Multi-Objective Optimization of Electromagnetic Device Based on Design Sensitivity Analysis and Reliability Analysis

Ziyan Ren, Dianhai Zhang, Chanhyuk Park, Chang Seop Koh

Abstract - In this paper, for constrained optimization problem, one multi-objective optimization algorithm that ensures both performance robustness and constraint feasibility is proposed when uncertainties are involved in design variables. In the proposed algorithm, the gradient index of objective function assisted by design sensitivity with the help of finite element method is applied to evaluate robustness; the reliability calculated by the sensitivity-assisted Monte Carlo simulation method is used to assess the feasibility of constraint function. As a demonstration, the performance and numerical efficiency of the proposed method is investigated through application to the optimal design of TEAM problem 22—a superconducting magnetic energy storage system.

Key Words: Design sensitivity analysis, Multi-objective design optimization, Reliability, Robustness, Sensitivity-assisted Monte Carlo simulation

1. Introduction

Until now, there are two kinds of optimization algorithms to deal with uncertainties in engineering design problems: robust design optimization (RDO) [1] and reliability-based design optimization (RBDO) [2]. The RDOs focus on minimizing performance fluctuation resulted from variation of design variables to improve the robustness of performance (objective function), while the RBDOs lay stress on keeping the design feasibility with respect to constraints at the required probabilistic level.

Among the existing RDO algorithms, the gradient index approach is very effective since the performance robustness can be improved by minimizing the maximum absolute derivative of objective function with respect to design variables without consideration of any statistical information. In the conventional RBDO algorithm, the uncertainty is considered in the constraint function and the nominal constraint is transferred into a probabilistic constraint. Through checking if the design satisfies the probabilistic constraint or not during optimization process, finally, the reliable optimal solution will be obtained. It is obvious that both the RDO and the RBDO can improve the quality of optimal solution by emphasizing different aspects (variation of objective function or possibility of violating constraints). However, until now, in the electromagnetic optimization area, there are scarcely any researches to combine robustness and reliability in one optimization model especially in the form of multi-objective optimization.

In this work, with the help of sensitivity analysis assisted by finite element method (FEM), a reliability-based robust design optimization formulation is proposed by considering both robustness and reliability, where the RDO and the RBDO are combined in the multi-objective optimization form. In order to improve the numerical efficiency of optimization process, the reliability calculation is implemented by the first-order sensitivity-assisted Monte Carlo simulation method.

2. Multi-Objective Reliability-Based Robust Design Optimization (MO-RBRDO) Algorithm

2.1 Classical Optimization

The classical constrained optimization problem without consideration of uncertainty in design variables is generally formulated as follows:

\[ \min f(\mathbf{d}) \]
\[ \text{s.t. } g_k(\mathbf{d}) \leq 0, \quad k = 1, \ldots, M \]

where \( M \) is the number of constraints, and \( \mathbf{d} = [d_1, d_2, \ldots, d_N]^T \) is the design variable vector with \( N \) components. The optimal solution of (1), which gives the best
performance, usually locates on the interface constructed by constraints. Consequently, even there is a tiny uncertainty in design variables, the classical optimum will be perturbed to give a worse performance and violate some constraints, which should be evitable in the design of a real engineering problem. Therefore, the uncertainty cannot be ignored. In order to describe the uncertainty sententiously, the following assumptions are made:

1. all design variables (deterministic and uncertain) are treated as independent uncertain ones and are described by $x=(x_1,x_2,...,x_N)^T$;

2. the random variables $X$ follow Gaussian distribution with mean value $x$ and standard deviation $\sigma$, $X \sim N(x, \sigma)$. For a deterministic design variable, $\sigma$ is set zero;

3. based on (2), the variation range of $x$ is defined as uncertainty set $U(x)$ as:

$$U(x) = \{ \xi \in R^N | x - k\sigma \leq \xi \leq x + k\sigma \}$$  \hspace{1cm} (2)

where $\xi$ is a random perturbed design and the constant $k$ is determined according to the required confidence level (CL). Table 1 shows some representative CL values.

### 2.2 Proposed Multi-Objective Reliability-Based Robust Design Optimization

#### 2.2.1. Performance (Objective function) robustness

The gradient index $GI(x)$ ($x \in R^N$) defined as the maximum absolute derivative of objective function with respect to design variables is applied to measure the performance robustness:

$$GI(x) = \max \left( \frac{\partial f(x)}{\partial x_i} \right), \hspace{1cm} i=1,...,N.$$  \hspace{1cm} (3)

In order to guarantee the better performance (minimum objective function value in (1)) and stronger robustness under uncertainty simultaneously, the gradient index is treated as a single objective function to be minimized. Furthermore, the (3) does not need any statistic information such as the standard deviation and mean value of uncertainty in design variables. Therefore, the optimization efficiency can be improved.

#### 2.2.2. Constraint feasibility (reliability)

When the uncertainty is included in optimization problem, the satisfactory performance cannot be absolutely obtained. Instead, assurance of satisfactory performance can be made based on the probability of success in satisfying some constraint criterions. This probabilistic assurance of performance is called reliability. Firstly, for problem (1), $g_k(x)<0$ and $g_k(x)>0$ represent feasible and infeasible regions, respectively. The surface $g_k(x)=0$, ($k=1, 2, ..., M$) defines a limit state between feasible and infeasible regions.

In brief, for a constrained problem, the reliability of a specified design is defined as the probability to be keeping in the feasible region constructed by constraint functions in the parameter variation space:

$$R = \text{Prob}(g_k(x) \leq 0) = \int_{g_k(x) \leq 0} \phi(x)dx$$  \hspace{1cm} (4)

where the $\phi(x)$ is the joint probability density function of random variables $x$. Since the calculation of integration in (4) is very complex and also difficult, which results in the development of various approximation methods such as the Monte Carlo simulation (MCS) and the first–order reliability method (FORM). The MCS needs as many samples (normally 1,000,000) as possible to guarantee higher accuracy. The FORM needs to solve sub-optimization problem for each random design. Furthermore, the determination of proper initial point and step size in FORM is very difficult [3]. Therefore, it is very expensive when the MCS and the FORM are directly applied to the engineering problem involving finite element analysis. In section 3, one sensitivity-assisted MCS (SA–MCS) method developed in [3] is introduced, which requires neither iteration nor searching for the most probable point of failure in the optimization based methods such as FORM so that it is convenient to be applied to reliability analysis.

In the reliability-based design optimization, due to introduction of reliability, the general constraint is transferred into probabilistic one, which should be greater than or equal to a predefined target reliability $R^t$:

$$R_k(x) \leq R^t_k, \hspace{1cm} k=1,...,M$$  \hspace{1cm} (5a)

and consequently, the conventional reliability-based design optimization (RBDO) problem is formulated as follows:

$$\min f(x)$$  \hspace{1cm} (5b)

subject to: $R_k(x) \leq R^t_k, \hspace{1cm} k=1,...,M$.

During the optimization process, the reliability of each design with respect to all constraints is checked per iteration. As the design moves further inside the feasible region, it becomes more feasible and reliable.

#### 2.2.3. Combination of robustness and reliability

It is obvious that (3) and (5) have their own advantages in certain areas, however, the independent application of them cannot guarantee comprehensive improvement of solution quality. Therefore, they should be combined in the optimization process. In this paper,
the multi-objective reliability-based robust design optimization algorithm, MO-RBRDO, is proposed and formulated as follows:

\[
\begin{align*}
\text{min } & \quad f(x) \\
\text{min } & \quad G(x) = \max \{ |f(x)| / \sigma_n, i = 1, \ldots, N \} \\
\text{s.t. } & \quad R_i(x) \leq C_{k_i}^i, k = 1, \ldots, M
\end{align*}
\]

(6)

By using the multi-objective global optimization algorithms such as the multi-objective particle swarm optimization (MOPSO), a set of optimal designs which are in sensitive to variations and reliable to constraints under uncertainty can be obtained. The designer can select a proper solution based on different requirements.

If the original constraint \( g_k(x) \leq 0 \) is considered in (6), it will become the normal multi-objective robust optimization based on gradient index (MORO-GI). To make a distinction between the MORO-GI and the MO-RBRDO, solution of the MORO-GI is called Pareto optimal front, while solution of the MO-RBRDO is called the reliable Pareto-front. A visualized comparison between the MORO-GI and the MO-RBRDO is shown in Fig. 1.

Since additional condition (reliability) is attached in the MO-RBRDO, therefore the corresponding solution set can be said as a sub-Pareto front of the MORO-GI problem. It can be seen that in the sensitive region, gradient indices of designs on the reliable Pareto-front are a little bigger than those on the Pareto optimal front. It is because that the MO-RBRDO lays particular emphasis on constraint feasibility, and it makes a balance among objective function, GI, and reliability.

3. Design Sensitivity Analysis and Reliability Analysis

3.1 Design Sensitivity Analysis by Finite Element Method

In the finite element analysis, the design sensitivity can be obtained by direct differentiation method and adjoint variable (AV) method. Considering the numerical efficiency, the AV method is adopted in this paper. For the electromagnetic problem, the design target is a function of design parameter \([p]\) (nodal mesh related with real design variable \([x]\)) and magnetic vector potential \(A\), which is also a function of \([p]\). Therefore, by using the chain rule of differentiation, the design sensitivity of objective function \(f(x)\) or constraint function \(g(x)\) with respect to design parameter \([p]\) can be written as [4]:

\[
\frac{df}{dp} = \frac{\partial f}{\partial A} \frac{\partial A}{\partial p} + \frac{\partial f}{\partial [A]} \frac{\partial [A]}{\partial p}
\]

(7)

and the residual vector \([R]\) obtained from Galerkin approximation is obtained based on the linear or non-linear FEM as follows:

\[
[R] = [K + K_\ell] [A] - [Q]
\]

(8)

where \([K]\) and \([K_\ell]\) are linear and non-linear parts of the system matrix, respectively; \([Q]\) is the forcing vector. Differentiating both sides of (8) with respect to \([p]\), the derivative of \([A]\) in (7) can be expressed as:

\[
\frac{d[A]}{dp} = -[K + K_\ell]^{-T} \left( \frac{\partial [R]}{\partial [A]} \frac{\partial [A]}{\partial p} + \frac{\partial [R]}{\partial [p]} \frac{\partial [A]}{\partial p} + \frac{\partial [B]}{\partial [A]} \frac{\partial [A]}{\partial p} + \frac{\partial [B]}{\partial [p]} \right)
\]

(9)

where \(\nu\) is the magnetic reluctivity and \(B\) is magnetic flux density. In order to efficiently calculate the sensitivity, one adjoint variable \([\lambda]\) is defined as:

\[
[K + K_\ell] [\lambda] = \frac{\partial f}{\partial [A]}^T
\]

(10)

Substituting (9) and (10) into (7), finally the design sensitivity can be obtained as follows:

\[
\frac{df}{dp} = -[\lambda]^{T} \left( \frac{\partial [R]}{\partial [p]} \frac{\partial [A]}{\partial p} + \frac{\partial [R]}{\partial [A]} \frac{\partial [A]}{\partial p} + \frac{\partial [B]}{\partial [A]} \frac{\partial [A]}{\partial p} + \frac{\partial [B]}{\partial [p]} \right)
\]

(11)

Due to the application of adjoint variable, the above method is named adjoint variable method (AV) [4]. It can be seen that the AV method only needs twice FEM analysis (one time for (8), and one time for (10)) to obtain both performance and gradient information.
3.2 Implementation of Reliability Calculation

As we know, the MCS can be easily applied to the analytic function, while for the real electromagnetic problem, it is impossible due to the expensive performance analysis. Therefore, in this paper, the value of performance constraint $g_k(x) \leq 0$ in the uncertainty set is approximated by using the first-order Taylor expansion as follows:

$$g_k(\xi) \approx g_k(x) + \nabla g_k(x) \cdot (\xi - x), \quad k = 1, \ldots, M$$ (12)

where $\xi$ is the randomly generated test point in the uncertainty set $U(x)$ and the gradient information $\nabla g_k(x)$ is calculated at the nominal design $x$ using sensitivity analysis by the FEM. Due to the approximation, the performance constraint can be treated as an analytic function in the uncertainty set. Then the MCS method can be applied to the approximated function in (12) [3], and the reliability is calculated as follows:

$$R[g_k(x) \leq 0] = n/m$$ (13)

where $m$ is the total number of test designs randomly generated in the uncertainty set of the specified design $x$, and $n$ is the number of test designs satisfying the $k$-th constraint $g_k(x) \leq 0$.

Since the sensitivity analysis and the Monte Carlo simulation method are combined to calculate reliability, so the above method is called sensitivity-assisted MCS (SA–MCS) method. Fig. 2 gives the flowchart of the SA–MCS method.

3.3 Computational Cost

Considering both sensitivity analysis by the adjoint variable method and the reliability analysis by the SA–MCS method, Table 2 compares the computational cost per optimization step for the robustness and reliability evaluations of different algorithms for a problem with $M$ constraint functions. In table 2, the former and latter values in sensitivity column represent number of sensitivity analysis for the objective function and constraints, respectively.

It can be seen that the independent application of the RBDO and the MORO–GI need $(1+M)$ and $2$ FEM analysis, respectively. However, the combination of them, MO–RBRDO costs $M$ more times FEM analysis than the MORO–GI, one more FEM analysis than the RBDO. It is true the multi–objective problem (MORO–GI and MO–RBRDO) is more complexity than the RBDO, so the total function calls may be larger. For a engineering optimization problem, normally the number of constraints is less than five or six. Therefore, the computational cost of the MO–RBRDO can be acceptable. On the other hand, from the view point of making balance among the optimality, robustness, and reliability, the MO–RBRDO is best than the others.

<table>
<thead>
<tr>
<th>Methods</th>
<th>No. of FEM calls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Performance</td>
</tr>
<tr>
<td>MORO–GI</td>
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</tr>
<tr>
<td>RBDO</td>
<td>1</td>
</tr>
<tr>
<td>MO–RBRDO</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 3 Configuration of TEAM problem 22

Fig. 4 Quenching curve of the NbTi–superconductor.

4. Optimal Design of Superconducting Magnetic Energy Storage System

TEAM problem 22 is a standard test problem to check different optimization algorithms [5], which is one superconducting magnetic energy storage system as shown in Fig. 3. The system consists of two concentric coils: inner main solenoid coil to generate magnetic energy and outer shielding solenoid to reduce the stray field, and the current directions of the two coils are opposite to each other. In Fig. 3, $R$, $H$, and $D$ represent radius, height, and diameter of inner and outer coils, respectively. The design of TEAM 22 should satisfy the following requirements:

1. the total system energy $E(x)$ should be as close as
to 180 MJ;

(2) the magnetic stray field \(B_{\text{stray}}\) measured from testing points on lines a and b in Fig. 3 should be as small as possible. Furthermore, since inside and outside coils are made of superconducting material (NbTi), to keep the superconductivity, one performance constraint should be considered as shown in Fig. 4. and is approximated by mathematical formulation as follows:

\[
g_k(x) = |B_{\text{m,k}} + 6.4R_{\text{n,k}}| - 54.0 \leq 0, \quad k = 1, 2
\]

(14)

where \(B_{\text{m,k}}\) and \(J_k\) is the maximum magnetic flux density and current density in the \(k\)-th coil, respectively. The deterministic design variables are the geometric parameters of outside coil: \(x = [R_2, H_2, D_2]^T\). The uncertainties are considered in the physical parameters (current densities, \(J_1\) and \(J_2\)) following Gaussian distribution as \(J_1 \sim N(\mu = 22.5, \sigma)\) MA/m² and \(J_2 \sim N(\mu = -22.5, \sigma)\) MA/m².

The two design targets are combined into one formulation to represent the objective function as follows:

\[
f(x) = \frac{B_{\text{stray}}^2}{B_{\text{norm}}^2} \frac{|E(x) - E_{\text{ref}}|}{E_{\text{ref}}}
\]

(15a)

\[
B_{\text{stray}}^2 = \frac{1}{22} \sum_{i=1}^{22} B_i^2
\]

(15b)

where the \(E_{\text{ref}}\) is the reference system energy \(E_{\text{ref}} = 180\) MJ; \(B_{\text{norm}}\) is the reference stray field \(B_{\text{norm}} = 3\) mT for the three–dimension problem; and \(B_i\) is the magnetic flux density of the \(i\)-th test point.

For objective function, only magnetic stray field includes uncertainty, therefore, the gradient index can be calculated as:

\[
\text{GI}(x) = \frac{1}{B_{\text{norm}}} \cdot \max[|\nabla B_{\text{stray}}|]
\]

(16)

For reliability calculation, the performance constraints should be approximated as follows:

\[
g_k(\xi) \approx g_k(x) + \nabla g_k(x) \cdot (\xi - x)
\]

(17)

with the help of sensitivity analysis by the FEM, the corresponding gradient parts in (16) and (17) are expressed as:

\[
\nabla B_{\text{stray}} = \frac{\partial B_{\text{stray}}}{\partial [\xi]} + [\lambda]^{\top} \frac{\partial Q}{\partial [\xi]}
\]

(18a)

\[
\nabla B_{\text{n,k}} = \frac{\partial B_{\text{n,k}}}{\partial [\xi]} + [\lambda_{\text{n,k}}]^{\top} \frac{\partial Q}{\partial [\xi]}, \quad k = 1, 2
\]

(18b)

where the adjoint variables \(\lambda\) and \(\lambda_{\text{n,k}}\) are obtained by solving the following equations:

\[
[K]^{\top} \lambda = \frac{\partial B_{\text{stray}}}{\partial [\xi]}
\]

(19a)

\[
[A]^{\top} \lambda_{\text{n,k}} = \frac{\partial B_{\text{n,k}}}{\partial [\xi]}, \quad k = 1, 2
\]

(19b)

The MOPSO with 50 particles and 300 maximum iterations is applied to solve the following MO–RBRDO problem:

\[
\min f(x) = \frac{B_{\text{stray}}^2}{B_{\text{norm}}^2} \frac{|E(x) - E_{\text{ref}}|}{E_{\text{ref}}}
\]
\[
\min \ G(x) = \frac{1}{R_{\text{sem}}} \max_{1 \leq i \leq 2} \left| \frac{dE_{\text{res}}}{dI_i} \right|
\]
\[
\text{s.t.} \quad R_i |J_i| + 6.4 B_{\text{norm}} |\leq 54.0 | \geq R^*_i, \quad k = 1, 2. \\
R_i |J_i + D_i/2 - (R_i - D_i/2) | \geq R^*_i
\]

where the third geometric constraint avoids overlap of the inside and outside coils.

Uncertainties in current densities are assumed as \( \sigma = 0.179 \text{ MA/m}^2 \) with a confidence level of 0.95. For reliability analysis, the maximum test designs in the SA-MCS and target reliabilities for all constraints are set as 10,000 and 0.9, respectively. The corresponding optimization results are shown in Fig. 5 and Fig. 6. It can be seen that the MO-RBRDO can supply a Pareto-front by trading-off performance and robustness, at the same time the reliability of each design with respect to constraint can also be improved. Furthermore, all the reliable Pareto optimal designs have enough margins even perturbed by uncertainties as shown in Fig. 6.

For further comparison, several optimal designs obtained by deterministic optimization algorithms without considering uncertainty are selected from recently published papers as shown in Table 3, where the word of ‘classical’ means optimal result of (1) obtained by single objective particle swarm optimization in this paper. All the quantities are recalculated by our FEM program. In order to keep the generality, the reference magnetic field of eight-dimension problem is also set as \( B_{\text{norm}} = 3 \text{ mT} \). If optimal designs in Table 3 are perturbed by same uncertainties as that in the MO-RBRDO, the corresponding quantities such as reliability and gradient index are calculated and listed in Table 4.

Through comparison between the proposed MO-RBRDO algorithm and the deterministic optimization algorithm, the following investigations can be obtained:

1. Form the view point of constraint feasibility
   It can be seen that all reliable designs in Fig. 5 satisfy the probabilistic constraints and reliabilities are greater than 0.9. However, the optimal designs obtained from deterministic optimization are not always reliable. For example, design [7]-1 and design of 8D problem in [5] show lower reliability, more than that, design [7]-2 nearly locates on the constraint surface and is very easily perturbed to the infeasible region. Therefore, the reliability analysis is very essential in engineering optimization, the proposed MO-RBRDO algorithm successfully implements this requirement.

2. From the view point of performance robustness
   With a deviation of \( \Delta J_a = [0.35, 0.35] \text{ MA/m}^2 \), the maximum perturbed objective function is approximated as follows:

\[
[f(x)]_{\text{perturbed}} \approx f(x) + \sqrt{2} GI \cdot \sqrt{(\Delta J_1)^2 + (\Delta J_2)^2}.
\]

Table 3 Optimal designs from published papers

<table>
<thead>
<tr>
<th>Ref</th>
<th>( R_1 )</th>
<th>( H_1/2 )</th>
<th>( D_1 )</th>
<th>( R_2 )</th>
<th>( H_2/2 )</th>
<th>( D_2 )</th>
<th>( J_1 )</th>
<th>( J_2 )</th>
<th>( B_{\text{norm}} )</th>
<th>( E(x) )</th>
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<tbody>
<tr>
<td>[5][3D]</td>
<td>2.0</td>
<td>0.8</td>
<td>0.27</td>
<td>3.08</td>
<td>0.239</td>
<td>0.394</td>
<td>22.5</td>
<td>-22.5</td>
<td>8.820E-2</td>
<td>7.9138E-7</td>
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<tr>
<td>[6]</td>
<td>2.0</td>
<td>0.8</td>
<td>0.27</td>
<td>3.0888</td>
<td>0.2404</td>
<td>0.3886</td>
<td>22.5</td>
<td>-22.5</td>
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<tr>
<td>[7]-1</td>
<td>2.0</td>
<td>0.8</td>
<td>0.27</td>
<td>3.044</td>
<td>0.2665</td>
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<td>0.27</td>
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<td>0.3557</td>
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<tr>
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<td>0.27</td>
<td>3.0819</td>
<td>0.2439</td>
<td>0.3849</td>
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<td>8.772E-2</td>
<td>7.8948E-7</td>
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<tr>
<td>[8]</td>
<td>1.320</td>
<td>1.070</td>
<td>0.590</td>
<td>1.800</td>
<td>1.48</td>
<td>0.15</td>
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<td>[5][8D]</td>
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<td>1.513</td>
<td>0.195</td>
<td>16.695</td>
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<tr>
<td>[9]</td>
<td>2.6616</td>
<td>1.0494</td>
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Table 4 Results of designs in Table 3 perturbed by uncertainty

<table>
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<tr>
<th>Ref</th>
<th>Nominal constraints</th>
<th>Reliability</th>
<th>Gradient index</th>
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<tr>
<td>([f(x)]_{\text{perturbed}} )</td>
<td>( GI )</td>
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<td></td>
</tr>
<tr>
<td>[5][3D]</td>
<td>-7.9321</td>
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<td>[6]</td>
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<td>-1.3876</td>
<td>1.00</td>
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<tr>
<td>[8]</td>
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<td>0.7230</td>
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<td>3.1849E-2</td>
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<tr>
<td>[9]</td>
<td>-19.5904</td>
<td>-25.0185</td>
<td>1.00</td>
</tr>
</tbody>
</table>

a-To keep generality with \( GI \) in Fig. 5, all the gradient values are multiplied by 100.
The perturbed reliable Pareto optimal designs are shown in Fig. 5. Since the MO-RBRDO tries to minimize the maximum deviation of objective function with respect to uncertain design variables, so that the performance robustness of all the optimal designs is improved by showing a little deviations as in Fig. 5. Fig. 7 shows that the perturbation of optimal designs from deterministic optimization. It is obvious that a smaller deviation in design variables will drive the design to give unacceptable performance such as design [9] with a bigger gradient index of 0.8526. Therefore, uncertainty in design variables is not negligible.

5. Conclusion

In order to deal with uncertainties in design variables, the robustness and reliability is integrated in a single optimization model, and one reliability–based robust design optimization algorithm is proposed. By applying the SA–MCS method to efficiently calculate the reliability, the computational cost and complexity of the reliability–based robust design optimization is greatly reduced. The numerical result of TEAM 22 verifies that the proposed reliability–based robust design optimization can improve not only the performance robustness but also the reliability of constraints. Therefore, this paper guides the research direction to obtain robust, reliable, and optimal design in the area of electrical engineering.

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참고 문헌


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