in multiuser MIMO broadcast channels where each user has more than one antenna in the presence of other cell interference (OCI). Unlike conventional zero-forcing (ZF) based methods, the proposed scheme takes the OCI plus noise into account when suppressing the inter-cell multiuser interference, which results in improvement of the received signal-to-interference-plus-noise ratio. Simulation results show that the proposed scheme outperforms conventional methods in terms of sum rate for various OCI configurations.

I. Introduction

Multiuser multiple-input multiple-output (MIMO) techniques are one of key features of next generation wireless communications\(^\text{[1]}\). In single-cell multiuser broadcast channels, it was extensively investigated that the sum capacity can be achieved by using dirty paper coding (DPC)\(^\text{[2]}\), and a zero-forcing DPC (ZFDPC) scheme has been proposed as a practical suboptimal DPC method for

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the single receive antenna case\textsuperscript{[1]}. Also, in [3], a successive zero-forcing DPC (SZFDPC) scheme has been introduced as a generalization of the ZFDPC for the multiuser MIMO case\textsuperscript{[4]}. In this paper, we consider multi-cell multiuser MIMO networks where users experience interference from neighboring cells\textsuperscript{[5-7]}. In this environment, the performance of the schemes in [1] and [3] can be degraded significantly due to other cell interference (OCI), since these ZF based schemes identify the multiuser interference (MUI) suppressing precoder without considering the OCI and noise.

In this paper, we propose a minimum mean-squared error (MMSE) based successive DPC precoding algorithm in the presence of the OCI. In contrast to the SZFDPC, the proposed method computes an orthonormal vector set of the MUI suppression precoding matrix based on the MMSE channel inversion (MMSE-CI) method in [8] which takes the OCI plus noise into account. Then, applying the transmit combining matrix obtained under an MMSE criterion to the orthonormal vector set, the proposed scheme is able to increase the signal-to-interference-plus-noise ratio (SINR) at each user’s receiver. Consequently, the proposed scheme can be considered as an extension of the MMSE based block diagonalization (MMSE-BD) method in [9] and [10] to apply the DPC technique. From simulation results, we show that the sum rate performance of the proposed scheme outperforms that of the conventional methods over all OCI configurations.

Throughout this letter, we employ uppercase boldface letters for matrices and lowercase boldface for vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^T$, $\mathbf{A}^H$, and $\text{Tr}(\mathbf{A})$ denote the transpose, the conjugate transpose, and the trace operation, respectively.

II. System Model

We consider multiuser MIMO downlink systems where the base station is equipped with $N_t$ transmit antennas and there are $K$ users with $N_r \geq 1$ receive antennas. In this system, we assume that each user experiences the interference transmitted from neighboring cells and denote the number of interference data streams which affect user $j$ as $N_{i,j}$. Also, we define $\mathbf{s}_j$ and $\mathbf{H}_{i,j}$ as the OCI signal vector of length $N_t$ with average power $P_{i,j}$ and the $N_r \times N_r$ OCI channel matrix, respectively.

According to this model, defining the $N_r \times N_r$ receive filter as $\mathbf{M}_j$, the $j$th user’s receive filter output vector $\mathbf{x}_j$ in the presence of OCI can be expressed as

$$\mathbf{x}_j = \mathbf{M}_j \mathbf{H}_j \mathbf{P}_j \mathbf{s}_j + \mathbf{M}_j \mathbf{H}_j \sum_{k \neq j} P_{k,j} \mathbf{s}_k + \mathbf{M}_j \mathbf{w}_j$$

(1)

where $\mathbf{H}_j$ is the $N_r \times N_t$ channel matrix, $\mathbf{s}_j$ represents the transmitted data symbol vector for user $j$, $\mathbf{P}_j$ indicates the corresponding MUI suppression precoding matrix, and $\mathbf{w}_j$ denotes the OCI plus noise vector as $\mathbf{w}_j = \mathbf{H}_{i,j} \mathbf{s}_{i,j} + \mathbf{z}_j$. Here, $\mathbf{z}_j$ is the noise vector of length $N_r$ with independent and identically distributed (i.i.d.) entries according to $N_r(0, \sigma_z^2)$. Also, we assume that each data symbol of $\mathbf{s}_j$ has unit variance. Then the total transmit power constraint is given by

$$\sum_{k=1}^K \text{Tr}(\mathbf{P}_k \mathbf{P}_j) \leq P_j.$$

Assuming that the interfering signal from other cell is independent with the noise, the $j$th user’s OCI plus noise covariance matrix is given by

$$\mathbb{E}[\mathbf{w}_j \mathbf{w}_j^H] = \mathbf{H}_{i,j} \mathbf{O}_{i,j} \mathbf{H}_{i,j}^H + \sigma_z^2 \mathbf{I}_{N_r}$$

(2)

where $\mathbf{O}_{i,j} = \mathbb{E}[\mathbf{s}_{i,j} \mathbf{s}_{i,j}^H]$ and $\text{Tr}(\mathbf{O}_{i,j}) = P_{i,j}$. In this paper, we assume that each user can estimate the OCI plus noise covariance matrix by various methods explained in [5] and the covariance information can be reported to the base station. The effect of imperfect covariance estimation is an
interesting topic, but is beyond the scope of this paper.

### III. Proposed MMSE Based Successive Precoding

In this section, with a given user ordering, we illustrate a procedure for identifying the proposed algorithm through the three stage process. In the first stage, based on the MMSE-CI\[8\], which takes the OCI plus noise into account, we find a set of orthonormal basis vectors of the MUI suppression precoding matrix. From (2), we can see that the OCI plus noise is correlated. Thus, in order to apply the MMSE-CI, we first whiten the OCI plus noise term using the Cholesky factorization as

\[
H_{t,i}O_{r,i}H_{t,i}^H + \sigma_N^2 I_{N_c} = L_{i}^H L_{i}.
\]

Then, denoting the \( j \) th user’s receive whitening filter \( \mathbf{M}_j = L_{j}^H \), the receive dsignal vector with the uncorrelated noise of user \( j \) can be written from (1) as

\[
\mathbf{x}_j = H_{e,j} \mathbf{P}_j \mathbf{s}_j + H_{e,j} \sum_{k < j} \mathbf{P}_k \mathbf{s}_k + H_{e,j} \sum_{k > j} \mathbf{P}_k \mathbf{s}_k + \mathbf{w}_e,j
\]

(3)

where \( H_{e,j} = \mathbf{M}_j H_j \) and \( \mathbf{w}_e,j = \mathbf{M}_j \mathbf{w}_j \).

In this case, one of the MUI terms \( H_{e,j} \sum_{k < j} \mathbf{P}_k \mathbf{s}_k \) can be canceled by using the DPC technique. Thus we identify the modified \( N_j \times N_r \) MMSE-CI matrix \( \mathbf{F}_j \) which suppresses the MUI for user \( 1,\ldots,j-1 \) as

\[
\mathbf{F}_j = (\bar{\mathbf{H}}_{e,j}^H \bar{\mathbf{H}}_{e,j} + \alpha \mathbf{I})^{-1} \bar{\mathbf{H}}_{e,j}^H
\]

(4)

where \( \alpha \) represents the ratio of the total noise variance to the total transmit power, i.e.,

\[
\alpha = \mathbb{E}[\| \mathbf{w}_e \|^2] / P_t = K \cdot N_c / P_t[8],
\]

and the other users’ channel \( \bar{\mathbf{H}}_{e,j} \) is defined as

\[
\bar{\mathbf{H}}_{e,j} = [\mathbf{H}_{e,1}^H \mathbf{H}_{e,2}^H \cdots \mathbf{H}_{e,j-1}^H]^T.
\]

Here, denoting \( \mathbf{f}_{k,j} \) as the \( k \) th column vector of \( \mathbf{F}_j \) for \( k = 1,\ldots,N_r \), \( \mathbf{f}_{k,j} \) not only mitigates other users’ interference, but also attempts to suppress the signal among antennas of the \( j \)th user. Thus, if we use \( \mathbf{F}_j \) as a precoding matrix, there should be a performance loss when each user has multiple receive antennas. To overcome this issue, we compensate the suppression of each antenna signal of the \( j \)th user by applying the orthogonalization procedure to \( \mathbf{F}_j \) [10]. For orthogonalization, we employ the QR decomposition as

\[
\mathbf{F}_j = \mathbf{Q}_j \mathbf{R}_j \quad \text{for} \quad j = 1,\ldots,K,
\]

where the \( N_r \times N_r \) matrix \( \mathbf{Q}_j \) is composed of \( N_r \) orthonormal basis vectors of \( \mathbf{F}_j \). Then, the \( j \)th user’s precoding matrix \( \mathbf{P}_j \) of the proposed scheme can be constructed by a linear combination of columns of \( \mathbf{Q}_j \) which will be described in detail later.

Note that if there is no consideration of the OCI signals, \( \mathbf{M}_j \) becomes \( \mathbf{I}_{N_c} \). In this case, equation (4) can be written as

\[
\mathbf{F}_j = (\bar{\mathbf{H}}_{e,j}^H \bar{\mathbf{H}}_{e,j} + \alpha I)^{-1} \bar{\mathbf{H}}_{e,j}^H
\]

where \( \bar{\mathbf{H}}_{e,j} \) is defined as \( \bar{\mathbf{H}}_{e,j} = [\mathbf{H}_{e,j}^H \cdots \mathbf{H}_{e,j-1}^H]^T \). From the matrix inversion lemma, this can be expressed as

\[
\mathbf{F}_j = \frac{1}{\alpha} \left( I - \bar{\mathbf{H}}_{e,j}^H (\bar{\mathbf{H}}_{e,j} \bar{\mathbf{H}}_{e,j}^H + \alpha \mathbf{I})^{-1} \bar{\mathbf{H}}_{e,j} \right) \bar{\mathbf{H}}_{e,j}^H.
\]

Noticing that the orthonormal basis vectors of \( \left( I - \bar{\mathbf{H}}_{e,j}^H (\bar{\mathbf{H}}_{e,j} \bar{\mathbf{H}}_{e,j}^H + \alpha \mathbf{I})^{-1} \bar{\mathbf{H}}_{e,j} \right) \) span the null space of \( \bar{\mathbf{H}}_{e,j} \), the \( j \)th user’s precoder constructed by a linear combination of \( \mathbf{Q}_j \) becomes equivalent to the conventional SZFDPC method in [3] when \( \mathbf{M}_j = \mathbf{I}_{N_c} \) for high signal-to-noise-ratio (SNR). In contrast, our scheme exploits the OCI plus noise to form the modified MMSE-CI in (4) and the MUI suppression precoding matrix is obtained based on this. Thus, the proposed method is able to improve the sum rate performance over the conventional SZFDPC method.
In the second stage, we determine the transmit combining matrix which is multiplied to $Q_j$. For the ZF based schemes in [1] and [3], the water-filling (WF) solution is utilized to maximize the sumrate. However, in the proposed method, the $j$th user’s precoder constructed by a linear combination of $Q_j$ generates residual interference, and the conventional WF is not feasible since the residual interference may vary according to the power allocation. Thus, as a counterpart of the WF solution for the ZF based scheme, we introduce a precoder combining method which minimizes the total mean squared error (MSE) in the following.

Denoting $T_j$ as $N_t \times N_r$ transmit combining matrix applied to $Q_j$, the known MUI canceled signal vector of the $j$th user is given from (3) as

$$x_j^{\text{dpc}} = H_{e_j}Q_jT_js_j + H_{e_j}\sum_{k:j\neq k}Q_kT_ks_k + w_{e,j}$$ (5)

Let us again define the $j$th user’s combining matrix $T_j$ as $\beta T_j$, where the scaling parameter $\beta$ is used to fulfill the total transmit power constraint as

$$\beta = \sqrt{P \left[ \sum_{j=1}^{K} \text{Tr} (T_j^HQ_j^HQ_j) \right]^{-1}}.$$

Then, the MSE of the $j$th user is represented as

$$\mathbb{E} \left[ \| \Omega_j s_j - \frac{1}{\beta} x_j^{\text{dpc}} \|^2 \right]$$ (7)

where $\Omega_j$ is the $N_t \times N_r$ target channel matrix based on the MMSE target channel matrix based on the MMSE criterion, which is defined later. Thus, from (5) and (7), the total MSE minimization problem on $T_j$ is written as

$$\min_{T_j} \sum_{j=1}^{K} \mathbb{E} \left[ \| \Omega_j s_j - H_{e_j} \sum_{k:j}Q_kT_k s_k - \frac{1}{\beta} w_{e,j} \|^2 \right].$$ (8)

Also, applying (6) to the cost function in (8), this problem can be formulated as

$$\min_{T_j} \sum_{j=1}^{K} \text{Tr} (H_{e_j} \sum_{k:j}Q_k \tilde{T}_j^HQ_k^HQ_kH_{e_j}^H)$$

$$+ \frac{1}{P_j} \sum_{k:j} \text{Tr} (T_k^HQ_k^HQ_k)$$

$$- H_{e_j} \sum_{k:j} \tilde{T}_j^HQ_k^HQ_kH_{e_j}$$

$$= \mathbb{E} \left[ \| \Omega_j s_j - \frac{1}{\beta} x_j^{\text{dpc}} \|^2 \right].$$ (8)

Let us again define the $j$th user’s combining matrix $T_j$ as $\beta T_j$, where the scaling parameter $\beta$ is used to fulfill the total transmit power constraint as

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Also, applying (6) to the cost function in (8), this problem can be formulated as

$$\min_{T_j} \sum_{j=1}^{K} \text{Tr} (H_{e_j} \sum_{k:j}Q_k \tilde{T}_j^HQ_k^HQ_kH_{e_j}^H)$$

$$+ \frac{1}{P_j} \sum_{k:j} \text{Tr} (T_k^HQ_k^HQ_k)$$

$$- H_{e_j} \sum_{k:j} \tilde{T}_j^HQ_k^HQ_kH_{e_j}$$

$$= \mathbb{E} \left[ \| \Omega_j s_j - \frac{1}{\beta} x_j^{\text{dpc}} \|^2 \right].$$ (8)

Now, we take a derivative of the above cost function with respect to $T_j$ and set it to zero. Then, this results in

$$T_j = (Q_j^HH_jH_{e_j}Q_j + \alpha \mathbf{I}_{N_r})^{-1}Q_j^HH_{e_j}H_{e_j}Q_j$$ (9)

Here, the target channel matrix of the $j$th user $\Omega_j$ is employed as $H_{e_j}Q_j \tilde{T}_{high,j}$ from (5) where $\tilde{T}_{high,j}$ is denoted as the $j$th user’s combining matrix at high SNR, which becomes $\mathbf{I}_{N_r}$. Since

$$Q_j^HH_jH_{e_j}Q_j$$ in (9) becomes $Q_j^HH_{e_j}H_{e_j}Q_j$ at high SNR, it is obvious that $\tilde{T}_j$ converges to an identity matrix as SNR increases.

In the final stage, after finding $Q_j$ and $T_j$, we now make each user’s received signal vector single symbol decodable. In (5), the term $H_{e_j}Q_jT_j$ represents the interference suppressed block channel of the $j$th user. In order to decompose this channel into parallel subchannels, we apply the singular value decomposition of $H_{e_j}Q_jT_j$ as

$$H_{e_j}Q_jT_j = U_jA_jV_j^H.$$

Then, the $j$th user’s MUI suppression precoding matrix and the receive filter of the proposed scheme are obtained as

$$P_j = Q_jT_jV_j$$ and $$M_j = U_j^HL_j^H.$$

Note that, the block channel decomposing matrices $V_j$ and $U_j$ do not affect the total MSE in
(8) since they are unitary.

Consequently, after applying the above solutions to (1) and employing the DPC technique, the known MUI canceled receive filter output signal vector at the $j$th user can be written as

$$x_{j}^{\text{DPC}} = \Lambda_j s_j + M_j H \sum_{k<j} P_k s_k + M_j w_j$$

and the SINR of each stream can be expressed as

$$\text{SINR}_{j,i} = \frac{\lambda_{j,i}^2}{\left| m_{j,i} w_j \right|^2 + \sum_{k<i} \left| m_{j,i} H P_k \right|^2}$$

where $m_{j,i}$ is the $i$th row vector of $M_j$ and $\lambda_{j,i}$ denotes the $i$th diagonal element of $\Lambda_j$. Then, the sum rate of the proposed scheme is given by [9,11]

$$R = \sum_{j=1}^{K} \sum_{i=1}^{N_j} \log_2(1 + \text{SINR}_{j,i})$$

Note that the complexity of the proposed scheme is comparable with that of the SZFDPC, but is higher than the BD methods.

IV. Numerical Results

In this section, we present the sum rate of the proposed DPC scheme under various OCI configurations compared to the SZFDPC with the WF solution in [3], the MMSE-BD method in [9] and the ZF-BD scheme in [5]. We use an arbitrary user ordering for both DPC schemes, and evaluate all simulations for the case of $K=3$, $N_r=6$ and $N_t=2$. All channel matrices are assumed to be an uncorrelated MIMO Rayleigh fading channel. Also, we define the OCI-to-noise ratio of the $j$th user’s as

$$\text{INR}_j = P_j / \sigma_j^2$$

in dB. We denote $\text{INR}_j$, and $N_i$ as

$$\text{INR}_j = [\text{INR}_{1j}, \text{INR}_{2j}, \text{INR}_{3j}]$$

and

$N_i = [N_{i,1}, N_{i,2}, N_{i,3}]$, respectively.

Figures 1 and 2 illustrate the sum rates as a function of SNR and INR, respectively. As shown in these figures, even though the proposed MMSE-based DPC method has a small gain over the SZFDPC because of the noise enhancement even in the system with no OCI, this gain grows as the OCI power increases. This comes from the fact that the ZF based scheme in [3] tries to cancel the MUI without consideration of OCI as well as noise, whereas the proposed method identifies the precoding matrix in an OCI-aware manner. Since the conventional BD methods in [9] and [5] also take the OCI into account, both the schemes without using the DPC encoding can be more robust than the SZFDPC scheme for large OCI power. In conclusion, we can confirm that the proposed method outperforms the conventional schemes for various OCI configurations and SNR regions.

V. Conclusion

This paper has studied an MMSE based
successive precoding method as a practical method of DPC for a multiuser downlink network equipped with multiple receive antennas. The proposed algorithm identifies an orthonormal vector set of the precoding matrix taking the noise and OCI into account. Finally, in simulation results, it has been shown that the proposed nonlinear algorithm provides the improved sum rate performance over the conventional schemes in multiuser MIMO broadcasting channels for various configurations.

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