POCS를 이용한 초광대역 무선통신의 펄스파형 설계

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Pulse Shape Design for Ultra-Wideband Radios Using Projections onto Convex Sets

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요 약

FCC 스펜트럼을 만족하는 초광대역(UWB) 무선을 위한 새로운 펄스 파형을 제안한다. POCS(projections onto convex sets) 기술은 UWB 신호의 제반특성(FCC 스펜트럼 마스크하에서의 효율적인 스펜트럼 이용, 시간 제한성, 좋은 자기상관)의 제약조건하에서 UWB 펄스의 시간 및 스펜트럼의 파형을 최적화한다. 시뮬레이션 결과에 의하면 펄스 파형의 모든 값에 대해 새로운 펄스 파형은 FCC 스펜트럼 마스크를 매우 효율적으로 만족할 뿐만 아니라 동일한 자기상관함수를 갖고 있음을 보여준다. 또한 동일한 펄스폭에 대해 제안된 펄스의 절단된(즉 엄격히 시간 제한된) 펄스 파형은 이진 TH-PPM(time-hoping pulse position modulation) 시스템의 BER 성능에서 절단된 가우시안 모노사이클(Gaussian monocycle)보다 우수하다. POCS 기술은 이 기술의 본질적인 설계 유연성 및 결합 최적화 능력 관점에서 UWB 펄스 파형 설계에 매우 효과적인 방법을 제공한다.

Key Words : Ultra-wideband (UWB), Impulse radio, Pulse design algorithm, Projections onto convex sets (POCS)

ABSTRACT

We propose new pulse shapes for FCC-compliant ultra-wideband (UWB) radios. The projections onto convex sets (POCS) technique is used to optimize temporal and spectral shapes of UWB pulses under the constraints of all of the desired UWB signal properties: efficient spectral utilization under the FCC spectral mask, time-limitedness, and good autocorrelation. Simulation results show that for all values of the pulse duration, the new pulse shapes not only meet the FCC spectral mask most efficiently, but also have nearly the same autocorrelation functions. It is also observed that our truncated (i.e., strictly time-limited) pulse shapes outperform the truncated Gaussian monocycle in the BER performance of binary TH-PPM systems for the same pulse durations. The POCS technique provides an effective method for designing UWB pulse shapes in terms of its inherent design flexibility and joint optimization capability.

I. Introduction

A Gaussian monocycle (or monopulse) is widely considered as a basic pulse shape in ultra-wideband (UWB) radios (or impulse radios)\(^{[1][2]}\). However, it does not meet the FCC spectral mask\(^{[3]}\) and causes a spectral overlay problem. Thus, it must be reshaped to be suitable for

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FCC-compliant UWB radios. The UWB pulse shape has an influence on the system performance of UWB radios such as bit-error rates and data rates. The pulse duration of the UWB pulse shape need to be shorter for the higher data rate and system capacity. The BER performance of UWB radios depends on the received signal-to-noise ratio (SNR) and the autocorrelation function (ACF) of the pulse shape used in UWB radios. It is also described in that given the stringent transmit power limitations, maximization of the received SNR requires efficient utilization of the bandwidth and power allowed by the FCC spectral mask.

Various pulse shape design methods for UWB radios have been proposed to resolve the problems described above. The methods proposed in are to filter the Gaussian monocycle using optimization techniques. This filtering method can increase the pulse duration. The digital filter design method using the Parks-McClellan algorithm can shape optimal UWB pulses which meet the FCC spectral mask, but can have a locally optimum problem. The globally optimal pulse design method based on semidefinite programming (SDP) can achieve optimal spectral utilization at a relatively low sampling rate of 25 GHz. On the other hand, the method presented in and the methods proposed in and are to design orthogonal pulse shapes using ideas of Hermite polynomials and prolate spheroidal wave functions, respectively. The method using ideas of prolate spheroidal wave functions can meet the FCC spectral mask without increasing the pulse duration. The resulting pulse durations are time-limited to 1 ns, and are less than the pulse durations of the UWB pulses generated using Hermite polynomials of order 3 or higher. However, the pulse shapes do not achieve the most efficient spectral utilization, and require a higher sampling rate of 64 GHz. Results of show that time-limited UWB pulse shapes using a novel algorithm not only meet the FCC spectral mask, but also provide good BER performance (i.e., comparable to the BER performance of the 6th-order Gaussian monocycle) in multiple access interference environments. Novel orthonormal pulses for high data rate communications proposed in not only meet the FCC spectral mask for indoor UWB systems, but also preserve orthogonality at the correlation receiver.

In this paper, we consider a convex projections-based optimization problem which satisfies all of the desired UWB signal properties: efficient spectral utilization under the FCC spectral mask, time-limitedness, and good autocorrelation. The projections onto convex sets (POCS) technique incorporates all these signal properties into an UWB pulse shape design. The goal is to design new pulses whose temporal and spectral shapes are optimized under the constraints of all these signal properties. The POCS technique is an iterative optimization method that finds a feasible solution consistent with a number of a priori constraints. Constraints are defined using a priori information about the actual signal properties. There has been a number of successful applications of the POCS technique to communications and signal processing.

This paper is organized as follows. Section II addresses the POCS-based pulse shape design for FCC-compliant UWB Radios. In Section III, we discuss simulation results. Section IV gives the conclusions.

II. POCS-Based Pulse Shape Design for FCC-Compliant UWB Radios

The POCS technique numerically generates new UWB pulse shapes using the Gaussian monocycle. The Gaussian monocycle can be written as

\[
g(t) = 2\sqrt{e} A \tau_u \cdot e^{-\frac{t^2}{2\tau_u^2}},
\]

where \( A \) denotes the peak amplitude of the monocycle, \( e \) is the Napierian or natural base of logarithms, and \( \tau_u \) is the time duration between
its minimum and maximum values. The Gaussian monocycle’s duration \( t_{go} \) is approximately given by \( t_{go} = 4\mu \). The Fourier transform of \( g(t) \) is given by
\[
G(f) = \frac{1}{2\sqrt{\pi}} \frac{f}{f_c} e^{-\frac{1}{2}\left(\frac{f}{f_c}\right)^2},
\]
(2)
where \( f_c = 1/\pi \mu \) is the center frequency at which the magnitude spectrum \( |G(f)| \) is maximized.

An UWB pulse shape design can be formulated as finding a pulse shape which satisfies all these signal properties: efficient spectral utilization under the FCC spectral mask, time-limitedness, and good autocorrelation. In this paper, we apply the POCS technique to this UWB pulse shape design. The convex sets used in the POCS technique will be described, based on the UWB signal properties.

We can view \( \lambda \) samples of the Gaussian monocycle \( g(t) \) to be optimized as a vector \( u = [u(0), u(1), \ldots, u(N-1)]^T \in \mathbb{R}^N \), where \( g(n) = g(n T_s) \), for \( n = 0, 1, \ldots, N-1 \), is the discrete-time signal of \( g(t) \). Here \( \mathbb{R}^N \), \( T_s \), and \( T_a \) denote the set of all real ordered \( N \)-tuples, the transpose of a matrix, and the sampling interval, respectively.

2.1 Convex Sets Associated with the UWB Signal Constraints

We use three closed convex sets associated with the UWB signal constraints. Let \( u = [u(0), u(1), \ldots, u(N-1)]^T \in \mathbb{R}^N \). Then the convex sets are described as follows:

- \( C_1 \) denotes the convex set of all vectors \( u \) whose discrete Fourier transforms \( U(k) \) have magnitude less than or equal to a prescribed nonnegative function \( M(k) \) over a set of discrete frequency indices \( I_{DF} \). Here \( M(k) \) is the normalized magnitude spectrum (i.e., Fourier transform mask), with \( \max \{M(k)\} = 1 \), associated with the FCC spectral mask, and the Fourier transform mask is obtained by the square root of the FCC spectral mask.

- \( C_2 \) denotes the convex set of all vectors \( u \) that vanishes outside a set of discrete time indices \( I_{DTi} \).

The discrete time indices \( I_{DTi} \) are associated with a control parameter of time-limitedness, \( t_g \), where \( t_g \) is defined as \( t_g = R_{sd} t_{go} \). Here \( R_{sd} \) is a positive scaling factor.

- \( C_3 \) denotes the convex set of all vectors \( u \) whose absolute projection onto the vector \( h_i = [h_i(0), h_i(1), \ldots, h_i(N-1)]^T \) is bounded by the scalar \( d_i = |g T h_i| \), for \( l = 0, 1, \ldots, N-1 \), over a set of discrete time indices \( I_{DTi} \). Here \( h_i = [g(l), g(l+1), \ldots, g(N-1), 0, 0, \ldots, 0]^T \), for \( l = 0, 1, \ldots, N-1 \), represents the vector \( g \) left-shifted by \( l \) positions and with \( l \) zero paddings in the tail. Also \( u^T h_i \) and \( d_i \) denote the estimated autocorrelation and the design goal control parameter (i.e., the absolute value of the autocorrelation of the Gaussian monocycle), respectively. Thus, this set can control the autocorrelation of the designed pulse, based on the autocorrelation property of the Gaussian monocycle.

2.2 Projection Operators Associated with the Convex Sets

The projection operators presented in [15], [17], and [18] are adapted for the convex sets given in Section 2.1. Let \( x(n) \) be an arbitrary element of the Hilbert space, possibly outside the convex set. Also let \( \hat{g}(n) \) denote the result of the projection of an arbitrary element \( x(n) \) onto a convex set and let \( x(n) \to X(k) \) denote a discrete Fourier transform pair. The projection operators associated with the convex sets are defined as follows.

- Projection operator \( P_1 \) onto \( C_1 \): Let \( I_{DF} = (0, 1, \ldots, N-1) \), then

\[
P_1 \ x(n) = \hat{g}(n) \iff \begin{cases} 
M(k), & \text{if } |X(k)| \leq M(k), \ k \in I_{DF} \\
\frac{M(k)}{|X(k)|} X(k), & \text{if } |X(k)| > M(k), \ k \in I_{DF} \\
0, & \text{otherwise.}
\end{cases}
\]

(3)
Table 1. Normalized effective signal power (NESP) as a function of the control parameter $t_g$.

<table>
<thead>
<tr>
<th>$R_{pd}$</th>
<th>$t_g$ (ns)</th>
<th>NESP (%)</th>
<th>$R_{pd}$</th>
<th>$t_g$ (ns)</th>
<th>NESP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.1115</td>
<td>99.5358</td>
<td>6.00</td>
<td>1.1152</td>
<td>99.1221</td>
</tr>
<tr>
<td>0.80</td>
<td>0.1487</td>
<td>99.5358</td>
<td>8.00</td>
<td>1.4870</td>
<td>98.6426</td>
</tr>
<tr>
<td>1.00</td>
<td>0.1859</td>
<td>99.4276</td>
<td>10.00</td>
<td>1.8587</td>
<td>99.1604</td>
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<tr>
<td>1.50</td>
<td>0.2788</td>
<td>98.9345</td>
<td>12.00</td>
<td>2.2305</td>
<td>99.3798</td>
</tr>
<tr>
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<td>99.0461</td>
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<td>2.6022</td>
<td>99.0195</td>
</tr>
<tr>
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<td>99.1277</td>
<td>16.00</td>
<td>2.9740</td>
<td>99.4267</td>
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<tr>
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<td>0.5576</td>
<td>95.0925</td>
<td>18.00</td>
<td>3.3457</td>
<td>99.4184</td>
</tr>
<tr>
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<td>99.1375</td>
<td>20.00</td>
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<tr>
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<td>98.8853</td>
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</tr>
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<td>5.00</td>
<td>0.9294</td>
<td>99.3766</td>
<td>26.00</td>
<td>4.8327</td>
<td>99.8090</td>
</tr>
</tbody>
</table>

### III. Simulation Results

The POCS-based method has been simulated using the Gaussian monocycle in (1), where $f_c$ is set to 6.85 GHz. The duration of the Gaussian monocycle $t_{go}$ thus becomes $t_{go} = 0.186$ ns. The sampling rate $f_s$ is set to 41.1 GHz with a negligible aliasing error. We use the FCC spectral mask for indoor UWB systems.$^{[9]}$

First, we use the spectrum utilization efficiency as a performance measure. The spectrum utilization efficiency can be measured by the normalized effective signal power (NESP)$^{[6]}$

$$\psi = \frac{\int |G(f)|^2 df}{\int S(f) df} \times 100\%,$$

which is defined as $\psi = \frac{\int |G(f)|^2 df}{\int S(f) df} \times 100\%,$ where $S(f)$ and $F_p$ are the FCC spectral mask and the UWB passband ranging between 3.1 and 10.6 GHz, respectively. Table 1 shows the NESP performance as a function of the control parameter of time-limitedness, $t_g$. The spectrum utilization efficiency is significantly high and nearly constant irrespective of $t_g$. The NESP value ranges from 95.1 to 99.8 % for $0.11 \leq t_g \leq 4.83$ ns. The previous method presented in [6] provides maximum NESP values of 83.8 %, 85.0 %, and 85.5 % for a pulse duration of 1.32 ns, 1.44 ns, and 1.48 ns, respectively, while the previous method proposed in [11] provides a NESP value of approximately 39 % for a pulse duration of 1 ns. In particular, the method presented in [6] gives NESP values of approximately 45 to 77 % for pulse durations of 0.40 to 1.00 ns. Since our designed pulse shapes are not strictly time-limited, it is not easy to compare the NESP values of the designed pulse shapes directly with those of the pulse shapes proposed in [6] and [11] for the same pulse durations. However, it is noteworthy that our new pulse shapes provide significantly high NESP values (i.e., nearly close to 100 %) for all values of $t_g$. This means that the spectral utilization efficiency of our new pulse shapes is actually maximized under the FCC spectral mask.

Next, Figures 1-3 show the designed pulse shapes and their magnitude spectra for the three
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그림 1. 설계된 펄스파형 및 이의 크기 스펙트럼 \( t_g = 0.11 \) ns인 경우
Fig. 1 Designed pulse shape and its magnitude spectrum for the case of \( t_g = 0.11 \) ns.

그림 2. 설계된 펄스파형 및 이의 크기 스펙트럼 \( t_g = 0.19 \) ns인 경우
Fig. 2 Designed pulse shape and its magnitude spectrum for the case of \( t_g = 0.19 \) ns.

그림 3. 설계된 펄스파형 및 이의 크기 스펙트럼 \( t_g = 4.83 \) ns인 경우
Fig. 3 Designed pulse shape and its magnitude spectrum for the case of \( t_g = 4.83 \) ns.
cases: \( t_g = 0.11 \) ns, \( t_g = 0.19 \) ns, and \( t_g = 4.83 \) ns, respectively. It is evident that the magnitude spectra of the designed pulse shapes match the FCC Fourier transform mask \( M(k) \) for these three pulse shapes because of their high NESP values: NESP = 99.54 %, NESP = 99.43 %, and NESP = 99.81 % for \( t_g = 0.11 \) ns, \( t_g = 0.19 \) ns, and \( t_g = 4.83 \) ns, respectively. The Gaussian monocycle, the first derivative of a Gaussian pulse, has a single zero crossing, while our pulse shapes have multiple zero crossings like the \( n \)th derivative of a Gaussian pulse with \( n \) zero crossings\(^{[19]}\). This suggests that our pulse shapes are similar to the \( n \)th derivative of a Gaussian pulse. Also our pulse shapes are DC-free and have significantly less low-frequency components than the Gaussian monocycle.

The normalized ACFs of the designed pulse shapes are illustrated in Fig. 4. The normalized ACFs of our pulse shapes are almost indistinguishable for all values of \( t_g \) and similar to those of the modified Hermite pulses (MHPs) of order \( n = 0, 1, 2, 3 \)\(^{[19]}\). The main difference is that the width of the main peak in the ACF of the MHP becomes narrower as the order of the MHP increases\(^{[19]}\), while the width of the main peak in the ACF of our pulse shape remains nearly unchanged for all values of \( t_g \). This implies that our pulse shapes are less sensitive to timing jitter.

Finally, we compare the bit-error rate (BER) performance for binary time-hopping pulse position modulation (TH-PPM) UWB systems using the designed pulse shapes and the Gaussian monocycle via Monte Carlo simulation. Since the designed pulse shapes and the Gaussian monocycle are not strictly time-limited, they are truncated using a rectangular window with the length of \( T_m = t_h = R_p t_g \) for fair performance comparison. The truncated pulse shapes are strictly time-limited to \( T_m \). The PPM time shift \( \Delta_{PPM} \) is set to the pulse duration \( T_m \). Fig. 5 shows the BER performance as a function of \( E_b/N_0 \) for the two cases; \( N_b = 1 \), \( N_h = 3 \), \( N_p = 5 \), and \( T_f = 100 t_g \): (a) \( T_m = 0.11 \) ns and (b) \( T_m = 0.19 \) ns. Fig. 5 (a) represents the case of \( T_f = 11.15 \) ns (i.e., \( R_s = 89.67 \) Mbps) and \( T_f = 3.72 \) ns, and Fig. 5 (b) represents the case of \( T_f = 18.59 \) ns (i.e., \( R_s = 53.8 \) Mbps) and \( T_f = 6.20 \) ns. Here \( E_b, N_0, N_s, N_b, N_p, T_f, T_s, R_s \), and \( T_c \) denote the signal energy per bit, the noise spectral density, the number of pulses per symbol (or bit), the cardinality of the TH code (or the number of chips over \( T_f \)), the periodicity of the TH code, the frame

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time (or pulse repetition time), the symbol (or bit) interval, the symbol (or bit) data rate, and the chip interval, respectively. It is seen that our pulse shapes outperform the Gaussian monocycle in BER performance. Since the BER performance highly depends on the ACF of the pulse shape, the optimum PPM time shift need to be chosen for more performance improvement.

IV. Conclusions

The POCS technique has been applied to the design of new UWB pulse shapes. It is numerically shown that for all values of the pulse duration, the new UWB pulse shapes not only meet the FCC spectral mask most efficiently, but also have nearly the same ACFs. Our truncated (i.e., strictly time-limited) pulse shapes are also shown to outperform the truncated Gaussian monocycle in the BER performance of binary TH-PPM systems for the same pulse durations. On the other hand, the sampling rate (i.e., 41 GHz) of our method is relatively moderate comparing with that (i.e. 25 GHz) of the method presented in [6] and that (i.e. 64 GHz) of the method proposed in [11]. Moreover, the POCS technique inherently has flexibility and modularity in modeling various constraints without the reformulation of the design problem. Therefore, it is suitable for the UWB pulse shape design in terms of its design flexibility and joint optimization capability.

References


