzzy functions $f_1$ and $f_2$ such that $f = f_1 \cup f_2$.
   - (II) the membership function $\mu_f : X \otimes Y \rightarrow [0,1]$ of $f$ satisfies the following conditions:
     i. for all $x = \{x_1 \text{ if } x \in X_1, x_2 \text{ if } x \in X_2\}$, $y = \{y_1 \text{ if } y \in Y_1, y_2 \text{ if } y \in Y_2\}$ such that $\mu_{f_1}(x_1,y_1) > 0$ and $\mu_{f_2}(x_2,y_2) > 0$, 
     ii. $\min[\mu_{f_1}(x_1,z_1), \mu_{f_2}(y_1,w_1), E_X(x_1,y_1)] \leq E_Y(z_1,w_1)$

and

$\min[\mu_{f_1}(x_2,z_2), \mu_{f_2}(y_2,w_2), E_X(x_2,y_2)] \leq E_Y(z_2,w_2)$

where $x = \{x_1 \text{ if } x \in X_1, x_2 \text{ if } x \in X_2\}$, $y = \{y_1 \text{ if } y \in Y_1, y_2 \text{ if } y \in Y_2\}

w = \{w_1 \text{ if } w \in Y_1, w_2 \text{ if } w \in Y_2\}$ and $z = \{z_1 \text{ if } z \in Z_1, z_2 \text{ if } z \in Z_2\}$.

2. A fuzzy bi-function $f$ is called a strongly fuzzy bi-function if it satisfies the following additional condition:

   (iii) for all $x = \{x_1 \text{ if } x \in X_1, x_2 \text{ if } x \in X_2\}$, $y = \{y_1 \text{ if } y \in Y_1, y_2 \text{ if } y \in Y_2\}$ such that $\mu_{f_1}(x_1,y_1) = 1$ and $\mu_{f_2}(x_2,y_2) = 1$.

**Definition 3.5** Let $X$ and $Y$ be crisp bisets and let $f = f_1 \cup f_2$ be a fuzzy bi-function with respect to $E_X$ on $X$ and $E_Y$ on $Y$.

1. A fuzzy bi-function $f$ is said to be surjective if and only if

   $\forall y = \{y_1 \text{ if } y \in Y_1, y_2 \text{ if } y \in Y_2\}$, $\exists x = \{x_1 \text{ if } x \in X_1, x_2 \text{ if } x \in X_2\}$ such that

   $\mu_{f_1}(x_1,y_1) > 0$ and $\mu_{f_2}(x_2,y_2) > 0$.

2. A fuzzy bi-function $f$ is said to be strong surjective if and only if

   $\forall y = \{y_1 \text{ if } y \in Y_1, y_2 \text{ if } y \in Y_2\}$, $\exists x = \{x_1 \text{ if } x \in X_1, x_2 \text{ if } x \in X_2\}$ such that

   $\mu_{f_1}(x_1,y_1) = 1$ and $\mu_{f_2}(x_2,y_2) = 1$.

3. A fuzzy bi-function $f$ is said to be injective if and only if

   $\forall x = \{x_1 \text{ if } x \in X_1, x_2 \text{ if } x \in X_2\}$, $y = \{y_1 \text{ if } y \in Y_1, y_2 \text{ if } y \in Y_2\}$

   $w = \{w_1 \text{ if } w \in Y_1, w_2 \text{ if } w \in Y_2\}$ and $z = \{z_1 \text{ if } z \in Z_1, z_2 \text{ if } z \in Z_2\}$

   $\min[\mu_{f_1}(x_1,z_1), \mu_{f_2}(y_1,w_1), E_X(x_1,y_1)] \leq E_Y(z_1,w_1)$

   and

   $\min[\mu_{f_1}(x_2,z_2), \mu_{f_2}(y_2,w_2), E_X(x_2,y_2)] \leq E_Y(z_2,w_2)$.

4. A fuzzy bi-function $f$ is said to be bijective if and only if it is surjective and injective.

5. A fuzzy bi-function $f$ is said to be strong bijective if and only if it is strong surjective and injective.

**Definition 3.6** Let $X = X_1 \cup X_2$ be a crisp bi-set. The fuzzy bi-relation $U = U_1 \cup U_2$ on $X \otimes X$ defined by

$\mu_{U_1}(x_1,y_1) = \begin{cases} 0 & \text{if } x_1 = y_1, \\ 1 & \text{if } x_1 \neq y_1, \end{cases}$

and

$\mu_{U_2}(x_2,y_2) = \begin{cases} 0 & \text{if } x_2 = y_2, \\ 1 & \text{if } x_2 \neq y_2, \end{cases}$

is called a unit fuzzy bi-function on $X = X_1 \cup X_2$ and is denoted by $U_X = U_X \cup U_Y$.

**Definition 3.7** Let $X = X_1 \cup X_2$, $Y = Y_1 \cup Y_2$, and $Z = Z_1 \cup Z_2$ be crisp bi-sets and let $R = R_1 \cup R_2$, $K = K_1 \cup K_2$, and $S = S_1 \cup S_2$ be fuzzy bi-relations on $X \otimes Y$, $Y \otimes X$, and $Y \otimes Z$, respectively.

1. The sup-min composition $R \cdot S$ of $R$ and $S$ on $X \otimes Z$ is defined by a fuzzy bi-relation with the membership function

   $\mu_{R \cdot S} = \mu_{R_1 \cdot S_1} \cup \mu_{R_2 \cdot S_2} : X \otimes Z \rightarrow I$

   given by
Theorem 3.4. Since \( g \circ f \) and \( g_{1} \circ f_{1} \) are fuzzy functions,
\[
\min\{\mu_{g_{1}} \circ f_{1}(x_{1},z_{1}), \mu_{g_{2}} \circ f_{2}(y_{2},z_{2})\} \leq E_{Z}(z_{2},w_{2})
\]
that is, \( g \circ f \) satisfies the condition (II) of Definition 3.4.

References


\[
\mu_{R_{i}}(x_{i},y_{i}) = \sup_{y_{i} \in y_{i}} \{\min[\mu_{R_{i}}(x_{i},y_{i}), \mu_{S_{i}}(y_{i},z_{i})]\}
\]
저 자 소 개

장이재 (Lee-Chae Jang)
1979년 2월 : 경북대 수학과 (이학사)
1981년 2월 : 경북대 대학원 수학과 (이학석사)
1987년 2월 : 경북대 대학원 수학과 (이학박사)
1987년 6월 ~ 1988년 6월 : 미국 Cincinnati 대 (교환교수)
1987년 3월 ~ 현재 : 건국대 전산수학과 교수
 관심분야 : 함수해석학, 쇼케이적분, p-진 해석학
 E-mail : leechae.jang@kku.ac.kr

김태균 (Taekyun Kim)
1994.03.25 : 규슈 대 학 교 (Kyushu Univ.)
 이학 박사학위 취득 (수학: 수론).
 1994.4.1~1998.01.31 : 경북대, 해군사관학교, 경남대, 대전대, 강사.
 1999.02.01~2000.02.29 : 민사고, 수학교사
 2000.03.01~2000.12.31 : 캐나다 Simon Fraser Univ., CECM의 Visitor
 2001.4.12~2006.08.31 : 공주대 과학교육연구소, 연구교수
 2004.03.02~2007.02.28 : 한남대 대학원 (수학), 강사
 2006.08.01~2008.02.29 : 경북대 전자기컴퓨터학부, 교수
 2008.03.01~현재 : 광운대학교 교양학부 (수학), 부교수
 1994. 06.01 : 큐미수학회 및 편미수학회지 연구소 설립

이병제 (Byungje Lee)
1988년 2월 : 경북대 전자공학과 (공학사)
1993년 12월 : Southern Illinois University, 전기 및 컴퓨터공학과 (공학석사)
1997년 5월 : Southern Illinois University, 전기 및 컴퓨터공학과 (공학박사)
1998년 3월 ~ 현재 : 광운대학교 전자공학과 교수
 관심분야 : 안테나 설계 및 해석, 전자과관련 수치해석

김원주 (Won-Joo Kim)
1998년 2월 : 신라대 수학과 (이학사)
2000년 2월 : 경희 대 학 원 수 학 과 (이학석사)
2005년 2월 : 경희 대 학 원 수 학 과 (이학박사)
2006년 3월 ~ 현재 : 건국대 자연과학연구소
 E-mail : piterfan2@kku.ac.kr