Abstract - We propose a phase optimization technique for sidelobe suppression in OFDM system. The technique is based on the idea that phase shift to some of the transmit symbols within the symbol constellation plane can lead to significant sidelobe suppression. The sidelobes are reduced by optimizing using iterative method the phases of some subcarriers on the left and right hand side of the used OFDM spectrum. The proposed technique requires a small amount of side information that needs to be transmitted. Simulation results show that the proposed technique can reduce the sidelobes by significant amount.

Key Words : sidelobe suppression, phase optimization, OFDM, subcarriers on left, subcarriers on right

1. INTRODUCTION

The demand for wireless communication capacities is continuously growing. Measurement campaigns [1] have shown that wide ranges of potential spectral resources are used very rarely. Therefore, a huge capacity gain for these frequencies should be possible with the implementation of advanced signal processing technology such as spectrum pooling approach. The spectrum pool basically represents the idea of merging spectral ranges from different spectrum owners (military, trunked radio, etc) into a common pool. From this common spectrum pool hosted by the licensed system, public rental users may temporarily rent spectral resources during idle periods of licensed users, with no change to licensed system. The potential rental system needs to be highly flexible with respect to the spectral shape of the transmitted signal.

Orthogonal frequency division multiplexing (OFDM) modulation is one of the candidates for rental user as it is possible to leave a set of subcarriers unused, thus providing a flexible spectral shape that fills the spectral gaps without interfering with the licensed system user. A detailed description of this approach is given in [2]. Many techniques have been proposed to reduce the out of band radiations in the OFDM system, including insertion of guard bands [3], which results in a waste of available bandwidth. Subcarrier weighting and cancellation carriers [4]-[5] involve the complex optimization techniques that add to system complexity when the number of subcarrier is large. Multiple choice sequences (MCS) [6] have been shown to be effective when the size of the set of is large at the cost of low throughput.

In this paper, we propose sidelobe suppression technique in OFDM System based on phase optimization of subcarriers. The motivation for the proposed phase optimization technique is explained in Section 2. We formulate the optimization problem for the technique to suppress sidelobes and propose the iterative method in Section 3 and Section 4, respectively. In Section 5 simulation results are given to demonstrate the effectiveness of the proposed technique. Finally conclusions are drawn in the Section 6.

2. MOTIVATION

The technique used for the sidelobe suppression in [6] performs the mapping of original transmission signal onto a set of sequences. A set of sequences...
$x^{(p)} = (x_0^{(p)}, x_1^{(p)}, \ldots, x_{N-1}^{(p)})$, $p = 1, \ldots, P$ is produced from the input sequence $x = (x_0, x_1, \ldots, x_{N-1})^T$. From this set a sequence which offers maximum reduction of out of band radiation is chosen. Various approaches are used for this mapping. In phase approach the MCS sequences are obtained by applying random phase shifts to the original symbols and hence the resulting MCS symbols are formed as

$$x_n^{(p)} = x_n \exp(j\phi_n^{(p)}), \quad n = 0, \ldots, N-1, \quad p = 1, \ldots, P \quad (1)$$

where phase shifts $\phi_n^{(p)}$ lie in the interval $[0, 2\pi]$ and are generated as

$$\phi_n^{(p)} = 2\pi r_n^{(p)} / L \quad (2)$$

where $L$ is a constant integer and $r_n^{(p)}$ is an integer randomly chosen from the set $r_n = \{0.1, \ldots, L-1\}$. Thus $\phi_n^{(p)}$ can take one of the $L$ discrete phase values. The same random seeds are used at the transmitter and the receiver. The resulting MCS symbols do not necessarily correspond to the original symbol. Shifting phases of the data sequence by different amount generates different in-phase and quadrature-phase components, hence reducing MCS symbols of the OFDM Signal. This approach is suitable for large sets of the multiple choice sequences and the transmitter and the receiver need to maintain a lookup table.

Motivated by phase approach in [6], we propose a technique based on the phase optimization which uses phases for the subcarriers on the left and right hand side of the OFDM spectrum using iterative method. The block diagram of the OFDM transmitter employing sidelobe suppression technique is shown in Fig. 1. The transmitter uses optimized phases for some data subcarriers to generate OFDM signal with reduced sidelobes. The input bits are symbol mapped by applying phase shift keying (PSK) and $N$ complex valued data symbols $\{x_n\}, \quad n = 0, 1, \ldots, N-1$ are generated. These symbols are serial-to-parallel converted resulting in an $N$-element data symbol array $x = (x_0, x_1, \ldots, x_{N-1})^T$. The array $x$ is fed into the phase optimization unit that generates $x_{\phi}$ which is formed as

$$x_{\phi} = (x_{\phi_0}, x_{\phi_1}, \ldots, x_{\phi_{N-1}})^T \quad (3)$$

where $x_{\phi_0} = x_0 e^{j\phi_0}$. The phase optimization block calculates phase vector $\phi = (e^{j\phi_0}, e^{j\phi_1}, \ldots, e^{j\phi_{N-1}})^T$ which is determined so that the sidelobes of the original signal are suppressed.

The resulting phases are multiplied with data symbols. The vector $x_{\phi}$ is modulated on $N$ subcarriers using inverse Fourier transform and finally a parallel-to-serial conversion is performed to produce the signal $q_n$. Prior to transmission, a guard interval, with a length greater than the channel delay spread is added to each OFDM symbol using cyclic prefix block in order to mitigate the effects of the intersymbol interference (ISI). The signal $q_n$ is then passed through the transmitter radio frequency which amplifies the signal and upconverts it to the desired center frequency.

3. PROPOSED PHASE OPTIMIZATION TECHNIQUE

The information data symbol is denoted by $\{x(n)\}, \quad n = 0, 1, \ldots, N-1$ and the transmitted OFDM signal in the phase optimization block of Fig. 1 is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1 \quad (4)$$

The corresponding $N$-point spectrum is given by

$$y(l) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{-j2\pi kl/N}, \quad l = 0, 1, \ldots, N-1 \quad (5)$$
Combining these two equations, we obtain the relation between $X(k)$ and $y(l)$

$$y(l,k) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} x(n) e^{-j \frac{2 \pi k (l-n)}{N}} , \quad l = 0, 1, ..., N-1 \quad (6)$$

Since our goal is to suppress the sidelobes in a certain frequency range for $y(l,k)$, we observe complex samples at the normalized frequencies $\{y(l,k)\}$, $l = 1, 2, ..., r$ where the optimization of the sidelobes is performed. Collecting these complex samples in a vector, we obtain

$$y_r(k) = (y(0,k), y(1,k), ..., y(r,k))^T \quad (7)$$

Stacking the vector $y_r(k)$ into matrix we get

$$Y = (y_r(0), y_r(1), ..., y_r(N-1)).$$

The objective is to find phase factors $\phi_k$ with the aim of minimizing the signal amplitude at sidelobe frequencies. We can write

$$\bar{y} = Y \phi = \begin{bmatrix} y(0,0) & y(0,1) & ... & y(0,N-1) \\ y(1,0) & y(1,1) & ... & y(1,N-1) \\ ... & ... & ... & ... \\ y(r,0) & y(r,1) & ... & y(r,N-1) \end{bmatrix} \begin{bmatrix} e^{j \phi_0} \\ e^{j \phi_1} \\ ... \\ e^{j \phi_r} \end{bmatrix} \quad (8)$$

where $\bar{y}$ contains sidelobe signal samples. The sidelobe suppression is related to the following optimization problem

$$\text{Min} ||\bar{y}||^2 \quad \text{subject to} \quad 0 \leq \phi_k \leq 2\pi \quad \text{for} \quad k = 1, ..., l_c$$

or $k = N-r_c-1, ..., N-1 \quad (9)$

where $||\bar{y}||^2$ denotes the squared norm of the vector $\bar{y}$. The $l_c$ and $r_c$ are the number of the subcarriers that are to be optimized on the left and right hand side of the OFDM spectrum, respectively. One of the important constraint is that $\phi_k$ can only take values between $0$ and $2\pi$. In order to solve such optimization problem many effective and reliable numerical methods such as differential evolution, Nelder mead, Random search, and simulated annealing are available.

### Suboptimal Exhaustive Search

For coherent demodulation, it is necessary to send $\phi_k$ to the receiver as side information. When $\phi_k$ is a continuous value, an infinite number of bits will be required as side information. The solution is to limit $\phi_k$ to a level from a finite number of predetermined levels which will provide suboptimal performance. Results show that limiting the number of the phase factors do not significantly effect the sidelobe suppression. Therefore the phase factors are restricted to a set with finite number of elements to reduce search complexity. The set of allowed phases is written as of value and hence (9) can be approximated as

$$P = \left\{ \frac{\pi l}{W} | l = 0, 1, ..., W-1 \right\} \quad (10)$$

where $P$ is a set of values such that it contains the allowed phase factors while $W$ is the number of allowed phase factors. Only $l_c + r_c$ free variables needs to be optimized. These $l_c + r_c$ variables can be selected on both sides of the OFDM signal and hence $W(l_c + r_c)$ distinct phases need to be tested. The iterations required are $W \times (l_c + r_c)$ and each iteration involves several complex additions and multiplications operations. The phase vector that is retained after all iterations are completed will be approximation to the global optimal solution.

### 4. Iterative Phase Optimization Technique

The proposed technique uses the phase optimization method to suppress the sidelobes through optimizing the phases of the some data subcarriers at both ends of the OFDM spectrum. Since the computational complexity of the algorithm in Section III can be very large we use an iterative technique which provides optimal phase combination vector with reduced complexity. In the iterative technique the phase optimization method is repeated for the whole vector until the maximum sidelobe suppression is achieved. In the iterative phase optimization algorithm, initially phase factors $\phi_k$ are set to zero and sidelobes are calculated. Then the first phase factor $\phi_0$ is changed among all the possible values in allowed phase set $P$ for the first subcarrier and recompute the sidelobe levels. Then a value is selected which achieves the
lowest sidelobe levels as a part of the final set of the phase vector. After that the phase factors for the second subcarrier are tested. This algorithm continues in this fashion until all phase factors have been explored for the \( l_c + r_c \) subcarriers. The complexity of this search method is equal to \( W \times (l_c + r_c) \).

The reduced complexity phase optimization technique is summarized as:

(i) An OFDM signal without sidelobe suppression is calculated.

(ii) The phase optimization process is carried out on each subcarrier. All the phase factors are tested for a subcarrier, and the sidelobe power is calculated. The phase factor with the lowest sidelobe levels is retained.

(iii) Step (ii) is repeated for each subcarrier. For the \( l_c + r_c \) subcarriers if the sidelobe power meets the target value, the phase factor is the target value of the proposed technique.

As the number of subcarriers are increased the performance of sidelobe suppression can be improved, but the processing time gets longer because of much iterations. To solve this problem, the threshold technique can be used. The threshold technique uses a comparator to check the sidelobe levels of the OFDM signal. If the sidelobes of the OFDM signal are lower than the threshold sidelobe level, then this OFDM signal is transmitted right away. Otherwise the data subcarriers phases are controlled so that the sidelobe levels of the OFDM signal may be lower than the threshold value.

One of the problems of this technique is the amount of side informations that needs to be transmitted. The receiver needs to know the set \( \phi_k, k = 1, ..., l_c + r_c \). Hence an unambiguous representation of it must be transmitted to the receiver. The bits needed for this side information represent the redundancy associated with the phase optimization technique for sidelobe suppression. To further lessen the problem, the quantization of the phase shifts to BPSK or QPSK can be carried out to reduce the amount of side information that needs to be transmitted. There is a possibility that can us refrain from explicitly transmitting side information if differential encoded modulation across the subcarriers is used. In this case, only the number of subcarriers must be known in the receiver and some subcarriers must be left unmodulated as the reference carriers. Fig 2 shows the flow chart of the proposed iterative phase optimization technique.

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**Fig. 2** Flow chart for iterative phase optimization technique

**Fig. 3** Illustration of phase optimization technique:
(a) OFDM signal without phase optimization and
(b) OFDM signal with phase optimization
The principle of the phase optimization technique is illustrated in Fig. 3 for the parameters \(N=8\), and random signal vector \(\mathbf{a}\). The spectra of the individual subcarriers as well as that of the sum signal of all subcarriers are shown in frequency domain. From Fig. 3 it can be seen that in the case of the phase optimization the subcarriers are adapted to cancel each other in the optimization range.

### 5. SIMULATION RESULTS

Numerical results that illustrate the proposed phase optimization technique with respect to sidelobe suppression are given. The modulation method used throughout the simulation results is QPSK. The cases for \(N=64\) and \(N=128\) subcarrier system are considered. In all cases, normalized spectrum plots for OFDM symbols are presented to illustrate the effectiveness of the proposed algorithm. The ratio of phase optimized subcarriers is set to 1:16. Optimized data subcarriers are used at each end of the OFDM spectra. The spectra of the OFDM signals with and without phase optimized subcarriers signal for a random symbol vector \(\mathbf{a}\) are illustrated in Fig. 4 for 64 subcarrier system. Threshold technique is not applied in any case and the subcarriers are allowed to traverse all the phases in the allowed phase set \(P\).

One of the limiting factors in phase optimized technique is the phase variance between the signals. To reduce the number of redundancy bits that are to be transmitted the phase factors must be limited to certain values. If the number of the phases that are to be traversed are increased we can achieve slight improvement in sidelobe suppression because the algorithm is able to search from a greater number sequences. However the effect on the sidelobe suppression is not so profound as compared to the increased complexity and extra side information. In this case the phase variance is set to \(\pi/4\). We can clearly see reduction in sidelobe suppression by using 4 phase rotated subcarriers for 64 subcarriers in Fig. 4 and 8 phase rotated subcarriers for the 128 subcarriers system in Fig. 5. A suppression of 9 dB is observed from Fig. 5. The reason for the improvement is the more freedom in cancelling the inphase and quadrature components of the subcarriers, where \(P\) is restricted to four phase values \(\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\).

![Fig. 4 Spectrum of OFDM signals with and without phase optimization](image)

![Fig. 5 Spectrum of OFDM signals with and without phase optimization: \(N=128\)](image)

When the proposed technique of phase optimization was applied to the system with 128 subcarriers, a suppression of 10 dB is observed.

A possible drawback of the phase optimization method is degradation of peak to average power ratio (PAPR). This can be due to the addition of the subcarriers with the same phase. The PAPR of the time–domain sample sequence \(\{x(n)\}, n=0,1,...,N-1\) is defined as

$$\text{PAPR}(x(n)) = \max_{n} \frac{|x(n)|^2} {E[|x^2(n)|]} / N$$

Application of the phase optimization technique shows that PAPR of the system with phase...
optimization and without phase optimization have slight difference. The PAPR of the phase optimized signal is increased by 1 dB as shown in the Fig. 6 for the case of $N = 64$ subcarriers QPSK modulated OFDM. Thus the PAPR characteristics are slightly degraded by phase optimization technique.

Fig. 6 PAPR of a $N = 64$ subcarrier QPSK modulated OFDM signal without phase optimization and with phase optimization

6. CONCLUSION

A technique based on phase optimization to suppress the sidelobes of OFDM transmission signals is proposed and investigated. The technique is based on idea that phase shift to some of the transmit symbols within the symbol constellation plane can lead to significant sidelobe suppression. The proposed sidelobe suppression technique has capability of easily reducing the sidelobes of OFDM transmission signals by several dB. The price to be paid for this achievement is a moderate loss in symbol energy used to transmit the side information, since the transmission of additional signaling information is required.

REFERENCES


