Transmission Network Expansion Planning Based on Dependent-chance Bi-level Programming Method

Hong Fan† and Hao-zhong Cheng*

Abstract – Transmission Network Expansion Planning method considers return rate based on Dependent-chance bi-level programming method is presented in the paper. The upper level objective of the model is maximum realization rate of return rate being bigger than one realization level. The lower level programs have two sub-problems, one is social profit maximization in normal operation, the other is load shedding being lower than one realization rate in N-1 operation environment. Monte-Carlo method is adopted to mimic uncertain parameters. A hybrid algorithm combining Genetic algorithm and prime-dual interior point method is adopted to solve the proposed model. 18-bus system has been tested in the paper, the results prove proposed method is valid.

Keywords: Electric power system, Transmission Expansion Planning, dependent-chance programming, bi-level programming

1. Introduction

Traditional transmission network expansion planning should ensure the optimal transmission network scheme with minimum investment cost and operation cost according to different load value in different planning period[1].

The perfect traditional transmission network expansion planning model is a complex uncertain dynamic multi-objective hybrid integer nonlinear programming model, the model is so complicated and hard to solve. So the model generally can be simplified from different factors[2]. With the development of mathematic algorithms and computing technologies, transmission network expansion planning model can be solved, it has two categories, one is mathematic programming methods, the other is heuristic algorithm.

In the process of transmission network expansion planning, some factors such as load and generators are variable, so certain programming method is hard to describe the problem, so traditional transmission network expansion planning has big challenges. In recent years, uncertain programming method[3] are used for market-based transmission network expansion planning problem[4,5], the optimal planning scheme has good flexibility to future environment.

† Corresponding Author Dept of Electrical Engineering Shanghai University of Electric Power, Shanghai, China
(fan_hongong@126.com)

* Department of Electrical Engineering Shanghai Jiaotong University, Shanghai, China

Received: November 19, 2013; Accepted: January 6, 2014

With the development of bi-level programing theory, it is also applied in transmission network expansion planning problem[6,7,8]. Dependent-chance bi-level programming method is a one of bi-level programming method, in the paper, it is used for transmission network expansion planning problem.

This paper is organized as follows: Section II describes the dependent-chance bi-level programming and gives transmission network expansion planning model based on dependent-chance bi-level programming. Section III provides a brief description of algorithm to solve TNEP problem. Section IV shows the computational results. Section V summarizes the conclusions of the work.

2. Mathematical modeling

2.1 Dependent-chance bi-level programming

Dependent-chance bi-level programming is a leader-follower structure. In this structure, the upper level program is the leader and the lower level program is the follower. The dependent-chance bi-level programming modeling is one of the uncertain bi-level programming which can be formulated as follows:

\[
(P_1) \quad \text{Max} \ Pr\{ F(x, v, \xi) \geq \bar{F} \} \\
\text{s.t.} \quad G(x, \xi) \leq 0 \\
(P_2) \quad \text{Max} \ Pr\{ f(x, y, \xi) \geq \bar{f} \} \\
\text{s.t.} \quad g(x, y, \xi) \leq 0
\]
Transmission Network Expansion Planning Based on Dependent-chance Bi-level Programming Method

Where \((P_1)\) is the leader program; \((P_2)\) is the follower program; Function (1) is the upper-level objective, function (2) is upper-level constraints, function (3) is lower-level objective, function (4) is lower-level constraints. \(x \in \mathbb{R}^n\) is upper-level deciding vectors, \(y \in \mathbb{R}^n\) is lower-level deciding vectors, \(\xi \in \mathbb{R}^n\) is stochastic vectors, \(\Pr\{\\}\) is probability function, \(G(x, \xi) \leq 0\) is upper-level constraints, \(g(x, y, \xi) \leq 0\) is lower-level constraints.

2.2 Transmission network expansion planning modeling

The above dependent-chance programming is applied to transmission network expansion planning problem, which can be formulated as follows:

The upper-level objective

\[
(P1) \quad \text{Max } f_1(x) = \Pr\{F \geq R\} + \alpha \cdot f_2
\]  
\[
F = (T_{c,t}(t_i) - I_{c,t}(t_i)) / I_{c,t}(t_i)
\]  
\[
I_{c,t}(t_i) = \sum_{l \in \Omega} c_l Z_l \frac{-(1+i)^{v_l}}{(1+i)^{v_l} - 1}
\]  
\[
\text{s.t. } 0 \leq Z_i \leq Z_i
\]  

In the model, function (5) is the upper level program objective, \(F\) is the investment rate of transmission network. \(R\) is one ideal value called realization value. Equation (6) is the function of \(F\), Equation (7) is investment per year. In equation (6) and (7), \(T_{c,t}(t_i)\) is transmission network operation profit per year, \(c_l\) is the cost of new-added line \(l\), \(Z_l\) is the number of new-added line \(l\), \(v_l\) is the transmission network investment callback period, \(\Omega\) is the set of new-added lines, \(I_{c,t}(t_i)\) is transmission network investment cost per year. In the upper-level program objective, the lower-level program objective is a part of the upper-level program objective, it realizes the upper-level program and lower-level program interaction.

The upper-level constraints:

\[
\text{s.t. } 0 \leq Z_i \leq Z_i
\]

Constraint (8) is new-added line \(l\) constraint, \(Z_i\) is the maximum number of new-added line \(l\).

The lower-lever program has two sub-problems, one is social profit maximization in normal operation, the other is the probability maximization of load curtailment less than one given value. Owe to the different operation condition, the two problems can be two isolate problems.

The first sub-problem objective

\[
(P21) \quad \text{Max } f_1(x, y) = \left(\sum_{t \in \Omega} B(t_i) - \sum_{t \in \Omega} C(t_i)\right)
\]  

Equation (9) is the objective of the first lower-level program, it is social profit maximization objective, deciding vectors includes the upper-level program deciding vectors and the first lower-level program deciding vectors, such as generation power, load, bus voltage, bus angle.

The first lower-level program constraints:

\[
\text{s.t. } 0 \leq P_{gj}(t_i) \leq P_{gj}
\]  
\[
0 \leq P_{l}(t_i) \leq P_{l}
\]  
\[
\sum_{t \in \Omega} P_{gj}(t_i) - \sum_{t \in \Omega} P_{l}(t_i) = 0
\]  
\[
|P_{l}(t_i)| \leq \overline{P}
\]  
\[
T_{RC}(t_i) \leq \omega \cdot T_{ES}(t_i)
\]

Equation (10) is the active power of generation bus \(l\), Equation (11) is the active load of load bus \(j\). Equation (12) is the power equilibrium. Equation (13) is the active power of line \(l\), equation (14) is the transmission jam-up constraints. Where, \(P_{gj}(t_i)\) is the active power of generation bus \(i\), \(P_{gj}\) is the maximum active power of generation bus \(i\), \(P_{l}(t_i)\) is the active load of load bus \(j\), \(P_{l}\) is the maximum active load of load bus \(j\), \(P_{l}(t_i)\) is the active power of line \(l\), \(\overline{P}\) is the maximum active power of line \(l\), \(T_{ES}(t_i)\) is the profit of transmission service, \(T_{RC}(t_i)\) is the profit of transmission jam-up, \(\omega\) is a given coefficient.

The second sub-problem objective:

\[
(P22) \quad \text{Max } f_2(x, y_2) = \Pr\left\{\sum_{t \in \Omega} P_{gj}^p(t_i) \leq \overline{P}_j\right\}
\]

Equation (15) is the second lower-level program objective, it is maximum probability of load curtailment less than an ideal value. \(\overline{P}_j\) is a given value of maximum load curtailment in N-1 operation condition.

The second sub-problem constraints:

\[
\text{s.t. } 0 \leq P_{gj}^p(t_i) \leq \overline{P}_j
\]  
\[
0 \leq P_{l}(t_i) \leq P_{l}
\]
Equation (16) is the active power constraint of the generation bus \( i \). Equation (17) is the active load constraint of the load bus \( j \). Equation (18) is the power flow equation. Equation (19) is the active power of line \( l \). Where \( P_g^p(t_i) \) is the active power of generation bus \( i \), \( P_l^p(t_l) \) is the active power flow of line \( l \). \( B^p(t_b) \) is the susceptance matrix, \( \theta^p(t_b) \) is the angle of buses.

From the above model, \( T_b(t_b) \) can be gotten from the first sub-problem and it acts on the upper-level program objective, the load curtailment probability which gotten from the second sub-problem and it also reacts to the upper-level program objective. While the upper-level program can get certain transmission network which have effect on the two sub-problems.

3. Algorithm for the proposed model

3.1 Uncertain factors simulation

1) Line cost simulation
Suppose line cost \( C_l \) variety obeys uniformity distribution \( C_l \in U[C_l^{\text{min}}, C_l^{\text{max}}] \).

2) New-added generation buses capacity
The site and capacity of new-added generation plant obeys discrete probability distribution.

\[
p(k = g, k = 1, \cdots, n_g) = p_k, \quad k = 1, \ldots, n_g
\]  

Equation (20) is the probability \( p_k \) in capacity \( g \) of generation bus \( i \). where \( 0 < p_k < 1, \quad \sum_{k=1}^{n_g} p_k = 1, \quad n_g \) is the number of new-added generation buses.

3) Load value of new-added load buses
The load variety of new-added load bus \( i \), its initial load value is \( P_{l0i} \), then its variable value \( \Delta P_{l0} \) obeys normal distribution \( \Delta P_{l0} \sim N(\mu_{l0}, \sigma_{l0}^2) \).

\[
P_{l0} = P_{l0i} + \Delta P_{l0}
\]  

3.2 Scenario simulation

Scenario simulation has four conditions:

- Scenario 1: suppose new-added generation bus capacities meet discrete probability distribution (20), new-added load values meet normal distribution (21).
- Scenario 2: base on Scenario 1, one part of new-added generation buses capacities meet normal distribution, such as \( P_{l0} \sim N(\mu, \delta) \).
- Scenario 3: base on Scenario 2, new-added load values are 90% discount of initial load values.
- Scenario 4: base on Scenario 2, new-added load values are 110% discount of initial load values.

3.3 Computation of upper-level function probability

The computation of the upper-level probability is as follows:

\[
U : \Pr \left\{ F \geq R \right\}
\]  

The computation steps are as follows:

1) A given transmission network planning scheme, compute transmission network investment \( I_{L,T}(t) \);
2) Set \( \text{sum} = 0 \);
3) Randomly setting \( M \) new-added load scenarios, \( \{\xi(1), \ldots, \xi(M)\} \);
4) Setting \( M \) new-added generation buses and capacities \( \{\xi(M+1), \ldots, \xi(2M)\} \);
5) Setting \( M \) line cost \( \{\xi(2M+1), \ldots, \xi(3M)\} \);
6) Computing the first sub-problem to get transmission profit \( T_b(t_b) \), the get the invest rate callback \( \{F(1), \ldots, F(M)\} \);
7) Judging \( F > R \), if it is tenable then \( \text{sum} = \text{sum} + 1 \), else it continues;
8) Getting \( \text{sum}/M \) to be \( \Pr \{ F \geq R \} \).

3.4 Computation of lower-level function probability

The computation of the lower-level probability is as follows:

\[
U : \Pr \left\{ \sum_{j=0}^{n_0} P_{l0j}(t_j) \leq \bar{P}_l \right\}
\]  

The computation steps are as follows:
Transmission Network Expansion Planning Based on Dependent-chance Bi-level Programming Method

1) A given transmission network planning scheme;
2) Set \( \sum = 0 \);
3) Randomly setting \( M \) new-added load scenarios, \( \{ \xi^{(1)}, \cdots, \xi^{(M)} \} \);
4) Setting \( M \) new-added generation buses and capacities \( \{ \xi^{(M+1)}, \cdots, \xi^{(2M)} \} \);
5) Setting \( M \) line cost \( \{ \xi^{(2M+1)}, \cdots, \xi^{(3M)} \} \);
6) Computing load curtailment in N-1 operation conditions;
7) Suppose \( \sum_{p \in P} P_{ij}^{(n)}(t_j) \leq P_i \) is tenable, then \( \sum = \sum + 1 \), else it continues;
8) Getting \( \sum / M \) to be \( \Pr \left( \sum_{p \in P} P_{ij}^{(n)}(t_j) \leq P_i \right) \).

3.5 Computation of lower-level function

The computation of the lower-level probability is as follows:
1) A given upper-level transmission network;
2) Prime-dual interior point is used to solve the first sub-problem, active power of generation buses are gotten, then transmission service profit and jam-up profit are gotten, then transmission profits are gotten;
3) Prime-dual interior point is used to solve the second sub-problem, total load curtailments in N-1 operation are gotten;
4) Probability of load curtailment less than a given value is gotten, then the second sub-problem objective is gotten.

3.6 Computation of individual objective

The computation of the individual objective is as follows:
1) A given upper-level transmission network;
2) Prime-dual interior point is used to solve the first sub-problem, active power of generation buses are gotten, then probability \( \Pr \{ F \geq R \} \) is gotten;
3) Prime-dual interior point is used to solve the second sub-problem, total load curtailments in N-1 probability \( \Pr \{ f \leq P_i \} \) is gotten;
4) Judging \( \Pr \{ F \geq R \} \), if it is 1, then adjusting \( R \), then computing \( \Pr \{ F \geq R \} \), then individual objective is gotten.

In the paper, genetic algorithm and prime-dual interior point method are mixed to solve the proposed models.

4. Case study

18-bus system is tested in the paper, the example is a simplified actual system in the western part of China with 18 buses and 27 branches which can be seen in [1]. In order to meet ‘N-1’ secure rule, all expandable branches in the network are supposed to have three candidate circuits. For simplification, the investment cost of one unit cost of one circuit is assumed to be 1 Million Yuan/km.

In the solution, two methods are used to solve the problem and are compared. One is certain method, the other is the proposed method. The result is shown in Fig. 1:

![Fig. 1. Comparison of two optimal networks in the 18-bus system.](image)

From the Fig. 1, we can see the planning scheme of proposed method has more lines than that of certain method, which adds 3-7, 5-6(2), 5-11, 5-12, 6-7(2), 6-13, 6-14, 11-12, 11-13, 14-15, which no adds 1-2(2), 4-7, 4-16, 7-9, 7-13(2), 7-15, 8-9(2), 17-18. The number of new-added lines of the proposed method is 31, while that of certain method is 30.

The investment callback and probability of certain method planning scheme in some random scenarios are compared with that of the proposed method. Two schemes are shown as follows:

Form the Table 1, we can see the investment cost of the proposed method scheme is bigger than that of certain method scheme. the callback of the certain method scheme is bigger than that of the proposed method. But probability of the proposed method scheme is bigger than that of that of the certain method scheme.
Table 1. Comparison of two planning methods in the 18-bus system

<table>
<thead>
<tr>
<th>Items</th>
<th>Static investment cost (Million Yuan)</th>
<th>Investment cost per year (Million Yuan)</th>
<th>Return (%)</th>
<th>Probability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The proposed method</td>
<td>29140</td>
<td>3422.8</td>
<td>15</td>
<td>100</td>
</tr>
<tr>
<td>Certain method scheme</td>
<td>28620</td>
<td>3361.7</td>
<td>28</td>
<td>10</td>
</tr>
</tbody>
</table>

5. Conclusions

Dependent-chance bi-level programming method is applied in transmission network expansion planning problem in the paper, it realizes transmission network planning modeling and solving, by case study, the proposed method can get high return transmission network planning scheme, it improves the flexibility and adaptability of transmission network planning scheme. The proposed method in the paper exploits the study idea in transmission network expansion planning, establishes the basis of bi-level programming theory for transmission network expansion planning.

References


Hong Fan was born in Hubei Province of China. She received the Ph.D. degree from Shanghai Jiao Tong University in electric power system automation, Shanghai, China. She is a lecture at Shanghai University of Electric Power, Shanghai, China. Her research area is power transmission network planning.

Hao-zhong Cheng was born in Zhejiang Province of China. He received B.Sc. and M.Sc and the Ph.D degrees in Dept. Electric Engineering Shanghai Jiao Tong University, and is a Professor of Shanghai Jiao Tong University now. His research covers power transmission network expansion planning, voltage stability, power quality.