A Frequency-Dependent Model of a Cabtyre Cable for a Transient Analysis

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Abstract – A frequency-dependent line model is very significant for an accurate analysis of line and cable transients. This paper has proposed a methodology to obtain a frequency-dependent distributed-parameter line model of a cabtyre cable by employing an exponential curve fitting from a measured transient voltage and an injected current. The characteristic admittance and propagation function are expressed by exponential functions in Semlyen line model of EMTP (Electro-magnetic transient program) and Maple software has been used as an optimization tool. The frequency dependence of the per unit series impedance and shunt admittance and their phase angles are derived from the obtained propagation parameters. Simulation results by the proposed model agree well with the measured results. The proposed approach is expected to be useful to analyze a transient associated with the cabtyre cable.

Keywords: Cabtyre cable, EMTP, Frequency-dependent effect, Line model, Wave propagation

1. Introduction

In the simulation of switching and fault transients on a distributed parameter line such as a transmission line, the frequency dependence of the line parameters contributes to wave distortion of surges on the line. A wave propagation characteristic at a high frequency is very significant for studying a surge voltage. For transmission line and cable, the frequency dependent parameters are characterized by the geometric and the physical constants. However, no such formula is available for a cabtyre cable and fitting is mainly done based on an experimental observation in a time-domain [1].

In this paper, a model of the cabtyre cable is developed under the assumption that an overhead line for its impedance can be calculated using the formula for neglecting the relative permittivity $\varepsilon_r$ of the cabtyre cable insulator i.e. $\varepsilon_r = 1$ for the air. The cable admittance is calculated using the formula for a pipe-type overhead cable [2], with multi-conductor matrix analysis [3]. Firstly, a code to calculate cabtyre cable parameters is developed using Maple. To incorporate the frequency-dependent effect in Semlyen’s line model [4] of EMTP [5], required data are produced in the Maple program.

There are three factors of the frequency-dependent effects of a multiphase cable: propagation constant, characteristic impedance and modal transformation matrix. The first two factors significantly affect transmission line transients, but the frequency-dependent effect of the modal transformation matrix can be neglected, if the three cores are symmetrically arranged. A frequency-dependent distributed-parameter model of a cabtyre cable is based on time-domain exponential fitting from a measured sending-end transient voltage and an injected current. The transient characteristic impedance and propagation function are expressed by exponential functions. The time-domain result is transformed into a frequency domain by Laplace transform. The frequency-dependent effects of the line parameters are derived from the results. A transient simulation by the proposed approach is carried out, and the result is compared with a measured result.

Finally, validity of model is investigated by practical inverter surge measurements. A study is carried out in comparison with conventional (Dommel’s) line model and frequency dependent Semlyen’s line model of EMTP.

2. Theory of Semlyen’s Line Model

2.1 Basic theory of semlyen’s line model

Semlyen’s frequency-dependent model is an approach to carry out a real time convolution recursively in the following manner.

$$v(t) = h(t) * e(t) = h(0)e(t) + \int_0^t h(t) e(t-\tau) d\tau$$  (1)

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where $v(t)$: voltage response including the frequency response of a line
$e(t)$: original voltage of which the frequency dependence to be included
$h(t)$: step response of the frequency dependence

If $h(t)$ is expressed by a combination of exponential functions, for example as in eq.(2), then eq.(1) is rewritten as in eq.(3).

$$h(t) = \beta \left[ 1 - \exp(-\alpha t) \right] / \alpha$$

$$v(t) = v(n_{10}) = a h_{n-1} + b h_n + c l_{n-1} + d l_n - 2$$

where

$$h_{n-1} = h[n-1] \cdot \varepsilon_n = e(n_{10}), \quad a = \exp(-\alpha_{10})$$

$$a_b = \beta \left[ (1 - \alpha) / (\alpha_{10})^2 - (3 - \alpha) / 2 \alpha_{10} + 1 \right]$$

$$a_c = \beta \left[ 2(1 - \alpha) / (\alpha_{10})^2 + 2(\alpha_{10} - \alpha) \right]$$

$$a_d = \beta \left[ (1 - \alpha) / (\alpha_{10})^2 - (1 + \alpha) / 2 \alpha_{10} \right]$$

2.2 Model parameters

It is clear from eqs. (2) and (3) that the frequency-dependent effect of a line can be included analytically in a transient simulation by the EMTP [4,5], if $\alpha$ and $\beta$ are given. The EMTP subroutines “Line Constants” and “Cable Constants” provide the necessary data such as the line impedance and the admittance to determine the above parameters in most overhead lines and underground cables [6,7]. In the case of a multi-core cabtyre cable, no impedance and admittance formula is available, and thus the parameters cannot be given by the Line Constants/Cable Constants. In such a case, the easiest way of determining the parameters is a measurement of voltages and currents [1]. If a measured result of $h(t)$ is obtained, the parameters $\alpha$ and $\beta$ can be easily determined by curve fitting of the measured $h(t)$. Also, it is possible to determine the parameters by transforming eq. (2) into a frequency domain.

$$H(s) = \beta / \left( 1 + s - 1/s + \alpha \right)$$

where $s = j \omega + \alpha$ : Laplace operator

The step response on a distributed-parameter line is defined theoretically when a step functions with a unit amplitude “1/s” by:

$$v(t) = \mathcal{L}^{-1} \left[ \Gamma(s) \right] 2 / s$$

where $\Gamma(s)$: propagation constant

If $Z$ and $Y$ cannot be given theoretically, then those can be obtained by measurements. Once eq. (5) is obtained, then $\alpha$ and $\beta$ in eq. (4) are determined by curve fitting of eq. (5).

3. Determinations of Line Parameters by Measured Results

There are various methods for determining line parameters such as a conductor impedance and an admittance [8].

1) In frequency domain:

By applying a sinusoidal current with frequency $f$[Hz], amplitude 1[A] and phase angle 0[deg] to the sending end of a line, a measured sending-end voltage gives the open–circuit impedance $Zf[\Omega]$ when the remote end being open-circuited, and the short-circuit impedance $Zs[\Omega]$ when the remote end being short-circuited. From $Zf$ and $Zs$, the propagation constant $\Gamma$ and the characteristic impedance $Zo$ are evaluated, and the series impedance $Z$ and the shunt admittance $Y$ are obtained as:

$$Z = Z_o \Gamma$$

$$Y = \Gamma / Z_o$$

With a variable frequency source, the frequency characteristics of $\Gamma$, $Z_o$, $Z$, and $Y$ are obtained. Note that the attenuation constant $\alpha$ and the phase constant $\beta$ are given as the real and the imaginary parts of $\Gamma$.

2) In time domain:

By applying a step function voltage of the amplitude 1[pu] to a line with length $l$, a measured voltage $v(t)$ at the sending end corresponds to the following formula when the remote end is open-circuited.

$$v(t) = \mathcal{L}^{-1} \left[ \Gamma(s) \right] 2 / s$$

$$v(t) = \mathcal{L}^{-1} H(s)$$

When the series impedance $Z$ and shunt admittance $Y$ are given theoretically, the propagation constant and the characteristics impedance $Z_o$ are obtained from $Z$ and $Y$ in the following form.

$$\Gamma^2(s) = Z(s)Y(s), \quad Z_o^2(s) = Z(s)Y(s)$$

If $Z$ and $Y$ cannot be given theoretically, then those can be obtained by measurements. Once eq. (5) is obtained, then $\alpha$ and $\beta$ in eq. (4) are determined by curve fitting of eq. (5).
where \( H(s) \) is given in eq. (5).

By curve fitting of measured \( v(t) \) following eq.(2), the parameters required for Semlyen’s line model can be obtained. If the measured \( v(t) \) is transformed into a frequency domain, curve fitting of \( V(s) \), according to eq.(4), gives the parameters.

\[
V(s) = L v(t)
\]  

(9)

Eq. (5) is rewritten to:

\[
H(s) = \exp(-2\alpha t) \exp(-2j\beta t) = \exp(-2\alpha t) \exp(-2\tau) s
\]

(10)

where \( \tau = \ell/c \) : traveling time for a length ‘\( \ell \’ \)

\( c = \omega/\beta \): propagation velocity

Considering that \( \exp(-sT) \) expresses a time delay of a wave traveling length “cT”, eq. (10) is rewritten by:

\[
H(s) = \exp(-2\alpha \ell) \exp(-2\tau) s
\]

(11)

The time delay “\( 2\tau \)” is easily evaluated from the measured result \( v(t) \), and also the attenuation “\( \alpha \)” is calculated from the measured results of \( v(t) \) and the applied voltage. Similarly from measured current \( i(t) \) at the sending end, the characteristic impedance is evaluated by:

\[
Z_0 = \frac{V(s)}{I(s)}
\]

(12)

where \( I(s) = \mathcal{L} i(s) \)

Having known \( \Gamma \) and \( Z_0 \), the line impedance \( Z \) and the admittance \( Y \) are evaluated by eq. (7)

### 3.2 Measurements of voltages and currents

Fig. 1 illustrates a cabtyre cable with three cores used in measurements. The cable is arranged at the height of 6.4mm above an aluminum plate. The cable length is taken to be \( l=30.2m \). Since the cabtyre cable is composed of multi cores, the above explained method has to be applied to modal components [3]. Because of the point symmetry of the three cores which are twisted, the transformation matrix between modal and phasor component is assumed to be not frequency-dependent and nearly equal to that given by a symmetrical component theory. Then, circuits to measure modal quantities are derived as Fig. 2.

In the frequency domain measurement, an impedance analyser (IA) (Agilent model 4294A 40Hz - 110MHz) is used. For a transient measurement, a pulse generator (PG) is used as a source voltage. A current is evaluated from source voltage \( v_s(t) \) and sending-end voltage \( v_t(t) \). All the voltages are measured by an oscilloscope, Tektronix DPO 4104, 1GHz.

### 4. Comparison of Measured and Simulation Results

#### 4.1 Line parameters

Fig. 3 to Fig. 6 show a comparison of measured results and simulation results by Semlyen’s line model for mode-1 and mode-2 by real line and dotted line respectively. In Fig. 3(a) for the attenuation, the maximum difference reaches 3dB/km, but the propagation velocity in Fig. 3(b) shows a satisfactory agreement in the whole frequency ranges. The characteristic impedance in Fig. 4 shows reasonable agreements.

The series impedance in Fig. 5 and the shunt admittance in Fig. 6 show reasonable agreement between the measured results and the simulation results by Semlyen’s model. Thus the proposed approach of determining Semlyen’s line model parameters can be said to have a reasonable accuracy. A comparison for mode-0 quantities is not given here, because the mode-0 does not contribute a transient phenomenon associated with cabtyre cable applications.
Fig. 3. Measured and Semlyen results of propagation constant.

Fig. 4. Measured and semlyen results of characteristic impedance.

Fig. 5. Measured and Semlyen results of series impedance.

Fig. 6. Measured and semlyen results of shunt admittance.
4.2 Transient simulation of inverter surge

Validity of modal measurements are investigated for an inverter surge for three conditions:
(1) No cable connection between the inverter and the motor
(2) With 2.18m cable and
(3) With 30.2m cable

![Experimental set-up.](image)

Fig. 7. Experimental set-up.

Fig. 7 illustrates an experimental circuit of measuring a surge in an inverter circuit connected by a cabtyre cable to a 3 phase squirrel cage induction motor (2.2 kW, 50Hz, 200V, 9.2A, 1430RPM). A 7.5A/3.0kVA PWM inverter (Type VFS7-2015P, Toshiba Corporation) is used as a 200V, 60Hz source. Terminal R, S and T represents three phase supply voltage. Converter is represented by D1-D6 diodes, capacitor C is DC link and inverter transistors are represented by Sw1-Sw6 switches.

1) Inverter side voltages

Fig. 8 shows voltages measured at inverter side i.e. supply voltage and phase voltages between phases U, V and W.

![Inverter side voltages.](image)

(a) No cable connection

(b) With 2.18m cabtyre cable

(c) With 30.2m cabtyre cable

Fig. 8. Inverter side voltages.

![Motor side voltages.](image)

(a) Supply AC Voltage

(b) Voltage between phases UV

(c) Voltage between phases UW

(d) Voltage between phases VW

Fig. 9. Motor side voltages.
2) Motor side phase voltages

Fig. 9 shows phase voltages measured at motor terminals for three conditions, only one phase is discussed in detail. A similar trend is observed for the other two phases. There is no significant difference in the phase voltages across the motor when there is no cable connection or a short cable (2.18m) is used for connection between the motor and the inverter. However, it is clear from the measured results that the cabtyre cable length affects the voltage waveform and its peak value. As the length increases, the waveform becomes more sinusoidal rather than square like, and the peak voltage becomes higher.

4.3 Comparison of measured and simulated voltages

A motor winding may partially discharge due to an inverter surge giving a significant impact on its insulation, and lead to a breakdown. A soft switching technology has been developed to suppress the surge voltage. It requires to evaluate the surge in advance. For this, an accurate prediction of the surge voltages at the terminals of the motor becomes very significant. Models for the cabtyre cable and the motor are required for the prediction.

An inverter surge is simulated using two cabtyre cable models and a simple motor model. One of the cable models is a conventional line mode (Dommel’s line model) and the other cable model is Semlyen’s frequency-dependent model of the EMTP. The motor model is expressed by an equivalent circuit consisting of three resonant circuits with stray capacitors.

A magnified result of Fig. 9 (c) in micro-secs is shown in Fig. 10. It is observed in the figure that the steady-state voltage is 280volts, when a step like voltage shown in Fig. 11 is used as an input voltage with the amplitude of 280 volts for a simulation.

Fig. 12 shows a comparison between measured and simulation results by the Dommel’s line model and by the frequency-dependent Semlyen’s line model. The steady state voltage is 280volts in each result. The measured peak voltage is 523volts, and 527volts by Semlyen’s line model, while it is 570volts by Dommel’s line model. It is observed that the Dommel’s model gives a much higher peak voltage, higher by about 50V, than the voltage calculated by the Semlyen’s model which agrees well with the measured one. This is estimated to be caused by the frequency dependence of the cable. The above observation has made it clear that a transient associated with a cabtyre cable can be simulated well by proposed method in co-operation with Semlyen’s line model of the EMTP.

Fig. 10. Measured micro-surge at motor terminal.

Fig. 11. Simulated input voltage.

Fig. 12. Comparison of measured and simulation results.
5. Conclusions

In this paper, measurements of wave propagation characteristic on a cabtyre cable were carried out in frequency and time domains. Based on the measured results, a frequency-dependent model of the cabtyre cable has been developed for a transient simulation by the EMTP. The developed model is directly incorporated with the Semlyen’s line model, and also with the Dommel’s distributed-line model prepared in the EMTP. A reasonable agreement has been obtained in measured results and EMTP simulation results by using the developed model. The developed cabtyre cable model is expected to be useful to an inverter surge analysis and other transients in a circuit involving the cabtyre cable.

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