A New Robust Discrete Static Output Feedback Variable Structure Controller with Disturbance Observer for Uncertain Discrete Systems

Abstract - In this paper, a new discrete static output feedback variable structure controller based on a new dynamic-type sliding surface and output feedback discrete version of the disturbance observer is suggested for the control of uncertain linear systems. The reaching phase is completely removed by introducing a new proposed dynamic-type sliding surface. The output feedback discrete version of disturbance observer is derived for effective compensation of uncertainties and disturbance. A corresponding control with disturbance compensation is selected to guarantee the quasi sliding mode on the predetermined dynamic-type sliding surface for guaranteeing the designed output in the dynamic-type sliding surface from any initial condition for all the parameter variations and disturbances. Using Lyapunov function, the closed loop stability and the existence condition of the quasi sliding mode is proved. Finally, an illustrative example is presented to show the effectiveness of the algorithm.

Key Words : Output feedback, Discrete variable structure system, Digital sliding mode control, Disturbance observer

1. Introduction

The theory of the variable structure system (VSS) or sliding mode control (SMC) can provide the effective means to the problem of controlling uncertain dynamical systems under parameter variations and external disturbances in case of the continuous[1]-[4] and discrete time system[5]-[18]. One of its essential advantages is the robust of the controlled system to variations of parameters and external disturbances in the quasi sliding mode on the predetermined sliding surface, s(k)=0[5][6]. The proper design of the sliding surface can determine the almost output dynamics and its performances. In the SMC for discrete time systems, a few issue is considered in the design of VSS controller, i.e. the stable design of the sliding surface[10][11], reachability from a given initial state to the fixed sliding surface[12][15][17], proof of existence condition of quasi sliding mode[3][7][8][15] to gather with closed loop stability[6], robustness against uncertainties and disturbance[6][15][16][18], etc.[19]. In 1985, Milosavljevic defined the quasi sliding mode and presented the condition for the existence of the quasi sliding mode in discrete VSS[5]. The sliding and convergence condition for controlling discrete-time system is suggested by Sarpturk et. al[7] which is modified from that of Milosavljevic’s where an absolute value condition for the reaching and the existence condition of the quasi sliding mode is imposed. Furuta in 1990 proposed the design methodology of discrete VSS by using the transformation matrix, and using Lyapunov function the quasi sliding and convergence condition is proposed and the sliding sector concept is introduced to design sliding mode controller for linear single-input discrete-time systems[8]. In [9], the problems of robust model following control of discrete-time uncertain systems is considered. Using equivalent control of the discrete VSS, the sliding surface is designed in [8] and [13], both are different. Using a candidate Lyapunov function, the coefficient of the sliding surface is designed[10]. By means of optimal theory to minimize the cost function, the optimal sliding surface is chosen with selection of the optimal switching gain[11]. Wang[12] designed the a simple sliding mode such that the robust stability of the uncertain system and reduced the chattering along the sliding mode. However a counterexample showing the instability of the control scheme proposed by Wang et al. was given in [13]. The band of the quasi sliding mode is rigorously defined and a new reaching condition is established in [14]. For multivariable system, Koshkouei and Zinober suggested a new condition for the existence of the discrete-time...
sliding mode and presented a design procedure such that the robust stability of the sliding motion is achieved in [15]. The fixed and adaptive sliding mode control in the presence of an unknown disturbance were in [16]. Hui and Zak compared the difference in the requirements for the sliding mode behavior for continuous- and discrete-time systems and discussed the limitations of discrete-time variable structure sliding mode control[17]. Cheng et al. provided a simple design technique of sliding mode controllers for a class of multi input uncertain discrete-time system with matching conditions[18]. For uncertain nonlinear system, the discrete-time implementation of a second-order sliding mode control scheme is analyzed in [20]. Using the suggested discrete version of the continuous disturbance observer in [22], a full state feedback VSS is proposed in [23]. The integral action is introduced to the discrete VSS to remove the reaching phase and improve the output performance[25]. In [28], using the free-weighting matrix approach, the output feedback control of a linear discrete-time system with time varying polytopic uncertainties is investigated. Dong designed the robust static output feedback controllers for quasi sliding mode on the predetermined sliding surface[24].

In [29], using the free-weighting matric approach, the output feedback control of a linear discrete-time system with time varying polytopic uncertainties in [29]. For the time-delay singular uncertain system, quasi-sliding mode variable structure controller is designed by Guo in [30].

Until now in most of discrete VSSs, the used sliding surface except [25] is only the linear combination of the full state $s(k) = c^T x(k)$ and fixed in state space. Because of this, the closed loop system has the reaching phase for the initial state far from the sliding surface and the quasi sliding mode for the robustness is not guaranteed during this phase.

In this paper, a new discrete output feedback variable structure controller based on the a new dynamic-type sliding surface and discrete version of the disturbance observer is suggested for the control of uncertain linear systems. The reaching phase is completely removed by introducing a new proposed dynamic-type sliding surface which stems from [24] in continuous time. The ideal sliding dynamics is exactly obtained. The output feedback discrete version of disturbance observer is introduced to the effective compensation of uncertainties and disturbance which stems from [22] in continuous time and [23] in full state feedback discrete time. A corresponding control with disturbance compensation is selected to guarantee the quasi sliding mode on the predetermined sliding surface for guaranteeing the designed output in the dynamic-type sliding surface from any initial condition for all the parameter variations and disturbances. The advantages obtained after removing the reaching phase are discussed. Finally, an illustrative example is presented to show the effectiveness of the algorithm.

### 2. A Discrete Integral Variable Structure Systems

#### 2.1 System Descriptions and Basic Backgrounds

Let the uncertain linear time invariant discrete plant to be controlled be given in the state space representation by

$$X_{k+1} = (A + \Delta A)X_k + (B + \Delta B)U_k + d_k$$

$$Y_k = CX_k$$

where $k \geq 0$ is an integer, $X_k \in \mathbb{R}^n$ is the state, $X_0 \in \mathbb{R}^n$ is its initial condition of the state, $Y_k \in \mathbb{R}^q$ is the output, $U_k \in \mathbb{R}^q$ is its initial condition of the output, $U_k \in \mathbb{R}^d$ is the input control to be determined, nominal matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$ is full rank, and $T_{ik}$ is the unknown lumped uncertainty to be estimated.

**Assumption:**

A1: $(A, B)$ is completely controllable

A2: The lumped uncertainty $T_{ik}$ is piecewise smooth and, bounded and satisfies the matching conditions[18]

A3: C and $T_{ik}$ satisfy this equation $CT_{ik} = 0$ for estimation of $T_{ik}$ in the output feedback.

#### 2.2 Dynamic-Type Sliding Surface

For the system (6), a dynamic-type sliding surface being modified from that of continuous case [24] is proposed as follows:

$$S_k = F[Y_k - A_k Y_{k-1}]$$

where a non zero $F$ satisfies that $F C B$ is invertible and

$$Y_{k+1} = A^{-1} Y_0$$

so that the dynamic-type sliding surface is zero at initial time $k=0$ for any initial condition $Y_0$ in output space. Therefore the reaching phase to the dynamic-type sliding surface is removed completely and there is no need of the consideration of the reaching condition in this suggested discrete VSS. The following definitions are introduced.

**Definition 1:** Discrete Ideal Sliding Mode[8][15]

If $S_k = 0$, $k \geq 0$ is satisfied, then it is called as a discrete ideal sliding mode.

**Definition 2:** Quasi Sliding Mode[5]

For any real number $\epsilon > 0$, if $|S_k| < \epsilon$ for all $k$ is satisfied because of the finite sampling time, then it is called as quasi sliding mode.

Substituting (6) in $S_{k+1} = 0$ yields the equivalent control[8][10]
which can not be implemented because of the disturbance and function of the state. The closed loop system by equivalent control is obtained as

\[ \begin{align*}
S_{k+1} &= F_1 [Y_{k+1} - A_r X_{k+1}] = F_1 [C X_{k+1} - A_r C X_k] \\
U_{t+k} &= - (F C B^{-1} + F C B C^{-1} F A_r C X_k)
\end{align*} \]  

(6)

(7)

where

\[ \begin{align*}
A_r &= [A - B F C^{-1} + B F C^{-1}] \cdot F A_r C \]  

(8)

(9)

The solution of (8) defines the surface in discrete ideal sliding mode of the proposed dynamic-type sliding surface. The dynamics of discrete ideal sliding mode is obtained from

\[ \begin{align*}
\dot{X}_k &= A_r X_k - X_0
\end{align*} \]  

(10)

The solution of (11) is identical to that of (9). To design the dynamic-type sliding surface to be stable, one chooses the matrix \(A_r\) to be stable and \(F\) is a non zero.

From (9), the following equation is obtained

\[ \begin{align*}
S_k &= [F_1 Y_k - A_r X_k] - (F C B^{-1} + F C B C^{-1} F A_r C X_k)
\end{align*} \]  

(12)

From (11), the following equation is obtained as

\[ \begin{align*}
\dot{S}_k &= A_r S_k - A_n C X_k
\end{align*} \]  

(13)

Therefore, the equation is obtained as

\[ \begin{align*}
A_n C &= CA_n = CA = C(A - B K C)
\end{align*} \]  

(14)

2.3 Control Input

Now, to estimate the lumped uncertainty (3) for compensation by the output feedback control input, a one step delay nonlinear disturbance observer of discrete version is modified from that in [22] of continuous version and [23] of full state version as follows

\[ \begin{align*}
\dot{X}_k &= A_n X_k - X_0
\end{align*} \]  

(11)

\[ \begin{align*}
\Delta V_k &= V_k - V_k - S_k - S_k \\ \text{on the integral dynamic-type sliding surface from the initial state and stability in the sense of Lyapunov.}
\end{align*} \]  

Theorem 1: If the dynamic-type sliding surface is designed in the stable, i.e., stable \(A_r\), the proposed input with disturbance observer satisfies the quasi sliding mode on the predetermined dynamic-type sliding surface from the initial state and stability in the sense of Lyapunov.

Proof: Take the discrete candidate Lyapunov function as

\[ \begin{align*}
V_k &= S_k^T S_k
\end{align*} \]  

(21)

then

\[ \begin{align*}
V_{k+1} &= S_{k+1}^T S_{k+1} \\
&= S_k^T F_1 CB S_k + \epsilon
\end{align*} \]  

(22)

\[ \begin{align*}
&= S_k^T F_1 CB S_k \\
&= S_k^T G S_k + \epsilon
\end{align*} \]  

(23)

\[ \begin{align*}
&= S_k^T M S_k
\end{align*} \]  

(24)

\[ \begin{align*}
&= S_k^T Q S_k
\end{align*} \]  

(25)

\[ \begin{align*}
&= S_k^T Q S_k < 0
\end{align*} \]  

(26)

which implies that

\[ \begin{align*}
&\|S_{k+1}\| < \|S_k\|
\end{align*} \]  

(27)

\[ \begin{align*}
&\|S_{k+1}\|^2 < \frac{1}{2} \|S_{k+1}\|^2, \|S_{k+1}\| = S_{k+1} - S_k
\end{align*} \]  

(28)

\[ \begin{align*}
&\|S_{k+1}\|^2 < \|S_k\|^2
\end{align*} \]  

(29)

are satisfied [15] which completes the proof of Theorem 1.

By the results of Theorem 1, the quasi sliding mode on the integral dynamic-type sliding surface for all \(k \geq 0\) is guaranteed. The performance designed in the dynamic-type sliding surface is almost also guaranteed.

3. Design Example and Simulation Studies

Consider 3rd order discrete system as

\[ \begin{align*}
X_{k+1} &= \begin{bmatrix} 1 & 0.01 & 0 \\ 0 & 1 & 0.01 \\ 0 & 0 & 0.4575 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ 0 \\ 12.246 \end{bmatrix} U_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T_{ck}
\end{align*} \]  

(27)

\[ \begin{align*}
Y_k &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} X_k
\end{align*} \]  

(28)

which satisfy the assumption A1-A3. To design the dynamic-type sliding surface, the closed loop system matrix in (4) or (11) \(A_r\) is selected as

\[ \begin{align*}
S_{k+1} &= F_1 [Y_{k+1} - A_r X_{k+1}] = F_1 [C X_{k+1} - A_r C X_k] \\
&= F_1 [C A X_k + C B U_{k+1} - A_r C X_k] \\
&= F_1 [C A X_k - C B K C X_k - C B F T_{ck}^T + C B G S_k + \epsilon]
\end{align*} \]  

(15)

(16)

(17)
\[ A = \begin{bmatrix} 1.0 & 0.01 \\ -0.6223 & 0.0642 \end{bmatrix} \] (29)

in order to assign the stable pole at 0.9933 and 0.0709.

And non zero \( F \) in (4) is determined as
\[ F = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix} \] (30)

Hence, the closed loop matrix \( A_s \) in (10) of the ideal sliding dynamics of (8) becomes
\[ A_s = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} = A_s = (A - BK) \] (31)

in order to assign the stable poles at 1.0, 0.9933, and 0.0709. Then, the feedback constant gain \( K \) is
\[ K = [0.0508, 0.0321] \] (32)

The disturbance observer (15) is implemented as
\[ T_d = y_d - 0.4577y_{d-1} - 12.246U_{d-1} \] (31)

The \( G \) is chosen as \( G = 0.001 \) which satisfy the relationship (19). As a results of the systematic design, \( M \) and \( Q \) are as follows:
\[ M = 0.0122 \] (32)
\[ Q = 0.9998 < 1 \] (33)

The \( Q \) satisfies the relationship of (19) and the stability condition (23). An initial condition for (27) is given as \( X_0 = [0 \ 90 \ 0]^T [\text{degree/sec} \ \text{degree} \ \text{degree/sec}]^T \) and \( Y_{-1} = [82.471 \ 790.2942]^T \) by (5). The simulation is carried out with \( 10[\text{msec}] \) sampling under \( T_d = 0.2544 \) load variation of disturbance from 1[sec] to 3[sec]. Fig. 1 shows three output responses (i) ideal sliding output, i.e. solution of (11), (ii) without disturbance, (iii) with disturbance. As can be seen, the three outputs are almost identical. The phase trajectories for the three cases (i) ideal sliding trajectory, (ii) without disturbance, (iii) with disturbance are depicted in Fig. 2. The phase trajectory under disturbance is disturbed because of one step delay estimation of disturbance observer and the quasi sliding mode of the discrete VSS, however fastly recovered by the suggested control input. Fig. 3 shows the two dynamic-type sliding surfaces for the two cases (i) without disturbance and (ii) with disturbance. The load variation of disturbance from 1[sec] to 3[sec] and its estimated value by means of the discrete one step delay disturbance observer are shown in Fig. 4. The control inputs for the two cases (i) without disturbance and (ii) with disturbance are depicted in Fig. 5. From the simulation studies, the usefulness of the proposed controller is verified.

**4. Conclusions**

In this paper, a systematic design of a new robust discrete output feedback VSS with disturbance observer is presented for control of uncertain linear discrete systems under lumped uncertainties. To successfully remove the reaching phase problems, a discrete dynamic-type sliding surface is suggested to define the hyper plane from any given initial condition. For the design of its dynamic-type sliding surface, the ideal sliding dynamics is obtained. The dynamic-type sliding surface is determined to have exactly that performance of the ideal sliding mode dynamics from a given initial condition to the origin. The output feedback discrete version of disturbance observer is presented to effectively estimate the lumped uncertainties. A corresponding control input with disturbance observer is also designed to almost guarantee the performance pre-determined in the dynamic-type sliding surface. The robustness of the pre-determined output for all the lumped uncertainties is investigated in Theorem 1 together with the existence condition of the quasi sliding mode of the discrete VSS and the stability of the closed loop system in the sense of Lyapunov. Through simulation studies, the usefulness of the proposed controller is verified.
Fig. 3 Two dynamic-type sliding surfaces for the two cases (i) without disturbance nd (ii) with disturbance

Fig. 4 Load variation of disturbance from 1[sec] to 5[sec] and its estimated value by means of the discrete one step delay disturbance observer

Fig. 5 Control inputs for the two cases (i) without disturbance and (ii) with disturbance

References


