On the Starvation Period of CDF-Based Scheduling over Markov Time-Varying Channels

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ABSTRACT

In this paper, we consider a cumulative distribution function (CDF)-based opportunistic scheduling for downlink transmission in a cellular network consisting of a base station and multiple mobile stations. We present a closed-form formula for the average starvation period of each mobile station (i.e., the length of the time interval between two successive scheduling points of a mobile station) over Markov time-varying channels. Based on our formula, we investigate the starvation period of the CDF-based scheduling for various system parameters.

Key Words : CDF-based scheduling, starvation period, multiuser diversity, Markov channel

I. Introduction

Opportunistic scheduling has been studied extensively in the last decade; it can maximize the sum throughput in wireless networks by selecting the user who has the largest channel gain at each time-slot[1]. As a result, a user having higher signal-to-noise ratio (SNR) on average is scheduled more frequently, which leads to unfairness[2]. A cumulative distribution function (CDF)-based opportunistic scheduling[3] was proposed to resolve the fairness problem while achieving high throughput by utilizing a set of weight parameters, and has been extended to various networks in recent work[1].

Along with the fairness, another important feature inherent in the opportunistic scheduling is starvation: a user may wait for a long time until it experiences peaks in its channel gain, which contrasts with non-opportunistic policies such as the round-robin scheduling. Such starvation can become severe in wireless networks with time-correlated channels. Note that the starvation period (i.e., the length of the time interval between two successive scheduling points of a user) determines the head-of-line packet delay, and thus understanding the starvation period helps to control delay performance.

In this paper, we present a closed-form expression for the average starvation period of the CDF-based scheduling over Markov time-varying channels. Through numerical studies, we investigate the starvation period for various system parameters.

II. System Model

We consider downlink in a cellular network consisting of one base station (BS) and N mobile stations (MSs). We assume that the BS always has traffic to send to each MS.

2.1 Wireless channel model

Let $\gamma_n(t)$ be the SNR of the wireless channel between the BS and MS $n$ at time-slot $t$. We assume that $\gamma_n(t)$ is a random variable having a general CDF $F_n(x) := P(\gamma_n(t) \leq x)$. To describe the time-correlated wireless channel, we use a two-state Markov model (also known as Gilbert-Elliott channel) as follows. Let $C_n(t)$ be the channel state of MS $n$ at time-slot $t$. Then, $C_n(t) = 0$ if $\gamma_n(t) < l$ and $C_n(t) = 1$ if $\gamma_n(t) \geq l$ for a threshold $l > 0$. When $C_n(t) = k$, $k \in \{0, 1\}$, and MS $n$ is scheduled at time-slot $t$, the BS transmits packets to MS $n$ with the transmission rate $r_k$ where $r_0 < r_1$. The process $\{C_n(t)\}_{t=0}^\infty$ is assumed to be stationary and forms a Markov chain with the transition probability matrix

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**Fig. 1. Example of starvation period:** $T_1 = 4$, $T_2 = 2$

period provided that its initial slot has the channel state $C_n(h) = k$. To find $E[T_n^o]$, we introduce three events $e_i (i = 0, 1, 2)$:

$$
\begin{align*}
    e_0 & := \{n^*(h+1) = n\}, \\
    e_1 & := \{n^*(h+1) = n\} \cap \{C_n(h+1) = 0\}, \\
    e_2 & := \{n^*(h+1) = n\} \cap \{C_n(h+1) = 1\}.
\end{align*}
$$

By the law of total probability, we have

$$
E[T_n^o] = E[T_n^o | C_n(h) = k] \cdot P(e_i | C_n(h) = k).
$$

Applying first-step analysis, we then obtain

$$
E[T_n^o | e_i, C_n(h) = k] = \begin{cases} 1 & i = 0, \\ 1 + E[T_n^o] & i = 2, \\ 1 + E[T_n^o] & i = 2, \\ 1 & i = 1, \\ 1 & i = 2. 
\end{cases}
$$

In matrix form, above equations are expressed as

$$
\begin{pmatrix}
    E[T_n^o] \\
    E[T_n^o]
\end{pmatrix} = \begin{pmatrix} 1 & 1 + E[T_n^o] & 1 + E[T_n^o] \\
\end{pmatrix} \begin{pmatrix} P_{0,0} s_n^0 + P_{0,1} s_n^1 & 0 \\
0 & 1 - s_n^1
\end{pmatrix} \begin{pmatrix} E[T_n^o] \\
E[T_n^o]
\end{pmatrix},
$$

from which we obtain

$$
\begin{pmatrix}
    E[T_n^o] \\
    E[T_n^o]
\end{pmatrix} = \left(I - A_n \begin{pmatrix} 1 - s_n^0 & 0 \\
0 & 1 - s_n^1
\end{pmatrix}\right)^{-1} \begin{pmatrix} 1 \\
1
\end{pmatrix},
$$

where $I$ is the $2 \times 2$ identity matrix. To find $A_n^k$, we introduce

$$
B_n := \{n^*(t-1) = n\} \cap \{n^*(t) \neq n\},
$$
i.e., the event that time-slot $t$ is the initial slot of the starvation. Due to Markov property and the stationarity, we have $\alpha_n^k = P(C_n(t) = k|B_t)$ for any $t$. Hence,

$$\alpha_n^k = \frac{P(C_n(t) = k; B_t)}{P(C_n(t) = 0; B_t) + P(C_n(t) = 1; B_t)} = \frac{\beta_n^k}{\beta_n^0 + \beta_n^1},$$

where $\beta_n^k := P(C_n(t) = k; B_t)$. By conditioning on the state pair $\{C_n(t), C_n(t-1)\}$ and then applying $P(C_n(t) = k; C_n(t-1) = k') = \pi_n^{k'} \cdot p_{k,k'}$, we obtain

$$\beta_n^k = \sum_{k' = 0}^{1} P(C_n(t) = k; C_n(t-1) = k', B_t) = \sum_{k' = 0}^{1} P(B_t|C_n(t) = k, C_n(t-1) = k') \cdot \pi_n^{k'} \cdot p_{k,k'}.$$
\( \langle w_1, \ldots, w_{10} \rangle = \frac{1}{55} (1, \ldots, 10) \) and \( \rho_n = 0.5 \) for all \( n \). Fig. 3 shows that the average starvation period decreases convexly with \( w_n \). Note that the weight \( w_n \) represents the channel access probability of MS \( n \) by (1). Hence, Fig. 3 also indicates that decreasing the channel access probability lower than \( 1/(2N) \) significantly increases the starvation period. Our analytic results match the simulation results well, verifying the correctness of our analysis.

V. Conclusion

In this paper, we present a formula for the average starvation period of the CDF-based scheduling. Based on the formula, we investigate the impact of the channel correlation and the weight on the starvation period. Extension of our analysis to a general finite-state Markov channel is our future work.

References


