New Adaptive Linear Combination Structure for Tracking/Estimating Phasor and Frequency of Power System

Choowong-Wattanasakpabal† and Teratum-Bunyagul*

Abstract – This paper presents new Adaptive Linear Combination Structure (ADALINE) for tracking/estimating voltage-current phasor and frequency of power system. To estimate the phasors and frequency from sampled data, the algorithm assumes that orthogonal coefficients and speed of angular frequency of power system are unknown parameters. With adequate sampled data, the estimation problem can be considered as a linear weighted least squares (LMS) problem. In addition to determining the phasors (orthogonal coefficients), the procedure estimates the power system frequency. The main algorithm is verified through a computer simulation and data from field. The proposed algorithm is tested with transient and dynamic behaviors during power swing, a step change of frequency upon islanding of small generators and disconnection of load. The algorithm shows a very high accuracy, robustness, fast response time and adaptive performance over a wide range of frequency, from 10 to 2000 Hz.

Keywords: Phasor, Frequency, Adaptive, ADALINE, Generator protection

1. Introduction

Voltage phasors and frequency of power system have always been of interest to power system engineers. The modern era of phasor and frequency measurement technology is driven by computer relaying of transmission lines and generator protection. In the past, phasors are applied in power system application with a steady state concept. However in reality, a power system never remains in a steady state. Frequency of voltage and current change consistently as a result of load and generation variations including fault events.

Most power systems operate in a relatively narrow band of frequency, i.e. within 0.5 Hz from a nominal value. Nonetheless, under some circumstances, islanding of a small generators and a disconnection of load can cause frequency variations as large as ±10 Hz, for example islanding condition with hydroelectric generator [1]-[2]. Such an extreme condition is usually resolved by available control actions. Phasor representation is only possible for a pure sinusoid. In practice, a waveform is often equipped with other signals of different frequencies. As a result, it becomes necessary to extract a fundamental frequency component of the signal before representing it by a phasor.

The phasor definition also implies that the signal is constant over time. Nonetheless, this assumption is only valid for a portion of time. This time window is known as “data window” and is a very important parameter in phasor and frequency estimation.

A number of numerical algorithms for measuring phasor and power system frequency have been published in many literatures [3]-[11]. In general, the high speed measurement made within one or two cycles by applying short data window tends to have greater errors than those techniques using a long data window. A well-known application of frequency measurement is for under-frequency load shedding. Normal operating time of an under-frequency relay is approximately 5-6 cycles. Nonetheless, excessively long data window is not a good way for improving the accuracy of the measurement. During transient, the frequency of power system may change rapidly. Consequently with a long data window length N, the process may combine significantly different frequency signals and result in significant errors.

Considering the process of selecting data window length N with sampling interval \(T\) such that the number of samples remains an integer value, the data window length N can be very large for real-time computations (77.00 samples at 20.78 Hz).

Method 1: This method estimates the frequency value and at the same time adjusts the sampling interval \(\Delta T\) such that the number of samples remains an integer value. Accordingly, N remains constant and \(\Delta T\) is varied [6]. This method is difficult to be implemented.

Method 2: This method estimates the frequency value and adjusts the data window length N. Hence, \(\Delta T\) remains constant and N are varied [7]-[11]. However, Method 2 cannot obtain an integer number of N and will result in errors. With, \(\Delta T = 0.000625\) s (32 samples per cycle at 50 Hz), the data window N for corresponding frequencies is shown below.

While adjusting data window length N is able to cover a large frequency range from 20.78 Hz to 57.14 Hz, the data window length N can be very large for real-time computations (77.00 samples at 20.78 Hz).
This paper presents new adaptive linear combination structure for tracking/estimating voltage-current phasor and frequency in power system. It is suitable for real-time application of phasor and frequency estimation providing high accuracy and can overcome the above mentioned problem of Method 1 and Method 2. In estimating phasors and frequency of power system from sampled data, the algorithm assumes that orthogonal coefficients and speed of angular frequency of power system are unknown parameters. With adequate sampled data, one can consider the estimation problem as a linear weighted least squares (LMS) problem. Hence, computation and accuracy of the new algorithm are not related to the data window length \( N \) as in the case of Discrete Fourier Transform (DFT) or other techniques that use the number of samples \( N \) for estimating the phasor and frequency of power system. Furthermore, the technique can measure phasors with a wide frequency range from 10 Hz to 2000 Hz with ease of implementation.

The paper is divided into six sections. Next section presents new adaptive linear combination structure for tracking/estimating voltage-current phasor and frequency in power system and shown theoretical derivation of the necessary equations. The following section presents simulations results for transient response during power swing, step change of frequency for islanding of small generators and disconnecting of load using Matlab program. After that, implementation and results from field data will be demonstrated in the next section and is followed by conclusion section. Finally, future work is presented in the final section.

### 2. The Methodology

The ADALINE structure was introduced as a powerful estimation tool [12] as present in the Appendix. However, this structure is not yet suitable for the problem of phasor and frequency tracking. Therefore, the structure of ADALINE has to be modified in order to be able to tracking phasor and frequency of power system. Fig. 1 illustrates the structure of ADALINE for tracking/estimating dynamic voltage-current phasor and frequency in power system. The structure is formed by the linear combination of time varying connection vector \( X(T) \). The time varying connection vector is multiplied by the weighting vector \( W(T) \), and then summed up to produce the linear output \( y(t) \).

The weighting vector can be adjusted using Least Mean Square algorithm (LMS). This will provide the output \( y(t) \) that is close to the input signal \( x(t) \) (voltage or current signal).

Unlike other methods, the proposed new algorithm assumes that the phasor and frequency are dynamic. The main procedure is divided into two stages. First, weighting vector is estimated. Then, the angular speed of fundamental frequency of the input vector will be adjusted by to obtain the actual value.

**Stages 1:** Let assume that the observation model of an input signal (voltage or current) at the measurement location can be expressed as (1).

\[
y(t) = A \cos(\theta + \Delta \omega t + \omega t)
\]

Where
- \( A \) is the magnitude of signal.
- \( \theta \) is the phase angle.
- \( \omega_0 \) is the angular speed of fundamental frequency.
- \( \Delta \omega \) is a small change in angular speed of fundamental frequency.
- \( t \) is the time of observation.

Applying the trigonometry relationship of \( \cos(a + b) = \cos(a) \cdot \cos(b) - \sin(a) \cdot \sin(b) \) to the above equation will result in

\[
y(t) = A \cos(\theta + \Delta \omega t) \cdot \cos(\omega_0 t)
- A \sin(\theta + \Delta \omega t) \cdot \sin(\omega_0 t)
\]

**Fig. 1.** ADALINE structure for tracking phasor and frequency.
Rearranging the above equations in matrix from will result in (3).

\[ y(t) = X(t)^T \cdot W(t) \]  
(3)

Where \( y(t) \) represents the output signal from the observation model. \( X(t) \) as (4) represents the time varying connection vector and \( W(t) \) as (5) represents the parameters to be estimated.

\[ X(t) = [\cos(\omega t), -\sin(\omega t)] \]  
(4)

\[ W(t) = [w_1, w_2] \]  
(5)

Where \( w_1 \) and \( w_2 \) are orthogonal coefficients of voltage or current signal which can be represented by (6)-(7).

\[ w_1 = A \cos(\theta + \Delta \omega t) \]  
(6)

\[ w_2 = A \sin(\theta + \Delta \omega t) \]  
(7)

Accordingly, \( w_1 \) represents real part of voltage or current phasor and \( w_2 \) represents imaginary part of voltage or current phasor.

For discrete-time signals, the weighting vector can be updated by using Widrow-Hoff delta rule as following.

\[ W(k+1) = W(k) + \alpha \cdot e(k) \cdot X(k) \]  
(8)

Where \( k \) is the index of iteration and \( \alpha \) is the learning parameter. Substituting \( X(k) \cdot X^T(k) = 1 \) into (8) will result in

\[ W(k+1) = W(k) + \alpha \cdot e(k) \cdot X(k) \]  
(9)

The perfect learning in (9) is completed when the error \( (|e|) \) is brought to zero. The weighting vector \( W(k) \) of voltage and current signal at the measurement location will be presented in phasor or complex form. As a result, the system can track voltage and current phasor.

**Stages 2:** A study of previous work has revealed that most of voltage and current phasor estimation are not suitable for extracting the fundamental frequency components. This is because the observation model neglects a small change in angular speed of fundamental frequency \( \Delta \omega \). In this paper, the effect of \( \Delta \omega \) is considered and the frequency is adjusted to its actual value. A small change in angular speed of the fundamental frequency is computed accurately from the weighting vector according to (10).

\[ \Delta \omega = \frac{w_1(k) \cdot w_2'(k) - w_1'(k) \cdot w_2(k)}{(w_1'(k) + w_2'(k))} \]  
(10)

From (10), \( w_1'(k) \) and \( w_2'(k) \) are the derivative of weighting vector with respect to time. A differentiation method can be used to compute \( w_1'(k) \) and \( w_2'(k) \) according to (11) and (12), where \( \Delta t \) is the sampling time interval.

\[ w_1'(k) = \frac{w_1(k) - w_1(k-1)}{\Delta t} \]  
(11)

\[ w_2'(k) = \frac{w_2(k) - w_2(k-1)}{\Delta t} \]  
(12)

For discrete-time signal, the angular speed of the fundamental frequency according to the observation model in (1) can be updated to the actual value by using (13).

\[ \omega_3(k+1) = \omega_3(k) + \alpha \cdot \Delta \omega \]  
(13)

### 3. Simulation Results

Comprehensive evaluation of the proposed algorithm was carried out by conducting several test cases. The first case focuses on a range of the measurement frequency. The second test deals mainly with examining the transient response of the algorithm in tracking/estimating phasor and frequency for all changes in amplitude, phase angle and frequency. The third test case considers a rate of change of frequency. The fourth and fifth test cases demonstrate a dynamic performance of the processing unit under power swing condition. The last test case is for testing noise property of the processing unit. All test scenarios applies a sampling rate of \( f_s = 4000 \) samples per second and a learning parameter \( \alpha = 0.12 \).

In this research, it is found that decreasing the learning factor will reduce the prediction error, but at the expense of increasing the convergence time. In contrast, increasing the learning factor will increase the prediction error dramatically while reducing the convergence time. Therefore, there is a trade-off in selecting the value of \( \alpha \) for fast convergence and fast response with a minimum prediction error. Much research has been conducted to determine a proper value of \( \alpha \). Nonetheless, most common approach in determining the value relies on trial and error method [12]-[13]. It was found that the value of the learning factor that gives the best phasor and frequency tracking performance is 0.12.

**CASE1:** The ability in estimating frequency value over a wide range of frequency changes is investigated using a sinusoidal test signal. The algorithm is tested by ramping up the system frequency (reference frequency of input signal) from 10 Hz to 2000 Hz with rate of change 50 Hz/s and a signal to noise ratio (SNR) =60 dB. The result in Fig. 2 shows that the values of estimated frequency can follow the change of reference frequency. In addition, it shows that the algorithm can satisfactorily perform frequency measurement over a wide range of frequency between 10 Hz to 2000 Hz.
CASE2: The transient responses of the algorithm are investigated. It is assumed that the power system is initially operating close to nominal frequency (50 Hz), and a sudden change in frequency occurs at $t = 0$ s. The frequency of input signal is changed from 50 Hz to 49 Hz with a signal to noise ratio (SNR) = 60 dB and 5% third harmonic.

The time response of the algorithm for a step change in frequency is shown in Fig. 3. The third harmonic of 5% of fundamental frequency magnitude and zero-mean Gaussian white noise with a signal to noise ratio of 60 dB are also included. Obviously, frequency estimation using the proposed algorithm with online adaptation results in less estimation errors within the range of 0.0075 Hz at a steady state as show in Fig. 4.

Next, a sudden change in frequency, magnitude and phase angle is applied at $t = 0$ s from 50 Hz to 45 Hz, 1 p.u. to 0.5 p.u. and -35 degree to -55 degree respectively. According to Fig. 5, the estimated frequency, magnitude and phase angle can track the actual value of the input signal with total delay time less than 60 ms. In addition, at steady-state, the values of frequency magnitude and phase angle are close to the actual value of the input signal.

CASE3: Performance for rate of change of frequency is tested using simulated voltage waveforms with different rates of frequency ramping (Hz/s). In Fig. 6, the reference frequency starts from 50 Hz with $+1$ Hz/s rate of change. The frequency estimation starts from 50 Hz and tracks the frequency variation. From the figure, the maximum error in the frequency tracking is less than 0.0125 Hz.

Subsequently, the rate of change of frequency is increased to $+20$ Hz/s. The frequency starts with a 50 Hz value and reaches 54 Hz after 0.2 s. The frequency estimation starts from 50 Hz and tracks frequency variation. According to Fig. 7, the maximum error in the frequency tracking is less than 0.25 Hz.
CASE4: The measurement performances under power swing case is presented in the fourth to the sixth test case. It is assumed that the power system is initially operating close to the nominal frequency (50 Hz) and a transient stability has been initiated. The magnitude of a corresponding voltage $v(t)$ signal is modulated with a frequency $f_m$ as follow.

$$v(t) = \sqrt{2} \left[1 + 0.2 \sin(2\pi f_m t)\right] \sin(\omega t)$$  \hspace{1cm} (14)$$

The frequency $f_m$ varies between 0-10 Hz. Fig.8 shows the result obtained from the phasor magnitude estimation according to the input in (14) with $\omega = 2\pi 50$ Hz and $f_m = 5$ Hz.

Fig. 7. Frequency estimation with rate of change + 20 Hz/s.

The frequency $f_m$ varies between 0-10 Hz. Fig.8 shows the result obtained from the phasor magnitude estimation according to the input in (14) with $\omega = 2\pi 50$ Hz and $f_m = 5$ Hz.

CASE5: During power swings condition, both magnitudes and frequency of voltage and current in a power system may be modulated. The equivalent test for dynamic performance is provided by using amplitude and frequency modulation function as follow.

$$v(t) = \sqrt{2} \left[1 + 0.2 \sin(2\pi f_m t)\right] \sin(\omega t)$$  \hspace{1cm} (15)$$

$$\omega = \omega_b + A \sin(2\pi f_t t)$$  \hspace{1cm} (16)$$

The frequency of input signal varies sinusoidally between $(\omega_b + A)$ and $(\omega_b - A)$ with a frequency $f_t$ and the amplitude of input signal varies sinusoidally between $(1+0.2)$ and $(1-0.2)$ with a frequency $f_m$. In Case 5, the frequency input is varied between 51 Hz and 49 Hz ($A = 1$, $\omega_b = 2\pi 50$), with $f_t = 1$ Hz and $f_m = 5$ Hz. The amplitude and frequency estimation from the proposed algorithm is shown in Fig.10 and Fig.11 respectively. From the figures, the proposed algorithm shows a high quality dynamic response. The estimated frequency fluctuates around the reference frequency according to Fig.10.
It can be seen from Fig. 12 that the maximum error of the frequency tracking is less than 0.28 Hz.

![Fig. 12. Absolute error.](image)

**CASE6:** The algorithm was tested under the presence of zero-mean Gaussian noise in the input signal (additive noise). The random noise standard deviation was selected to obtain a prescribed Signal-to-Noise-Ratio (SNR) defined as:

\[
SNR = 20 \log \frac{A}{\sqrt{2} \cdot \sigma}
\]

Where
- \(A\) is the magnitude of the signal.
- \(\sigma\) is the noise standard deviation.

As shown in Fig. 13, a better sensitivity is obtained when additive noise is added. Accordingly, the maximum error increases considerably. Nevertheless, for SNR>40dB, the maximum error is less than 0.1 Hz.

![Fig. 13. Maximum error of estimated frequency.](image)

### 4. Implementation

Instantaneous phasor and frequency metering was developed by using the techniques described previously and is shown in Fig. 14. The data acquisition system is based on a personal computer being composed of an analog to digital converter (A/D) with a sampling rate at 4 kHz. This A/D digit is 12 bits providing a non-anti-aliasing RC filter. The current signal is delivered to the data acquisition system from a 600/5 A current transformer. The LabView software package is applied to the system implementation [14]. The LabView environment is very friendly for the program development using graphical programming language. It uses terminology, icons and ideas being familiar to scientists and engineers, and relies on graphical symbols rather than textual for language programming. In addition, it provides extensive galleries of functions and subroutines for most programming tasks, including data acquisition.

The ability of amplitude and frequency estimation during a large power disturbance is investigated using a real data from a 115 kV transmission system shown in Fig.15. The voltage waveform consists of amplitude variation, frequency deviation and harmonics. Under these conditions, it is clear from Fig.15 that the calculated phasor magnitude can follow the modulated amplitude perfectly.

![Fig. 14. Instantaneous phasor and frequency meter.](image)

![Fig. 15. Voltage waveform and magnitude estimation.](image)
paper compares the proposed method with the widely used zero-crossing method. From Fig. 16, it can be seen that the behaviors of the two methods are very similar in trend, but the method proposed by this paper shows a smooth frequency drop and the results are matched well with the voltage waveform.

![Fig. 16. Frequency estimation.](image)

5. Conclusion

The paper describes and demonstrates the new adaptive linear combination structure for estimating electrical parameters of sinusoidal signals. The proposed technique is suitable for tracking phasor and frequency over a wide range of frequency variations which can be caused by changes in load and generation, power swing or fault events. The algorithm provides several benefits, which are:

1. The algorithm applies a fixed sampling interval $\Delta T$ and can estimate phasor and frequency accurately. In addition, it allows ease of implementation into a relay or other phasor measurement devices.
2. The algorithm possesses high quality dynamic responses. It is able to track phasor and frequency during situations below:
   - Transient events, i.e. a sudden or step changes in phasor magnitude, frequency and phase angle.
   - During power swing events, i.e. both magnitudes and frequency of voltage and current signal may be modulated.
3. The algorithm is able to track frequency change over a wide range from 10 Hz to 2000 Hz with a sampling rate $f_s = 4$ kHz.
4. Good noise property, i.e. for SNR $> 40$ dB, the maximum error is less than 0.1 Hz.

6. Future Work

The authors intend to study in details about the implementation of the techniques presented in this paper for utilities. This system can be a very useful tool for fault locator devices.

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Appendix

ADALINE, LMS and Widro-Hoff delta rule [12].

The ADALINE structure is shown in Fig. 17. Its output is constructed from the linear combination of its input vector $X_i = [x_1, x_2, \ldots, x_n]^T$ at any given time. The input vector is multiplied by the weighting vector $W_i = [w_1, w_2, \ldots, w_n]^T$, and then these weighted inputs are summed to produce the linear output $y(k) = X(k)^T \cdot W(k)$.

![Fig. 17. Basic ADALINE structure.](image)

In order for the ADALINE output $y(k)$ to precisely mimic the desired value of $x(k)$, the weighting vector is adjusted utilizing an adaptation rule based mainly on LMS algorithm. This rule is also known as Widro-Hoff delta rule, and is given by (17).

$$W(k+1) = W(k) + \alpha \cdot e(k) \cdot X(k)$$

$$\frac{X(k)}{X(k) \cdot X^T(k)}$$

(17)

Where $e(k) = x(k) - y(k)$ is the prediction error at any time $k$, $y(k)$ is the estimated signal magnitude at time $k$ and $\alpha$ is the learning parameter (reduction factor). Perfect learning is achieved when the error is brought to zero.
References


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