Unbalanced Magnetic Forces in Rotational Unsymmetrical Transverse Flux Machine

Salwa Baserrah†, Keno Rixen** and Bernd Orlik*

Abstract – The torque and unbalanced magnetic forces in permanent magnet machines are resultants of the tangential, axial and normal magnetic forces, respectively. Those are in general influenced by pole-teeth-winding configuration. A study of the torque and unbalanced magnetic forces of a small flux concentrating permanent magnet transverse flux machine (FCPM-TFM) in segmented compact structure is presented in this paper. By using FLUX3D software from Cedrat, Maxwell stress tensor has been solved. Finite element (FE-) magneto static study followed by transient analysis has been conducted to investigate the influence of unsymmetrical winding pattern, in respect to the rotor, on the performance of the FCPM-TFM. Calculating the magnetic field components in the air gap has required an introduction of a 2D grid in the middle of the air gap, whereby good estimations of the forces are obtained. In this machine, the axial magnetic forces reveal relatively higher amplitudes compared to the normal forces. Practical results of a prototype motor are demonstrated through the analysis.

Keywords: Unbalanced force, Normal component, Axial force, Distributed winding, Tangential force, Transverse flux, TFM, Permanent magnet

1. Introduction

Estimation of electromagnetic torque and normal forces has been a subject of interest in segmented FCPM-TFM with concentric saddle windings due to the nonlinearity introduced by magnetic saturation and the unsymmetrical structure of the windings with respect to the rotor, respectively. The main disadvantage of FCPM-TFM, even in conventional layered-structure symmetrical constructions [1], is the high level of torque pulsations as a function of rotor position, which is the main cause of noise and vibration. Therefore, studying and estimating the level of vibration due to the normal forces is necessary to improve the drive controller to compensate these torque pulsations.

Magnetic force analysis has already been addressed by a number of investigators. Several different methods to calculate the torque at different rotor angular positions based on the quasi static magnetic field are presented in [2] and [3], where experimental verifications have been illustrated. The driving frequencies of magnetic force, cogging torque and commutating torque are expressed in [4] as a function of pole, teeth and phase. A study of the torque and unbalanced magnetic forces in unsymmetrical design of brushless DC motors is conducted in [5], where it is analytically proved that the normal magnetic forces are eliminated, due to rotational symmetry, in case of symmetrical rotational design. The normal forces in surface permanent magnet synchronous linear motor with Halbach magnet array is studied in [6] by means of an analytical method with magnetic scalar potential, where minimization of their magnitudes have been achieved from \( \approx 144 \) N to \( \approx 673 \) N with the aid of genetic algorithm.

Distribution of the windings around the stator breaks the constructional symmetry in FCPM-TFM that is built in segmented configuration. Due to this asymmetry and since the generated magnetic forces on the stator are revolving as the rotor turns; its resultants exist in the form of torque and unbalanced magnetic force, where the later causes the ripples on the torque waveform and therefore, increases the level of noises in the machine. The unsymmetrical segmented structure of the three-phase FCPM-TFM is shown in Fig. 1, where the maximum and minimum positions of the normal forces acting on outer rotor with respect to the stator are illustrated.

In this paper, the characteristics of torque and unbalanced magnetic forces in rotational unsymmetrical segmented FCPM-TFM with distributed windings, which achieves higher torque density with lower cogging torque compared to other PM-machines, is investigated through solving Maxwell stress tensor. Torque, normal and axial magnetic force components are derived. The estimated no-load induced voltage is shown to agree with the practical results obtained from a prototype execution in generator mode of operation.

2. Method of Analysis

FLUX3D, an FE- solver for magnetic fields from Cedrat
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is used to calculate the magnetic flux density, $B$. The normal, tangential and axial forces, $f_r$, $f_\theta$ and $f_z$, respectively for the tooth face or on a surface, $S$ at the center of the air gap are computed by the following equations:

$$f_r \approx \sigma_r = \frac{1}{2\mu_{\text{air}}} (B_r^2 - B_\theta^2 - B_z^2) \rightarrow T = \oint_S f_r dS$$  

(1)

$$f_\theta \approx \sigma_\theta = \frac{1}{\mu_{\text{air}}} B_r B_\theta \rightarrow F_\theta = \oint_S f_\theta dS$$  

(2)

$$f_z \approx \sigma_z = \frac{1}{\mu_{\text{air}}} B_r B_z \rightarrow F_z = \oint_S f_z dS$$  

(3)

where, $B_r$, $B_\theta$ and $B_z$ are radial, tangential and axial components of $B$.

The resultant of magnetic forces acting on the stator at one position is expressed by torque $T$ and unbalanced normal magnetic and axial forces by $F_\theta$ and $F_z$, respectively.

The stator is built of non-oriented grain electric steel M270-50A laminations. The permanent magnet (PM) are of sintered NdFeB with a residual flux density of 1.25 Tesla and a coercivity force of 1989 KA/m, with tangential magnetization direction as implemented in TFM that utilizes flux concentrating topology of construction. The current density of Y-connected winding is of 1.575 A.turn/mm² and of a total winding section area $\approx$100 mm².

The radial length of the air gap and permanent magnet are 0.38, 10 mm, respectively. The radius of the stator is 41 mm and stator stack length is 40 mm. The gap between the phases is around 36.92 degrees, which corresponds to 2.66×$\tau_p$. Other dimensions of the machine are shown in Table 1 and the construction of the machine is illustrated by photos of the tested prototype in Fig. 2. A more thorough description of the machine construction and the mathematical design are shown in [8, 9].

### Table 1. Dimensions of Prototype FCPM-TFM

<table>
<thead>
<tr>
<th>General machine dimensions (mm)</th>
<th>Pole Number</th>
<th>Air gap diameter</th>
<th>Pole pitch, $\tau_p$</th>
<th>Air gap length</th>
<th>Axial length</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>82.38</td>
<td>10</td>
<td>0.38</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

**Stator and rotor dimensions**

<table>
<thead>
<tr>
<th>Height (radial)</th>
<th>Width (tangential)</th>
<th>Length (axial)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>Rotor pole</td>
<td>Centre</td>
<td>10.7</td>
</tr>
<tr>
<td>Stator pole</td>
<td>Sides</td>
<td>10.7</td>
</tr>
<tr>
<td>Between phases</td>
<td>14.45</td>
<td>26.67</td>
</tr>
<tr>
<td>Slot</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Mounting layers</td>
<td>30</td>
<td>(Inner/outer radius)</td>
</tr>
</tbody>
</table>

During the prototype construction, it became evident that the air gap surface from the rotor side will be of non-smooth shape due to the existence of the trapezoidal rotor poles and the parallelogram shapes of the PMs in the outer rotor. Thus, a deleterious effect on both the detent torque and the magnetic flux variations will be expected.

A full axial model was required to analyze the rotational unsymmetrical motor, which did not have periodical boundary condition around the shaft. Since the magnetic flux is concentrated in the air gap, particularly at the corner of a tooth, it is necessary to have a fine mesh in the air gap and the tooth corners. The total number of nodes and volume elements are $\approx$ 637400 and $\approx$ 464600, respectively.

The magnetic field produced as the rotor moves, are observed as a series of magneto static fields, or quasi static magnetic fields. A two-dimensional grid is placed in the middle of the air gap, where the flux density components in cylindrical coordinates are plotted and used in the calculation of the force components. The average output torque and the output torque pulsations of PM motors depend on air gap flux density waveform produced by the magnets. This waveform is affected by magnet shape (arc and height), air gap length, number of poles, and the direction of magnetization of the magnets [10].
In order to get accurate knowledge of flux density waveform, the mesh should be relatively fine on the surface of the grid, so that it can correctly predict the flux density components. Precisely speaking, $B_\theta$ has to be correctly estimated by FE-method, that is because it is of a very high irregular shape when the number of mesh elements is small, whereas the normal component is less sensitive to the discretization density.

The axial component is of a very small value that will not have much effect on the output if it is ignored. The number of discrete elements on both $\theta$- and $z$- directions of the grid are set to 400 elements.

Simulating half of the axial length of two pole pitch segment will give relatively correct prediction of the normal and tangential components of the flux density in the air gap, whereas the axial component will not be correctly estimated as the boundary conditions, in case of half axial length simulation, will be incorrect. Therefore, it is recommended not to apply the symmetry when calculating the axial component, though estimation errors will occur, as well as in calculation of the normal and axial force components, when Maxwell equations are to be utilized as a solving tool.

The back electromagnetic force EMF, $e$, at no-load condition is calculated by using Faraday’s law of electromagnetic induction based on the time variation of magnetic flux, $\Phi$ induced in a coil of N-turns. The magnetic flux is obtained by the software through calculating the derivative of the magnetic co-energy with respect to a current, $i$ in the coil. The time variation of magnetic flux is calculated considering constant angular velocity.

$$
e = -\frac{d\Phi}{dt} \hspace{1cm} (4)$$

$$\Phi = \oint \vec{B} \cdot \vec{h}_{G} \, d\Omega \hspace{1cm} (5)$$

where, $\vec{B}$ is the flux density vector on cross section surface of the coil, $\vec{h}_{G}$ is the field vector created by the coil in a vacuum, when carrying a current of 1A and $\Omega$ is the study domain [8].

### 3. Results and Analysis

The magneto static study covered by FE- in this section is divided into two parts, where the flux density in the air gap is studied as well as their contributions to the force components.

#### 3.1 Flux density components in cylinder coordinates

In the middle of the air gap, the components of flux density are expressed in radial, tangential and axial components, i.e. in cylindrical coordinates. The normal and axial components are of small values. Therefore, the radial component, $B_r$, and the modulus of the air gap flux density, $B_m$, over the grid are shown for three positions of the outer rotor, i.e. aligned, unaligned and intermediate positions, as illustrated in Fig. 3. The flux densities are shown for three
cases: Fig. 3(a) shows the effect of the current only, Fig. 3(b) shows the effect of PMs and Fig. 3(c), shows the effect of both current and PMs on the flux density. Obviously, the flux distribution occur over the areas that face the stator teeth i.e., poles, otherwise, zero quantities of the flux density components will show up. Therefore, the effective area of the air gap stands for the stator poles. With this winding configuration, there is a negligible magnetic interaction between the phases. The flux density profiles are symmetric across the surface of the stator tooth at the aligned and unaligned positions. For position values between the aligned and unaligned positions i.e., overlap position, the flux levels are unsymmetrical, with a slope that varies with rotor position and phase current. This agrees with the investigations of switched reluctance motors in [11].

The flux density components over the grid are plotted as the rotor turns for two pole pitch period in Fig. 4. Since the product of the normal and tangential components is required for solving Maxwell stress tensor, the product expressed in element wise approach is shown on the same plot too. The results are very sensitive to discretization density and integration grid. This sensitivity is linked mainly to the very high irregularity of the small tangential magnetic flux density component. It is clear that normal component distribution is less sensitive to discretization density and the location of grid as well. The product of normal and tangential components is even more sensitive to discretization density and grid location than each component separately, which appears as a reassuring result to what references [2, 12] indicate for calculation over a closed contour.

The modulus of the flux density is minimum at the aligned position and achieves maximum values in the air gap at the unaligned position as shown in Fig. 4(b) and 4(c). The tangential field component mirrored itself over the aligned position and it is hypothesized that this component for approximating Maxwell stress tensor is proportional to the gradient in the magnetic field magnitude. The normal and tangential flux density components will change for all cases in Fig. 4, from odd to even functions, respectively.

### 3.2 Tangential, normal, and axial force components

Eqs. (1), (2) and (3) have been used to calculate the tangential, normal and axial forces. The axial force component is the same as its corresponding value obtained in Cartesian coordinate system. The plot of the forces over two pole pitches has been calculated and is shown in Fig. 5, when a 3-phase sinusoidal current of 120° phase shift is applied with a peak current of 24 A.

The tangential force component in TFM is of three components, cogging, reluctance and interaction or commutating. Via integrating the product of the radial and tangential components of the flux densities in the middle of the air gap, the torque components of one phase are calculated. The results agree with those obtained by FLUX3D software as shown in Fig. 5(a). The torque components can be depicted analytically from the co-energy principle as in (6), when the losses in iron and copper are ignored [13].

\[
T_o \approx -T_{\text{cogging}}(\theta) + \frac{1}{\partial \theta} \sum_{i=1}^{\infty} \sum_{j=1}^{m} L_{ik} \theta \cdot \frac{\partial \psi_{\text{PM}}}{\partial \theta} \cdot i_k
\]

(6)

when linear case is considered, (6) will reduces into (7).

\[
T_o \approx -T_{\text{cogging}}(\theta) + \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j=1}^{m} \frac{dL_{ik}}{d\theta} \cdot i_j \cdot i_k + \sum_{i=1}^{\infty} \frac{\partial \psi_{\text{PM}}}{\partial \theta} \cdot i_k
\]

(7)

Tangential force component depends on the radial and tangential components of air gap flux density. Half axial length simulation can correctly estimate these two
components of flux density. Therefore, the torque is obtained from integrating the product of radial and tangential components flux density over the area of a grid located in the middle of the air gap of half the axial length. The normal magnetic force in the air gap produces the unbalanced magnetic resultant only for the rotational unsymmetrical design. The normal forces can balance each other by introducing segmented structure, where each phase is split into two parts facing each other as introduced in [8]. These forces are obtained via evaluating the integral in (3) over a full axial length grid.

The maximum position of the normal force rises, when the rotor approaches the overlap position in one pole pitch, while the minimum position occurs at the same position, however, in the other pole pitch. In Fig. 5(b), the forces at the aligned position that are produced by PMs are shown. A DC component of 0.375 N appears owing to the attraction forces at the aligned and unaligned positions of PMs. The normal forces are expressed in polar form as the rotor finishes $2\tau_p$ of one revolution period, and the patterns are shown in the appendix for the no-load, currents, fields and total effect cases. The three phase resultant of the normal components shows a sinusoidal pulsating pattern.

As the axial force decreases with the increase in the air gap length and due to the existence of the PMs in the rotor, which act as an air gap extension, these may aggravate the damage to the bearing. Therefore assessment of the axial forces is worthwhile. In general, the existence of the axial magnetic forces is due to the variation of permeance of mutual and leakage flux paths; and its nature depends on the inductance variation.

Eq. (3) is used for plotting Fig. 5(c) with full axial length investigation. The axial forces oscillate in random behavior with big peak values. Thus, approximated filtered curves for the axial forces have been plotted in Fig. 5(c), too. The axial forces can be determined by the virtual work method as well as done in [18] and can be defined as algebraic sum of the forces due to displacement of the rotor in axial direction. In FCPM-TFM in sector-wise configuration the axial forces show higher instantaneous amplitudes compared to the normal forces.

Generally, the torque in FCPM-TFM, is proportional to square of the modulus of the air gap flux density [19-21]. The flux concentrating effect [18] can be easily shown by replacing the steel poles in the rotor by air volumes. The output torque with steel poles will be of $C^2$ times the torque produced as air volumes are considered instead their corresponding of steel. The constant, $C$, is calculated from equivalent circuit of the machine as in (8):

\[
\frac{B_{e_{\text{steel-air, poles}}}}{B_{e_{\text{air-air, poles}}}} = \frac{\mu_0 \cdot h_{PM} + 2 (I_0 + I_{air}) \cdot \mu_{PM} \cdot K_M}{\mu_0 \cdot h_{PM} + 2 I_0 \cdot \mu_{PM} \cdot K_M} = C
\] 

(8)

Only 5% of the mean length of air volumes that replaces the steel ones will be followed by the flux lines so that they find the minimum reluctance path to enter the stator. The peak value of the torque reaches $\approx 10.5$ Nm with steel poles, while with air poles, the torque reaches 3.27 Nm, which is of factor $\approx 3$, agrees with the result obtained from (8). This effect is shown in Fig. 6.

### 3.3 Induced voltage waveforms

The induced flux in the coil is being calculated by the
The induced flux waveforms due to PMs as well as due to the armature current are individually of relatively sinusoidal shape and of almost 180° phase shift. The no-load PM flux linkage waveform can be found by using core back flux by rotating the rotor for different positions [19]. The introduction of the current has introduced a distortion of the flux waveform with a phase shift of \( \approx 2^\circ \) from the aligned position. The interaction of the rotor and stator fields has brought a waveform, with a dominant third harmonic level that causes this modulation effect.

The induced voltage waveforms at no-load due to the change of flux with time for different operating speeds are shown in Fig. 8. Normally, for the no-load condition, the machine is not saturated or is about to saturate. The output from transient investigation is shown on the same plot for operating speed of 100 rpm, which verifies the magneto-static result. Experimental results show as the machine is driven to \( \approx 145 \) rpm that the induced voltage is quite favorably with what is found from FE- method.

3.4 Three phase torque variations

TFM works with optimum performance, when the controller succeeds to align the current vector over the PM and shape current waveform to mitigate the torque ripple at each angular position [20]. In Fig. 9(a), the torque waveforms are shown for two pole pitch period. The torque

![Fig. 6. Flux concentrating effect illustration](image)

![Fig. 7. Estimated induced flux waveforms](image)

![Fig. 8. Induced no-load voltage waveforms: (a) Estimated; (b) Measured](image)

![Fig. 9. Three phase torque waveforms: (a) Estimated from magneto-static and transient analysis, (b) Measured](image)
variations are obtained by magneto static approach by applying sinusoidal current waveform. Transient result is shown on the figure too, where the current is obtained by block commutation [21]. The ripple factor is reduced with sinusoidal current source; however a current controller is required to bring TFM into smoother operation.

The measured three phase torque variations at 100 rpm are shown in Fig. 9(b). For this measurement, the machine has been supplied with 20A peak sinusoidal currents waveforms. The measurement has shown a good agreement to the expected calculated value.

4. Conclusion

This paper presents finite element modeling and analysis results that are performed during the design process of a high power density, FCPM-TFM having a radial air gap orientation and 3D-field configuration. The analysis includes study of the magnetic forces, i.e. normal and axial force components in the machine that tend to excite undesirable mechanical modes resulting in acoustic noise besides the ripples in the tangential component.

The study shows that the tangential force component or equivalently, the output torque can be easily estimated by placing a 2D-grid of half the machine axial length, in the middle of the air gap, and evaluating the integral of the product of the radial and tangential components of the flux density over the area of the grid. This method gives efficient results that are consistent with what Flux3D software reveals for the cogging, reluctance and commutating torque components.

For studying the normal and axial force components, a 2D-grid of total axial length of the machine has been required, so that the axial component of the flux density can be correctly calculated. In the study, end turn effects are neglected because the winding length in a slot is large relative to the end turn length and its small influence on the output torque has been already shown in [8, 9]. The calculated induced voltage waveforms and torque variations are compared with the measurements, and the comparison study shows the validity of the estimated values.

Appendix

Normal forces faced by the rotor as it turns over two pole pitch period expressed in polar form.

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References


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