Some Remarks on the $s$-plane to $w$-plane Correlations of $w$-transform

김 려 화*・김 영 철†

(Lihua Jin・Young Chol Kim)

Abstract - In this paper, we present some remarks on the correlations between $s$ and $w$ domains when a discrete-time transfer function is converted from $z$-plane by using the $w$-transform. With time response specifications, when a digital filter or controller is designed in $z$-plane, the $w$-transform is useful for the purpose if only the $w$-transformed system closely approximates to the continuous-time system. It will be shown that the approximation is accomplished only in the specific region depending on sampling time. Also, it is noted that such an approximation should be carefully dealt with for the case where a discrete-time reference transfer function is synthesized for the use of direct digital design.

Key Words: $w$-transform, $z$, $s$ and $w$ domains, Root space, Time and frequency responses

1. Introduction

There are various approaches that may be used in analyzing the stability and time response characteristics of discrete-time control systems. These approaches may be divided into two categories: (1) Direct and (2) digitization techniques. The first category involves carrying out the analysis entirely in the $z$-plane. The second category permits the analysis and synthesis of the sampled-data system to be carried out entirely in the $w$-plane[1]. The $w$-transform is a bilinear transform that converts from the $z$-domain to the complex $w$-domain. It is well known that the stability of a $z$-domain polynomial can be exactly analyzed by using the Routh-Hurwitz criteria on the $w$-transformed polynomial. However, it is important to note that the $w$-domain is not the same as the Laplace domain (i.e., $s$-domain), because the $w$-transform is a first-order approximation of the natural logarithm function, which means:

$$
\begin{align*}
  s &= \frac{1}{T_s} \ln(z) = \frac{2}{T_s} \left( \frac{z-1}{z+1} \right) + \frac{1}{5} \left( \frac{z-1}{z+1} \right)^3 + \cdots \quad (1) \\
  &\approx \frac{2}{T_s} \left( \frac{z-1}{z+1} \right) = w.
\end{align*}
$$

In many industrial controller design problems, time response specifications such as overshoot and settling time are frequently desired. For a continuous-time system, it has been presented that the characteristic ratio assignment (CRA)[2-5] and coefficient diagram method (CDM)[6] can be effectively applied to such a transient response design problem. On the other hand, there are few approaches that can use for the purpose of direct digital design associated with transient response control entirely in $s$-domain. In [7, 8], a direct digital design method combining the $w$-transform with the continuous-time CRA has been recently proposed. However, this approach can be applied to the case where the prospective closed-loop poles shall lie in a specific region of $s$-plane and $w$-plane. Because a transfer function in the specific $w$-plane is approximately identical to the corresponding transfer function in $s$-plane. In [1], it is pointed out that the similarities of transfer functions between $s$ and $w$ domains hold when $|s| < \frac{\pi}{2T_s}$, $|w| < \frac{2}{T_s}$ for a given sampling time $T_s$. Furthermore, it states that $\omega_w \approx \omega$ if $\omega \leq \frac{0.594}{T_s}$, and $\sigma \approx \sigma_w$ if $\left( \frac{\sigma T_s}{2} \right)^2 < < 2$ independent on $\omega$, for $s = \sigma + j\omega$ and $w = \sigma_w + j\omega_w$.

As for the frequency response of a discrete-time transfer function, $H(z)$, it is evaluated at $z = e^{j\omega T_s}$ which is on the unit circle. When the actual frequency of $\omega$ is input to the transfer function $H(z)$ designed by use of the $w$-transform, it is desired to know at what frequency, $\omega_w$, for the continuous-time transfer function that this $\omega$ is mapped to. The discrete-time to continuous-time frequency mapping of the $w$-transform is
The $w$-transform between $w$ and $z$ planes is defined as follows:

$$w := \frac{2}{T_s} \frac{z-1}{z+1}, \quad z := \frac{2 + T_s w}{2 - T_s w}.$$  \hspace{1cm} (3)

where $T_s$ is the sampling time. Since $z = e^{Ts}$, (3) is written by

$$w = \frac{2}{T_s} \frac{e^{Ts/2} - e^{-Ts/2}}{e^{Ts/2} + e^{-Ts/2}} = \frac{2}{T_s} \tanh \left( \frac{T_s s}{2} \right).$$ \hspace{1cm} (4)

Letting $s = \sigma + j\omega$, (4) can be expressed in terms of $\sigma$ and $\omega$ as follows.

$$w = \frac{2}{T_s} \frac{\tan(\frac{T_s \sigma}{2} + j\frac{T_s \omega}{2})}{\tan(\frac{T_s \sigma}{2} - j\frac{T_s \omega}{2})} = \frac{2}{T_s} \frac{\sinh(\gamma(1 + \tan^2\gamma)) - \sinh(\gamma(1 - \tan^2\gamma))}{\sinh(\gamma(1 + \tan^2\gamma)) + \sinh(\gamma(1 - \tan^2\gamma))} = \sigma + j\omega,$$

where $\gamma = \frac{T_s \sigma}{2}$ and $\beta = -\frac{T_s \omega}{2}$.

From (5), It is obvious that the $w$-transform is highly nonlinear mapping of $s$. When $s = j\omega$ (i.e., $\sigma = 0$), (2) results from (5) by using $\tanh(j\omega) = j\tan(\omega)$. Note that the second equation of (5) should be used if both $\sigma$ and $\omega$ are not zero. It is also easy to know from (4) that $w$ approaches $s$ as $T_s$ goes to zero. This means that complex variable $w$ is identical to $s$ when the sampling time becomes very small.

3. Relationships of the $w$-transform between $s$ and $\omega$ domains

In this section, we investigate the correlations between $s$ and $\omega$ domains. A condition for the good approximation of the transfer function between the $s$ and $\omega$ domains will be addressed in section 3.1. In section 3.2, several remarks on the time and frequency responses according to the pole locations will be discussed.

3.1 Condition for rigorous approximation of $s$- and $\omega$-domain transfer functions

According to the Appendix H in Houpis' book[1], the powerful approximation between $s$ and $\omega$ domain transfer functions exist for $|s| < \frac{\pi}{2T_s}$, and $|\omega| < \frac{\pi}{2T_s}$ for a given sampling time $T_s$. As more rigorous conditions than these, they represent that $\omega_c \approx \omega$ if $\omega \leq \frac{0.994}{T_s}$, and $\sigma \approx \sigma_w$ if $\left( \frac{\sigma T_s}{2} \right)^2 << 2$ independent upon $\omega$.

In order to investigate how precise this region gives the similarities between $s$ and $\omega$ domains, we are supposed to consider a specific region in $s$-domain, which is similar to the Houpis' region as follows:

$$\Gamma_i := \left\{ \begin{array}{l} \Re(s) \in \left[ 0, -\frac{\pi}{2T_s} \right] \quad \text{and} \quad \Im(s) \in \left[ 0, -\frac{\pi}{2T_s} \right] \end{array} \right\}.$$ \hspace{1cm} (6)

In Fig. 1, dots indicate grid points of the strip $\Gamma_i$ with $T_s = 1$ and the “+” marked points denote the corresponding ones in $\omega$-domain which are determined by (5). It is remarkable that the $w$-transformed points are quite different from the most points in $s$-domain except for region near origin.

Fig. 1 Region mapping $s$-plane(dotted points) on $\omega$-plane (+marked points) over $s \in \Gamma_i$, normalized by $T_s$. 

\[ \omega_c = \frac{2}{T_s} \tan \left( \frac{\omega T_s}{2} \right). \] \hspace{1cm} (2)
Here we define the mapping error of the $w$-transform between two complex variables, $s$ and $w$. The mapping error $\epsilon$ is defined as follows:

$$\epsilon := \frac{|s-w|}{|s|} \times 100\%.$$  

(7)

Both $s$ and $w$ regions that achieve the above mapping errors are depicted in Fig. 3. The arc in the second quadrant indicates a circle of radius 0.6 and 0.3, respectively. Let these regions in (i) be $\Gamma_{s}^{3\%}$ and $\Gamma_{w}^{3\%}$. That is,

$$\Gamma_{s}^{3\%} := \{s| |s| \leq 0.6/T_s, \Re(s) < 0 \},$$  

(8)

$$\Gamma_{w}^{3\%} := \{w| |w| \leq 0.6/T_w, \Re(w) < 0 \}. $$  

(9)

Remark 1:

For a given sampling time $T_s$, if the closed loop poles are placed outside the radius of $0.6/T_s$, the approximations are not so good. Reversely, if a specific region $\Gamma_s$ for desired closed-loop pole locations can be represented by a subset of either $\Gamma_{s}^{3\%}$ or $\Gamma_{w}^{3\%}$, a proper sampling time resulting good approximation can be chosen by $T_s \leq 0.6/|s|, s' \in \Gamma_s$.

Fig. 2 Comparisons of mapping to 4 subregions in $\Gamma_s$.

Now, we observe the mapping similarity for four specific subregions in the region $\Gamma_s$. The results are shown in Fig. 2. It turns out that the Houpis’ region, $|s| \leq \frac{\pi}{2T_s}$, may result in large mapping error.

As a result of the observation above using (5), we propose a more rigorous region than Houpis’ one, which is given by

(i) $H(s) \approx H(w)$ within 3% mapping error, if $|s| \leq 0.6/T_s$ and $|w| \leq 0.6/T_w$,

(ii) $H(s) \approx H(w)$ within 1% mapping error, if $|s| \leq 0.34/T_s$ and $|w| \leq 0.34/T_w$.

Remark 1:

Since $z = e^{T_s}$, the region in $z$-domain that corresponds to $\Gamma_{s}^{3\%}$ or $\Gamma_{w}^{3\%}$ are shown in Fig. 4.

3.2 Remarks on the time and frequency responses

The time and frequency responses of the overall system are often analyzed on the basis of the locations of pole and zero. In this subsection, we will examine about how much the mapping error between $s$ and $w$ planes makes the effect on time and frequency responses of the $w$-transformed transfer functions. Using the continuous-time plant for the unity feedback control system, such a problem can be boiled down to the problem of investigating the effect of sampling time on the degree of correlation between $s$ and $w$ domains. This comparative observation will be shown by a numerical example.

Consider a unit feedback system in Fig. 5(a), where the continuous-time plant is given by

$$G_c(s) = \frac{s+5}{s(s+1)}.$$  

(10)
The continuous-time closed-loop transfer function is
\[
H_c(s) = \frac{G_c(s)}{1 + G_c(s)} = \frac{s + 5}{s^2 + 2s + 5}.
\]
(11)

As shown in Fig. 5(b), the discrete-time transfer function, \( G_d(z) \), is derived by \( z \)-transform of \( G_c(s) \) with zero-order hold (ZOH).

\[
G_d(z) = \left(1 - z^{-1}\right)G_c(s) = \frac{s^2 + 5}{s(s + 1)}
\]
\[
= \left(4e^{-T} + 5T - 4\right)z + \left(5T e^{-T} - 4e^{-T} + 4\right)
\]
\[
z^2 - (1 + e^{-T})z + e^{-T}.
\]
(12)

Then the discrete-time closed-loop transfer function is
\[
H_d(z) = \frac{Y(z)}{R(z)} = \frac{G_d(z)}{1 + G_d(z)}
\]
\[
= \frac{\left(4e^{-T} + 5T - 4\right)z + \left(5T e^{-T} - 4e^{-T} + 4\right)}{z^2 + (3e^{-T} + 5T - 5)z + \left(5T e^{-T} - 3e^{-T} + 4\right)}.
\]
(13)

Substituting (3) into (13), the \( w \)-transformed transfer function of \( H_d(z) \) is obtained by
\[
H_w(w) = \frac{H_w(w)}{H_w(w)}.
\]
(14)

end of the section. If we intend to compare the similarity of \( H_w(w) \) with a \( s \)-domain transfer function, the Laplace transformed function of \( H_c(z) \), \( H_s(s) \), is necessary instead of \( H_c(s) \). Since the Laplace transform of the sampled signal has the periodicity, there is no unique point in \( s \)-plane which corresponds to any point in \( z \)-plane. However, if a transfer function in \( z \)-domain is transformed into the primary strip between \( \omega_s = \pm \frac{\pi}{T_s} \) in \( s \)-plane, the transfer function can be regarded as a function which is equivalent to the \( z \)-domain function in the sense that the \( z \)-transform of such a function, \( H_s(s) \), results in the same transfer function, \( H_c(z) \). In addition, we are supposed to assume that all the roots of both \( H_s(s) \) and \( H_c(z) \) lie in the primary strip of \( s \)-plane. To obtain this, let us first rewrite (13) in the simple form as follows.

\[
H_c(z) = \frac{b_1z + b_0}{z^2 + a_1z + a_0} = \frac{b_1z + b_0}{z - \lambda_1}(z - \lambda_2)
\]
(15)

where
\[
\left\{ \begin{array}{ll}
a_1 &= 3e^{-T} + 5T - 5, \\
a_0 &= -5Te^{-T} - 3e^{-T} + 4, \\
b_1 &= 4e^{-T} + 5T - 4, \\
b_0 &= -5Te^{-T} - 3e^{-T} + 4.
\end{array} \right.
\]
(16)

Using the partial fraction expansion and Laplace transform table, we have
\[
H_w(w) = \frac{c_1}{(s + r_1)} + \frac{c_2}{(s + r_2)} = \frac{n_1 s + n_0}{s^2 + d_1 s + d_0}
\]
(17)

where
\[
\left\{ \begin{array}{ll}
c_1 &= b_1\lambda_1 + b_0 \\
c_2 &= b_1\lambda_2 + b_0 \\
d_1 &= -\frac{1}{T_s} (\ln\lambda_1 + \ln\lambda_2) \\
d_2 &= \frac{1}{T_s} (\ln\lambda_1 \ln\lambda_2), \\
n_1 &= c_1 + c_2 = -\frac{b_1}{\lambda_0} \\
n_0 &= -\frac{1}{T_s^2} (\lambda_1 \ln\lambda_1 + \lambda_2 \ln\lambda_2), \\
r_1 &= -\frac{\ln\lambda_1}{T_s} \\
r_2 &= -\frac{\ln\lambda_2}{T_s}.
\end{array} \right.
\]
(18)

A. Time response correlation

As mentioned in the Remark 1 in section 3.1, the sampling time should satisfy the condition \( T_s \leq 0.6/|\omega_s| \), \( s \in \Gamma_s \), if one wants to make \( H_w(w) \approx H_c(z) \). By computing the poles of \( H_w(s) \) or \( H_w(w) \) with various sampling times

\[
\frac{G_i(t)}{r} - \frac{G_c(s)}{1 + G_c(s)} - \frac{G_s(s)}{1 + G_s(s)}.
\]
iteratively, we can easily obtain the boundary value of such sampling times, which is $T_s \leq 0.2513$. Thus, we have chosen the sampling time, $T_s = 0.25$, by which the resulting roots ($|s'| = 0.6/|T_s| = 2.4$) is placed just inside the boundary of root region ($F_{Bw}$ or $F_{Bw}$). It implies that this sampling time $T_s = 0.25$ will result in the mapping error between $H_s(z)$ and $H_s(w)$ less than 3%. For the purpose of comparison, the other sampling time is chosen as $T_s = \frac{\pi}{2s'} = 0.65$, which is given by the Houpis’ condition.

Now, we will investigate the time response characteristics of the $w$-transformed transfer function by comparing poles, zeros, mapping errors, and step responses of three transfer functions, $H_s(z)$, $H_s(s)$, and $H_s(w)$, respectively.

From (13), (14), and (17) with two sampling times, $T_s = 0.25$ and $T_s = 0.65$, their poles, zeros, and mapping errors are given in Table 1. The mapping error has been determined by substituting the poles of $H_s(z)$ and $H_s(w)$ into (7). Although $T_s$ is larger by 2.6 times than $T_s$, the mapping error of $H_s(w)$ becomes much larger by 7 times than that of $H_s(w)$. Step responses of $H_s(z)$, $H_s(s)$, and $H_s(w)$ are shown in Fig. 6. Time response characteristics such as the maximum overshoot, rise time, and settling time are given in Table 2. It is seen that the time responses of three transfer functions coincide very closely when $T_s = 0.25$, but these are quite different one another when $T_s = 0.65$. This is due to the disagreement of poles in $s$ and $w$ domains as shown in Table 1. Therefore, this example shows that we can synthesize a discrete-time transfer function having the almost same time response as the continuous-time transfer function if the sampling time for the $w$-transform is selected by means of the condition, $T_s \leq \frac{0.6}{|s'|}$, $s' \in \Gamma_s$. Note that the sampling time chosen by the Houpis’ condition may be not sufficient for good approximation $s \approx w$.

### Table 1 Poles and zeros of $H_s(z)$, $H_s(w)$, and $H_s(s)$ for $i = 1, 2$

<table>
<thead>
<tr>
<th>Models</th>
<th>$T_s$ [sec]</th>
<th>poles in $s$, $w$, $z$ domains</th>
<th>mapping error $\epsilon$ [%]</th>
<th>zeros in $s$, $w$, $z$ domains</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s(z)$</td>
<td>0.25</td>
<td>$0.7088 \pm 0.4365i$</td>
<td>-</td>
<td>0.2429</td>
</tr>
<tr>
<td>$H_s(w)$</td>
<td>-0.7088 $\pm 2.2502i$</td>
<td>0.92</td>
<td>8</td>
<td>-11.5628</td>
</tr>
<tr>
<td>$H_s(s)$</td>
<td>-0.7418 $\pm 2.2139i$</td>
<td>-</td>
<td>-11.5628</td>
<td></td>
</tr>
<tr>
<td>$H_s(z)$</td>
<td>0.65</td>
<td>$0.0019 \pm 0.8537i$</td>
<td>-</td>
<td>-0.1608</td>
</tr>
<tr>
<td>$H_s(w)$</td>
<td>-0.4209 $\pm 2.7346i$</td>
<td>20.52</td>
<td>-4.2500</td>
<td>3.0760</td>
</tr>
<tr>
<td>$H_s(s)$</td>
<td>-0.2345 $\pm 2.2516i$</td>
<td>-</td>
<td>12.1007</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2 Time responses characteristics of $H_s(z)$, $H_s(w)$, and $H_s(s)$ for $i = 1, 2$

<table>
<thead>
<tr>
<th>Models</th>
<th>$T_s$ [sec]</th>
<th>maximum overshoot [%]</th>
<th>rise time [sec]</th>
<th>settling time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s(z)$</td>
<td>0.25</td>
<td>39.20</td>
<td>0.508</td>
<td>4.60</td>
</tr>
<tr>
<td>$H_s(w)$</td>
<td>38.00</td>
<td>0.548</td>
<td>4.90</td>
<td></td>
</tr>
<tr>
<td>$H_s(s)$</td>
<td>35.65</td>
<td>0.559</td>
<td>4.70</td>
<td></td>
</tr>
<tr>
<td>$H_s(z)$</td>
<td>0.65</td>
<td>79.00</td>
<td>0.389</td>
<td>15.40</td>
</tr>
<tr>
<td>$H_s(w)$</td>
<td>97.80</td>
<td>0.426</td>
<td>10.43</td>
<td></td>
</tr>
<tr>
<td>$H_s(s)$</td>
<td>73.31</td>
<td>0.479</td>
<td>14.51</td>
<td></td>
</tr>
</tbody>
</table>

### B. Frequency response correlation

Let us consider the unit feedback control system shown in Fig. 5 again. We will investigate the frequency response characteristics of the $w$-transformed transfer function by comparing Bode plots of three transfer functions, $H_s(z)$, $H_s(s)$, and $H_s(w)$ respectively, where two sampling times are applied to.

The frequency responses of $H_s(z)$, $H_s(w)$, and $H_s(s)$ are given by

$$H_s(z)_{\omega = j \omega_c} = M_s \angle \theta_s,$$  \hspace{1cm} (19)

$$H_s(w)_{\omega = j \omega_c} = M_s \angle \theta_s,$$  \hspace{1cm} (20)

$$H_s(s)_{\omega = j \omega_c} = M_s \angle \theta_s.$$  \hspace{1cm} (21)
where $M_r$ is the magnitude function of $H_r(\cdot)$ and $\theta_r$ is the phase angle.

The closed-loop bode diagrams in $s$ and $w$ domains for $T_{s1} = 0.25$ and $T_{s2} = 0.65$ are depicted in Fig. 7. Table 3 illustrates the resonant peaks and bandwidths of the frequency responses. As seen in Fig. 7, when $T_{s1} = 0.25$, frequency responses of all three transfer functions, $H_{s1}(e^{j\omega T})$, $H_{s2}(j\omega)$, and $H_{s3}(j\omega)$, are very similar. On the other hand, the bandwidth of $H_{s2}(j\omega)$ is considerably increased, while the resonant peaks of $H_{s1}(e^{j\omega T})$ and $H_{s2}(j\omega)$ are of the same. As a result, the approximation condition in section 3.1 is well adopted to the frequency response. However, it has been shown that the Houpis’s condition[4] is not enough for time and frequency domain characteristics.

C. Remark for the closed-loop design

When we go to design a digital controller for the sampled-data system by using direct approach, we first obtain a discrete-time plant model by means of the $z$-transform with ZOH. Then the model $G(z)$ is exactly matched with the continuous-time model, $G(s)$, at every sample instance whatever the sample time is. One of the most popular closed-loop design methods is the model matching. In this approach, a reference transfer function, $H^r(z)$, that satisfies the desired performances has to be determined. Since there are many methods for obtaining such a target model in continuous-time domain, we often use the $w$-transform and then may regard the resulting transfer function in $w$-plane as the continuous-time closed-loop system in $s$-plane. But, it is important to note that $H_w(s)$ may be quite different from $H(s)$ even though the sampling time is sufficiently small. The main reason comes from the fact that $H(s)$ is a nonlinear function of $H(s)$ as seen in (13). Fig. 8 illustrates qualitatively, the interrelationships between these three domain transfer functions.

![Fig. 7 Frequency responses of $H_1(s)$, $H_2(s)$, and $H_3(s)$.
](image)

![Table 3 Resonant peak and bandwidth of $H_1(z)$, $H_2(w)$, and $H_3(s)$ for $i = 1, 2$.](image)

![Table 3 Resonant peak and bandwidth of $H_1(z)$, $H_2(w)$, and $H_3(s)$ for $i = 1, 2$.](image)

In other words, the actual closed-loop transfer function of the continuous-time domain, $H(s)$, is not identical with $H_s(s)$. Here, we show this furthermore through the same example as the above.

Let us compare the continuous-time closed-loop transfer function, $H(s)$, with the $H_w(s)$ for $T_{s1} = 0.25$. The time and frequency responses are shown in Figs. 9–10 and Tables 4–5, respectively. The mapping error of the closed-loop poles results in $\epsilon_1 = 14.36\%$, which is larger than 3%. This makes the closed-loop system be different each other although the sampling time is $T_{s1} = 0.25$, which is the same as the previous example. Therefore, it is remarkable again that either $H_w(s)$ or $H_s(w)$ should be used for the selection of sampling time,
but $H_1(s)$ be not and neither does the open-loop system, $G(s)$.

![Figure 9](image_url)  
**Fig. 9** Step responses of $H_1(s)$ and $H_1(s)$.

![Figure 10](image_url)  
**Fig. 10** Frequency responses of $H_1(s)$ and $H_1(s)$.

### Table 4 Poles, zeros, and time responses of $H_1(s)$ and $H_1(s)$.

<table>
<thead>
<tr>
<th>Models</th>
<th>$H_1(s)$</th>
<th>$H_1(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>poles</td>
<td>$-1 \pm 2j$</td>
<td>$-0.7988 \pm 2.2502j$</td>
</tr>
<tr>
<td>zeros</td>
<td>$-5$</td>
<td>8. $-4.8732$</td>
</tr>
<tr>
<td>maximum overshoot [%]</td>
<td>23.44</td>
<td>38.90</td>
</tr>
<tr>
<td>rise time [sec]</td>
<td>0.6</td>
<td>0.548</td>
</tr>
<tr>
<td>settling time [sec]</td>
<td>3.55</td>
<td>4.60</td>
</tr>
</tbody>
</table>

### Table 5 Resonant peak and bandwidth of $H_1(s)$ and $H_1(s)$.

<table>
<thead>
<tr>
<th>Models</th>
<th>resonant peak [dB]</th>
<th>bandwidth [rad/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1(s)$</td>
<td>2.43</td>
<td>3.22</td>
</tr>
<tr>
<td>$H_1(s)$</td>
<td>5.04</td>
<td>3.92</td>
</tr>
</tbody>
</table>

### 4. Concluding Remarks

The $\omega$-transform is often used in analyzing stability and time response characteristics of discrete–time control systems because the rich theories developed in continuous–time domain can be easily applied to. In [1], a condition that accomplishes $H(s)$ $\approx H(\omega)$ has been presented. In this paper, we have pointed out that the Houpis’ condition is not sufficient for good approximation between $s$ and $\omega$ planes. A more rigorous condition has been proposed and examined by an example. Based on the new approximation condition, we showed how systematically the sampling time can be chosen so that the resulting closed loop systems coincide closely.

When a discrete–time reference transfer function is synthesized for use of direct digital design, one may regard a transfer function composed in $\omega$–plane as the continuous–time closed loop system in $s$–plane. Here, it has been pointed out that $H_1(s)$ may be quite different from $H_1(s)$ even though the sampling time is sufficiently small. According to this remark, we state that either $H_1(s)$ or $H_1(s)$ should be used for the selection of sampling time, but $H_1(s)$ be not and neither does the open–loop system, $G(s)$.

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**References**


저자 소개

Lihua Jin (金 麗 花)
was born in Jilin, China in 1978. She received the B.S. degree in the department of Electronic Engineering from Yanbian University, China in June 1999, and the M.S. degree in the department of Electronics Engineering from Chungbuk National University, Korea in August 2007. She is currently working towards Ph.D. degree on Electronics Engineering. Her research interests include low-order controller design, model-free/non-parametric model design, and system identification.

Young Chol Kim (金 永 喆)
received the B.S. degree in Electrical Engineering from Korea University, Korea in 1981, and the M.S. and the Ph.D. degrees in Electrical Engineering from Seoul University, Korea in 1983 and 1987 respectively. Since March 1988, he has been with the College of Electrical & Computer Engineering, Chungbuk National University, Korea, where he is currently a professor. He was a post-doctoral fellow at Texas A&M University in 1992 and a visiting research fellow at COE-ISM, Tennessee State Univ./Vanderbilt Univ. in 2001. He served the chairman of the KIEE Control and Instrument Society 2009–2010. His research interests include parametric robust control, low-order controller design, and transient response control via characteristic ratio assignment.