A New Robust Variable Structure Controller for Uncertain Affine Nonlinear Systems with Mismatched Uncertainties

이정훈*
(Jung-Hoon Lee)

Abstract - In this paper, a systematic design of a new robust nonlinear variable structure controller based on state dependent nonlinear form is presented for the control of uncertain affine nonlinear systems with mismatched uncertainties and matched disturbance. After an affine uncertain nonlinear system is represented in the form of state dependent nonlinear system, a systematic design of a new robust nonlinear variable structure controller is presented. To be linear in the closed loop resultant dynamics, the linear sliding surface is applied. A corresponding control input is proposed to satisfy the closed loop exponential stability and the existence condition of the sliding mode on the linear sliding surface, which will be investigated in Theorem 1. Through a design example and simulation study, the usefulness of the proposed controller is verified.

Key Words : Uncertain nonlinear system, Variable structure system, Sliding mode control, Mismatched uncertainties

1. Introduction

Stability analysis and controller design for uncertain nonlinear systems is open problems now[1]. So far numerous design methodologies exist for the controller design of nonlinear systems[2]. These include any of a huge number of linear design techniques[3][4] used in conjunction with gain scheduling[5]; nonlinear design methodologies such as Lyapunov function approach [1][2][6][7][10][11], feedback linearization method[8][9][10], dynamics inversion[10], backstepping[11], adaptive technique which encompass both linear adaptive[13] and nonlinear adaptive control[14], and sliding mode control[15]-[26] et[27]-[29].

The sliding mode control(SMC) can provide the effective means to the problem of controlling uncertain nonlinear systems under parameter variations and external disturbances[15][16][17]. One of its essential advantages is the robustness of the controlled system to variations of parameters and external disturbances in the sliding mode on the predetermined sliding surface, \( s=0 \)[18]. In [19], for nonlinear output regulator scheme, sliding mode approach is applied. The underlying concept is that of designing sliding submanifold which contains the zero tracking error submanifold. The convergence to a sliding manifold can be attained relying on a control strategy still based on a simplex of control vectors are identified for multi input uncertain nonlinear systems in [20]. Lu and Spurgeon in 1997 considered the robustness of dynamic sliding mode control of nonlinear systems which are in differential input-out form with additive uncertainties in the model[21]. The discrete-time implementation of a second-order sliding mode control scheme is analyzed for uncertain nonlinear system, in [22]. Flemming surveyed so called soft variable structure controls, compared them to other[23]. For 2nd order uncertain nonlinear system with mismatched uncertainties, a switching control law between a first order sliding mode control and a second order sliding mode control is proposed to obtain the globally or locally asymptotic stability[24]. The optimal SMC for nonlinear system with time-delay is suggested in [25]. The nonlinear time varying sliding sector is designed for a single input nonlinear time varying input affine system which can be represented in the form of state dependent linear time variant system with matched uncertainties[26].

For uncertain affine nonlinear system with mismatched uncertainties and matched disturbance, the systematic design of the SMC is not reported until now. In this technical note, a systematic design of a new nonlinear variable structure controller based on state dependent nonlinear form is presented for the control of uncertain affine nonlinear systems with mismatched uncertainties and matched disturbances. After an affine
uncertain nonlinear system is represented in the form of state dependent nonlinear system, a systematic design of a new nonlinear variable structure controller is presented. To be linear in the closed loop resultant dynamics, the linear sliding surface is applied. A corresponding control input is proposed to satisfy the closed loop exponential stability and the existence condition of the sliding mode on the linear sliding surface, which will be investigated in Theorem 1. Through a design example and simulation study, the usefulness of the proposed controller is verified.

2. A Nonlinear Variable Structure Systems

2.1 Description of plants

Consider an affine uncertain nonlinear system

\[ \dot{x} = f(x,t) + g(x,t)u + d(x,t), \quad x(0) \]  

where \( x \in \mathbb{R}^n \) is the state, \( x(0) \) is its initial state, \( u \in \mathbb{R}^k \) is the control input, \( f(x,t) \in C^k \) and \( g(x,t) \in C^k, k \geq 1 \), \( g(x,t) \neq 0 \), for all \( x \in \mathbb{R}^n \) and for all \( t \geq 0 \) are of suitable dimensions, and \( d(x,t) \) implies bounded matched external disturbances.

**Assumption**

\[ A_1: f(x,t) \text{ is continuously differentiable and } f(0,t) = 0 \quad \text{for all } t \geq 0. \]

Then, uncertain nonlinear system (1) can be represented in more affine nonlinear system of state dependent coefficient form [26]-[28]

\[ \dot{x} = f_0(x,t) + \Delta f(x,t)x + g_0(x,t)u + d_0(x,t), \quad x(0) \]  

where \( f_0(x,t) \) and \( g_0(x,t) \) is each nominal value such that \( f_0(x,t) + \Delta f(x,t) \) and \( g_0(x,t) = [g_0(x,t) - \Delta g(x,t)] \), respectively, \( \Delta f(x,t) \) and \( \Delta g(x,t) \) are matched or mismatched uncertainties, and \( d_0(x,t) \) is the lumped uncertainties.

**Assumption**

\[ A_2: \text{The pair } (f_0(x,t),g_0(x,t)) \text{ is controllable for all } x \in \mathbb{R}^n \text{ and for all } t \geq 0 \]

\[ A_3: \text{The lumped uncertainties } d(x,t) \text{ is bounded} \]

\[ A_4: \dot{x} \text{ is bounded if } \dot{u} \text{ is bounded.} \]

2.2 Sliding Surface

To control uncertain nonlinear system (1) or (2) with a linear closed loop dynamics, the sliding surface used in this design is as follows:

\[ s = C^Tz(0) \]  

In (4), \( C \) is a nonzero element as the design parameter such that the following assumption is satisfied.

**Assumption**

\[ A_5: C^Tg(x,t) \text{ and } C^Tg_0(x,t) \text{ have the full rank, i.e invertible} \]

\[ A_6: C^T\Delta g(x,t) \left[ C^Tg_0(x,t) \right]^{-1} = \Delta f \text{ and } |\Delta f| < \delta \leq 1, \quad 0 < \delta < 1. \]

The equivalent control input is obtained using \( \dot{s} = 0 \) [15][16] as

\[ u_x = -[C^Tg(x,t)]^{-1}C^Tf_0(x,t) \]  

where \( \Delta f \) is the lumped external disturbances. This control input cannot be implemented because of the uncertainties, but used to obtaining the ideal sliding dynamics. The ideal sliding mode dynamics of the sliding surface (4) can be derived by the equivalent control approach [16] as

\[ \dot{x} = f_0(x,t)x + g_0(x,t)u, \quad u = -K(x)z \]

To select the stable gain, take a Lyapunov function candidate as

\[ V(x) = \frac{1}{2}x^TPx, \quad P > 0 \]

The derivative of (10) becomes

\[ \dot{V}(x) = x^Tf_0(x,t)^TP + Pf_0(x,t)x + u^Tg_0(x,t)Px + x^TPg_0(x,t)u \]

If take the control input as

\[ u = -g_0^T(x,t)Px \]

and \( Q(x,t) > 0 \) for all \( x \in \mathbb{R}^n \) and for all \( t \geq 0 \) is

\[ f_0(x,t)^TP + Pf_0(x,t) = -Q(x,t) \]

then

\[ V(x) = -x^TQ(x,t)x - 2x^TPg_0(x,t)g_0^T(x,t)Px \]

\[ = -x^TQ(x,t) + 2Pg_0(x,t)g_0^T(x,t)P^TQx \]

\[ = -x^Tf_0^T(x,t) + Pf_0(x,t)x \]

\[ = -x^TQ(x,t)x \]

\[ \leq -\lambda_{\min}(Q(x,t))x^2 \leq 0 \]

Therefore the stable gain is chosen as

\[ K(x) = g_0^T(x,t)P_\delta x = \left[ C^Tg_0(x,t) \right]^{-1}C^Tf_0(x,t) \]

2.3 Stabilizing Control Input

A corresponding control input is proposed as follows:

\[ u = -K(x)x - \Delta Kx - Ks - Ksign(s) \]
where $K(x)$ is a nonlinear feedback gain, $\Delta K$ is a state dependent switching gain, $K_i$ is a feedback gain of the sliding surface, and $K_2$ is a switching gain, respectively as

\[ K(x) = [C^Tg_0(x,t)]^{-1}C^Tf_0(x,t) \quad \text{or} \quad g_0^*(x,t) \]  
\[ \Delta K = [C^Tg_0(x,t)]^{-1} \Delta K \]  
\[ \Delta K_j = \left\{ \begin{array}{ll} \frac{\max\{C^T\Delta f(x,t) - \Delta ICf_0(x,t)\}}{\min\{I - \Delta I\}} \varsigma_j > 0 \\ \frac{\min\{C^T\Delta f(x,t) - \Delta ICf_0(x,t)\}}{\min\{I - \Delta I\}} \varsigma_j < 0 \end{array} \right. \]  
\[ j = 1, \ldots, n \]  
\[ K_i = [C^Tg_0(x,t)]^{-1}K_i^* \]  
\[ K^*_i > 0 \]  
\[ K_2 = \frac{\max\{C^T\hat{d}(x,t)\}}{\min\{I - \Delta I\}} \]  
\[ V(t) \leq V(0)e^{-2\Delta K_i} \]  
\[ \text{and the second order derivative of } V(t) \text{ becomes} \]  
\[ \ddot{V}(t) = \dddot{s} + \ddot{s} = (s)^2 + sC^T \]  
\[ \text{and by Assumption A4 } \dddot{V}(t) \text{ is bounded which completes the proof of Theorem 1.} \]

To show the effectiveness of the algorithm, an example will be presented.

3. Design Example and Simulation Studies

Consider a second order affine uncertain nonlinear system with mismatched uncertainties and matched disturbance

\[ x_1 = x_1 + x_2 \sin(x_1) + x_2 + 0.02 \sin(2x_1) \]  
\[ x_2 = x_2 + x_2 \sin(x_2) + (2.05 \sin(2t))u + d(x,t) \]  
\[ d(x,t) = 0.7 \sin(x_1) - 0.8 \sin(x_2) + 0.2(x_1^2 + x_2^2) + 2 \sin(5t) \]

Since (29) satisfy the Assumption A1, (29) can be represented in state dependent coefficient form as

\[ x_1 = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \]  
\[ x_2 = \begin{bmatrix} 1 + \sin^2(x_1) & 0 \\ 0 & 1 + \sin^2(x_2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin(x_2) + \begin{bmatrix} 0.02 \sin(x_1) \\ 0.5 \sin(2t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]

where the nominal parameter $f_0(x,t)$ and $g_0(x,t)$ and mismatched uncertainties $\Delta f(x,t)$ and $\Delta g(x,t)$ are

\[ f_0(x,t) = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}, \quad g_0(x,t) = \begin{bmatrix} 0 & 0 \end{bmatrix} \]
\[ \Delta f(x,t) = \begin{bmatrix} \sin^2(x_1) & 0 \\ 0 & \sin^2(x_2) \end{bmatrix}, \quad \Delta g(x,t) = \begin{bmatrix} 0.02 \sin(x_1) \\ 0.5 \sin(2t) \end{bmatrix} \]

To design the sliding surface, $f_s(x,t)$ is selected as

\[ f_s(x,t) = f_0(x,t) - g_0(x,t)K(x) = \begin{bmatrix} -1 & 1 \\ 0 & 21 \end{bmatrix} \]

in order to have the two poles at -21.9545 and -0.455. The P in (10) is chosen as

\[ P = \begin{bmatrix} 10 & -5 \\ -5 & 5.5 \end{bmatrix} \]

so as to be

\[ f(x,t)^T P + P f_s(x,t) = \begin{bmatrix} -220 & 230 \\ 230 & -241 \end{bmatrix} < 0 \]

Hence, the continuous feedback gain is chosen as

\[ K(x) = g_0^*(x,t)P = \begin{bmatrix} 10 & 11 \end{bmatrix} \]

Therefore, the coefficient of the sliding surface is determined as

\[ C = \begin{bmatrix} 20 & 1 \end{bmatrix} \]

The selected gains in the control input are as follows:

\[ \Delta K_1 = \begin{bmatrix} 26.5 \quad \text{if} \quad sx_1 > 0 \\ -26.5 \quad \text{if} \quad sx_1 < 0 \end{bmatrix}, \quad \Delta K_2 = \begin{bmatrix} 10 \quad \text{if} \quad sx_2 > 0 \\ -10 \quad \text{if} \quad sx_2 < 0 \end{bmatrix} \]

\[ K_1 = 5 \]  
\[ K_2 = 1.75 + 0.1(x_1^2 + x_2^2) \]

The simulation is carried out under 1[msec] sampling time and with $x(0) = [5 - 5]^T$ initial state. Fig. 1 shows (i) $x_1$ and (ii) $x_2$ time trajectories. The phase trajectory(i) and ideal sliding trajectory(ii) are depicted in Fig. 2.
The sliding surface $s(t)$ is shown in Fig. 3. The control input is depicted in Fig. 4. From the simulation studies, the effectiveness of the proposed SMC is proven.

4. Conclusions

In this note, a systematic design of a new robust nonlinear variable structure controller based on state dependent nonlinear form is presented for the control of uncertain affine nonlinear systems with mismatched uncertainties and matched disturbance. After an affine uncertain nonlinear system is represented in the form of state dependent nonlinear system, a systematic design of a new robust nonlinear variable structure controller with the linear sliding surface is suggested. A corresponding control input is proposed. The closed loop stability by the proposed control input with linear sliding surface together with the existence condition of the sliding mode on the selected sliding surface will be investigated in Theorem 1 for all mismatched uncertainties and matched disturbance. Through a design example and simulation studies, the usefulness of the proposed controller is verified.

Acknowledgement

This work was supported by the fund of Research Promotion Program(RPP-2009-025), Gyeongsang National University and also this research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology

References


저 자 소 개

이 정 준 (李 政 勻)

1966년 2월 1일생. 1988년 경북대학교 전자공학과 졸업(공학사), 1990년 한국과학기술원 전기 및 전자공학과 졸업(석사).
Tel : +82-591-751-5368, Fax : +82-591-757-3974
E-mail : jhleew@gnu.ac.kr