Input Power Normalization of Zero-Error Probability based Algorithms

Chong-il Kim*, Namyong Kim*

ABSTRACT

The maximum zero error probability (MZEP) algorithm outperforms MSE (mean squared error)-based algorithms in impulsive noise environment. The magnitude controlled input (MCI) which is inherent in that algorithm is known to plays the role in keeping the algorithm undisturbed from impulsive noise. In this paper, a new approach to normalize the step size of the MZEP with average power of the MCI is proposed. In the simulation under impulsive noise with the impulse incident rate of 0.03, the performance enhancement in steady state MSE of the proposed algorithm, compared to the MZEP, is shown to be by about 2 dB.

I. Introduction

In supervised signal processing such as in most equalization applications, the least mean square (LMS) algorithm is widely used for its simplicity and effectiveness[1]. One of the problems of the LMS algorithm is the step size parameter fixed at every iteration time. The fixed step size requires information of the statistics of the input signal such as signal input power and amplitude that affect its performance[2]. As an extension of the LMS algorithm, the normalized least mean square algorithm (NLMS) where its step size is proportional to the inverse of the dot product of the input vector with itself has been proven to have more enhanced performance[3,4].

As an information theory based criterion, unlike the MSE criterion, the zero-error probability (ZEP) and its related algorithms outperform MSE-based algorithms and yield superior and stable convergence in impulsive noise environment[5]. The nonlinear version of MZEP has been proposed for underwater
communication channels and known to compensate successfully for ISI without error propagation\cite{6}. Also for practical implementation a recursive gradient estimation method has been proposed for the MZEP algorithm to reduce its computational complexity significantly\cite{7}. In the recent study\cite{8}, it has been revealed that the ZEP criterion has equivalent optimum solution of MSE criterion and the magnitude controlled input (MCI) of MZEP algorithm plays the role of keeping the optimum solution undisturbed from impulsive noise.

Though the role of MCI in the MZEP algorithm under impulsive noise has been introduced, any application of the MCI to performance enhancement has not been carried out. In this paper, based on the NLMS approach, a normalized step size for the MZEP is investigated where the normalized step size is proportional to the inverse of the dot product of the MCI with itself. And then, through simulation under impulsive noise with different rates of impulse occurrence, it will be shown that the normalized step size employing the information of MCI can enhance the performance of MZEP significantly.

II. MSE Criterion and LMS algorithm

In communication systems, a symbol point $d_k$ at symbol time $k$ is transmitted through the wireless channel $H(z) = \sum h_z z^{-i}$ and noise $n_k$ is added to the channel output. Then the received signal $x_k$ is composed of the channel output and noise $n_k$ as

$$x_k = \sum h_z d_{k-i} + n_k \quad (1)$$

Assuming that the equalizer has the TDL (tapped delay line) structure with weights, $W_k = [w_{0,k}, w_{1,k}, \ldots, w_{L-1,k}]^T$ and input buffer $X_k = [x_k, x_{k-1}, \ldots, x_{k-L+1}]^T$, the equalizer output $y_k$ at time $k$ becomes $y_k = W_k^T X_k$. Then the error $e_k$ and the MSE criterion $P_{MSE}$ is defined in $[3]$ as

$$e_k = d_k - y_k = d_k - W_k^T X_k \quad (2)$$

$P_{MSE} = E[e_k^2] \quad (3)$

The expectation or mean operation $E[\cdot]$ in (3) can mitigate the influence of the Gaussian noise. But in case of impulsive noise, a single large impulse can dominate the mean operation so that the averaging operation may not be effective to defeat the impulsive noise.

As a practical algorithm developed based on the MSE criterion, the LMS (least mean square) is to use the instant error power $e_k^2$ instead of $E[e_k^2]$\cite{3}. With the instant gradient $\frac{\partial e_k^2}{\partial W}$ and a step size $\mu$, the LMS algorithm can be expressed as

$$\frac{\partial e_k^2}{\partial W} = 2e_k \frac{\partial (d_k - y_k)}{\partial W}$$

$$= -2e_k X_k = -2(d_k - X_k W_k^T)X_k \quad (4)$$

$$W_{k+1} = W_k - \mu \frac{\partial e_k^2}{\partial W}$$

$$= W_k + 2\mu e_k X_k \quad (5)$$

By letting the gradient $\frac{\partial e_k^2}{\partial W}$ in (3) be zero, the optimum weight $W_{LMS}$ can be obtained.

$$W_{LMS}^* = \frac{d_k X_k}{X_k^T X_k} \quad (6)$$

III. Normalized LMS algorithm

One of the problems of the LMS algorithm is the step size parameter $\mu$ fixed at every iteration time. The fixed step size requires information of the statistics of the input signal such as signal input power and amplitude that affect its performance\cite{2}. The step size $\mu_{NLMS}$ in the normalized least mean square algorithm (NLMS) as an extension of the LMS algorithm is defined in [3] as proportional to the inverse of the dot product of the input vector with itself.
The denominator \( \sum_{n=0}^{k} x_{k-n} \) is also equivalent to the sum of the expected energies of the input samples, \( \mu \sum_{n=0}^{k} x_{k-n} \). With an additional parameter \( \mu \) for freedom of adaptation, the NLMS algorithm can be expressed as

\[
W_{k+1} = W_k + \mu_{NLMS} \cdot e_k X_k
\]  

where the magnitude-controlled input (MCI), \( X^d_k \) is defined in [8] as

\[
X^d_k = G_e(e_k) \cdot X_k
\]

Since the Gaussian kernel \( G_e(e_k) \) is a function of an exponential decay with the error power \( e_k^2 \), large error power due largely to strong impulses involved within the input \( X_k \) is mitigated through \( G_e(e_k) \). This indicates that the MCI, \( X^d_k \) through the magnitude control process \( G_e(e_k) \cdot X_k \) in (12) can prevent system instability that might be induced by large input values contaminated with impulsive noise[8].

On the other hand, the gradient in (11),

\[
\frac{\partial P_{ZEP}}{\partial W} = -\frac{1}{\sigma^2 N \sum_{j=k-N+1}^{k} e_j X^d_j} \text{ can be written as}
\]

\[
\frac{\partial P_{ZEP}}{\partial W} = \frac{1}{\sigma^2 N \sum_{j=k-N+1}^{k} (d_j - X^d_j W_i) \cdot X^d_j}
\]

Letting the gradient (13) be zero, we obtain the optimum weight \( W_{MZE}^\omega \) for MZEP algorithm as

\[
W_{MZE}^\omega = \frac{\sum_{j=k-N+1}^{k} d_j X^d_j}{\sum_{j=k-N+1}^{k} X^d_j X^d_j}
\]

In the work[8], it has been proven that the statistical average of the optimum weights of MZEP and LMS are equal in the steady state that most error samples are assumed to be at around zero. That is,

\[
E[W_{MZE}^\omega] = E[W_{LMS}^\omega]
\]  

While MZEP based on ZEP criterion and LMS based on MSE have the same optimum solution, the
behavior of steady state weight under impulsive noise situations have been shown to be significantly different in the work\[8\]. The trace of the steady state weight for MZEP algorithm has stayed in the value of $W_{\text{ZEP}}$ thanks to MCI $X_{d}^{\text{tr}}$, whereas that of LMS algorithm being afflicted directly by large impulses has fluctuated wildly.

V. Power Estimation of MCI for Normalized Step Size

Similar to the NLMS where the step size is normalized by the averaged power of the current input samples as presented in Section 3, we propose the data-dependent step size which is normalized by the average of the expected energies of the current MCI $X_{\text{MCI}}^{\text{tr}}$ for greater stability with unknown signals and robustness against impulsive noise as well.

$$W_{k+1} = W_{k} + \frac{\mu_{\text{ZEP}}}{\sum_{i=k-N+1}^{k} |x_{i}^{d}|^{2}} \frac{1}{\sigma^{2}} \sum_{i=k-N+1}^{k} e_{i} X_{i}^{d} \quad (16)$$

where $x_{i}^{d} = G_{\text{MCI}}(e_{i})x_{i}$ is from (12).

Noticing the fact that averaging operation may not be effective to defeat the impulsive noise as explained in Section 3, we may observe that the denominator of (16) may be sensitive to impulsive noise. To avoid this kind of situation, we propose to track the averaged energy $E(k)$ recursively as

$$E(k) = \beta \cdot E(k-1) + (1-\beta) \cdot |x_{i}^{d}|^{2} \quad (17)$$

The equations (17) can be expressed as a $z$-transformed system $A(z)$ with its input $|x_{i}^{d}|^{2}$ and output $E(k)$.

$$A(z) = (1-\beta) \frac{z}{z-\beta} \quad (18)$$

The time constant of the single-pole low-pass filter $A(z)$ is controlled by the parameter $\beta$ ($0 < \beta < 1$). Then the normalized MZEP (NMZEP) algorithm with a constant $\mu_{\text{NMZEP}}$ becomes

$$W_{k+1} = W_{k} + \frac{\mu_{\text{NMZEP}}}{E(k)} \sum_{i=k-N+1}^{k} e_{i} X_{i}^{d} \quad (19)$$

VI. Results and Discussion

In this section, it will be investigated how the NMZEP equipped with MCI behave under different impulsive noise situations. The impulsive noise $n_{i}$ in (1) is composed of the background Gaussian noise with its variance $\sigma_{\text{GN}}^{2}$ and impulses. The amplitude distribution of impulses is a Gaussian with variance $\sigma_{\text{IN}}^{2}$ and the impulses are generated according to a Poisson process with its incident rate $\epsilon$\[9\].

$$f_{\epsilon}(n_{i}) = \frac{1-\epsilon}{\sigma_{\text{GN}} \sqrt{2\pi}} \exp\left[-\frac{n_{i}^{2}}{2\sigma_{\text{GN}}^{2}}\right]$$

$$+ \frac{\epsilon}{\sqrt{2\pi(\sigma_{\text{GN}}^{2} + \sigma_{\text{IN}}^{2})}} \exp\left[-\frac{n_{i}^{2}}{2(\sigma_{\text{GN}}^{2} + \sigma_{\text{IN}}^{2})}\right] \quad (20)$$

We take the same simulation environment as in [8] except that impulse noise models have different incident rates, $\epsilon = 0.01$ depicted as a sample in Fig. 1 and $\epsilon = 0.03$ in Fig. 4, respectively. The random symbol point $d_{i}$ from the set $\{d_{i} = -3, d_{z} = -1, d_{s} = 1, d_{4} = 3\}$ is transmitted through the multipath channel $H(z) = 0.304 + 0.903 z^{-1} + 0.304 z^{-2}$\[10\]. The TDL equalizer has 11 weights ($L = 11$). The sample size $N$, the kernel size $\sigma$ for the MZEP and NMZEP is 20 and 0.7, respectively. The step-size $\mu_{\text{ZEP}}$, $\mu_{\text{NMZEP}}$ and $\mu_{\text{LMS}}$ are 0.004, 0.008 and 0.0002, respectively. The forgetting factor $\beta$ and the initial $|x_{0}^{d}|^{2}$ for NMZEP are set to be 0.9 and 1.0, respectively.

We observe in Fig. 2 that due to the large impulse noise spikes shown in Fig. 1, the LMS fails to converge below -20 dB even for the small step-size. On the other hand, the MZEP type...
algorithms converge well. Compared to the MZEP, the curve of the proposed NMZEP algorithm reaches lower steady state MSE than the MZEP showing about 1 dB enhancement. In the Fig. 3, this performance difference can be noticed in the aspect of error probability density.

On the other hand, in the situation of the impulsive noise with \( \varepsilon = 0.03 \) as in Fig. 4, the performance difference between NMZEP and MZEP is clearer as in Fig. 5 and 6. The difference of steady state MSE for the MZEP and NMZEP algorithms is about 2 dB. In the comparison of error probability density, the performance difference can be observed more clearly.

From the results of MSE learning curves and error distribution, we find that the normalized step size with the information of MCI \( X_{m,k}^{MCI} \) can serve
Fig. 6. Error distribution for the impulsive noise with $\varepsilon = 0.03$ (red: LMS, green: MZEP, blue: NMZEP).

MZEP based algorithms to produce performance enhancement significantly. And the MCI contributes more to performance enhancement in the NMZEP algorithm as the impulsive noise is severer.

VII. Conclusion

The MZEP algorithm outperforms MSE-based algorithms in impulsive noise environment and the magnitude controlled input, MCI inherent in that algorithm, plays the role in keeping the algorithm undisturbed from impulsive noise. In this paper, a normalization approach to the step size of the MZEP based on the MCI has been proposed. From the simulation results, it can be concluded that the normalized step size employing the information of MCI can enhance the performance of MZEP significantly.

References


김 종 일 (Chong-il Kim)
1987년 : 서강대학교 전자공학과 공학사
1989년 : 연세대학교 전자공학과 공학석사
1993년 : 연세대학교 전자공학과 공학박사
1993년~현재 : 가톨릭관동대학교 전자공학과 교수.
관심분야 : 이동통신, STC, TCM, OFDM, CDMA, 주파수 합성기 등

김 남 용 (Namyong Kim)
1986년 2월 : 연세대학교 전자공학과 졸업
1988년 2월 : 연세대학교 전자공학과 석사
1991년 8월 : 연세대학교 전자공학과 박사
1992년 8월~1998년 2월 : 관동대학교 전자통신공학과 부교수
1998년 3월~현재 : 강원대학교 공학대학 전자정보통신공학부 교수
관심분야 : Adaptive equalization.