Segmentation of Welding Defects using Level Set Methods

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Abstract – Non-destructive testing (NDT) is a technique used in science and industry to evaluate the properties of a material without causing damage. In this paper we propose a method for segmenting radiographic images of welding in order to extract the welding defects which may occur during the welding process. We study different methods of level set and choose the model adapted to our application. The methods presented here take the property of local segmentation geodesic active contours and have the ability to change the topology automatically. The computation time is considerably reduced after taking into account a new level set function which eliminates the re-initialization procedure. Satisfactory results are obtained after applying this algorithm both on synthetic and real images.

Keywords: Image segmentation, Level set, Weld defects, Radiographic image.

1. Introduction

There are numerous applications of radiography (non-destructive test method) in engineering applications. Radiographic inspection is widely used for weld inspection. Based on X-Ray, it strongly depends on the mass of the material and is precise in detecting defects. This is a radiographic test method used to reveal the presence and nature of internal defects in a weld, such as cracks, slag, blowholes, and zones where proper fusion is lacking. In practice, an X-ray tube is placed on one side of the welded plate and an X-ray film on the other side.

Segmentation processing is a crucial step in radiographic image analysis of weld defect. It consists mainly on detecting and visualizing the common boundaries of distinct weld defect in the image. Nowadays, the popular approaches of image segmentation are the active contour models. These models are widely used in many applications, including edge detection, shape modeling and motion tracking. The first model used is the classical snake formulated by Kass and al [1]. It is a method of surrounding the boundary of an object in an image by closed curve. In this model a closed curve deforms under the influence of internal forces, image forces and external constraint forces.

But this model has its drawbacks. For example, it is sensitive to initial curve position and initial curve shape and doesn't have the ability of changing with topology.

Consequently, the researchers focused on the use of partial differential equations in image processing and computer vision. In particular, the use of level set method and dynamic implicit surfaces has increased dramatically in recent years. Traditionally the two closely fields, image processing and computer vision, have developed inde-

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pendently. However, level set and related PDE-based methods have served to provide new set of tools that have led to increased interaction.

The level set method overcomes the drawbacks of snake model. It is a numerical and theoretical tool for propagating interfaces. It was first introduced by Osher and Sethian [2], and has become a more and more popular theoretical and numerical framework within image processing, fluid mechanics, graphics, computer vision, etc. The level set approach is able to handle complex topological changes automatically.

All level set based image segmentation methods are based on an assumption that the level set function is or close to a signed distance function (SDF). Small time step and costly re-initialization procedure must be applied to guarantee this assumption, and in order to calculate the gradient, simple numerical schemes, based on finite differences, are applied. In this paper, in order to achieve higher order accuracy in the temporal discretization, we have used Total Variation Diminishing (TVD) Runge Kutta (RK) methods. The spatial derivatives are determined by using the Weighted Essentially Non-Oscillatory methods (WENO-5) [3] that accurately capture the formation of sharp gradients in the moving fronts. In the other hand, we have used the level set method without the re-initialization procedure in order to speed up the evolutionary process. Experiments results show that we have obtained good results. For all experiments, we have used a PC (DELL OptiLex 780, Processor Intel Core 2 Duo (E7500) 2.93GHz.

2. Front Propagation

Consider a curve moving in a plane. Let γ(0) be the initial curve. γ(t) is obtained by moving γ(0) along its normal with a speed F which may depend on local properties (such as curvature and normal direction), global
properties of the front (such as integrals along the front or associated differential equations), and independent properties (such as underlying fluid velocity). The main idea in the level set approach is to represent the front \( \gamma(t) \) as the level set \( \{ \phi = 0 \} \) of a higher dimensional function \( \phi \).

### 2.1 Implicit function and signed distance function

There are various implicit functions which can be used to represent interface. In our case, we have used a signed distance function to implicitly represent the interface. It is defined as follows [4]:

\[
\begin{align*}
\phi < 0 & \quad \text{if} \quad x \in \Omega^- \\
\phi = 0 & \quad \text{if} \quad x \in \partial\Omega \\
\phi > 0 & \quad \text{if} \quad x \in \Omega^+
\end{align*}
\]

(1)

Thus, \( \Omega^- \) is related to the region inside the curve and \( \Omega^+ \) is related to the region outside the curve, the interface between them is represented by the function \( \phi = 0 \), which is also called zero level set function.

The level set function is defined as the signed distance function. The value is the distance to the nearest point on the front which is negative inside and positive outside (of course zero at the boundaries). The existence of the front means that the signed distance level set function has positive and negative parts. This property should be kept through the iterations in order not to lose the front. There are several approaches in the literature for the re-initialization of the level set function. The level set function is usually updated by the following equation [4]:

\[
\phi_t = \text{sgn}(\phi) \left( 1 - |\nabla \phi| \right)
\]

(2)

Solving this equation frequently often keeps the function with a gradient magnitude equal to one at the steady state (\( |\nabla \phi| = 1 \)).

### 2.2 Initial formulation

In the previous subsection, we presented the representation for the interface, which is denoted as the zero isocountour of a function \( \phi \). We can link the evolution of this function to the propagation of the front itself through a time dependent initial value problem. At any time, the front is given by the zero level set of the time-dependent level set function \( \phi \). Then we get the following equation [5]:

\[
\phi(x(t), t) = 0
\]

(3)

Taking the time derivative for both sides of the equation, by the chain rule, we get

\[
\phi_t + \nabla \phi(x(t), t) \cdot x'(t) = 0
\]

(4)

Since \( F \) is defined as the speed in the outward normal direction, then \( x'(t) \cdot n = F \), where \( n = \frac{\nabla \phi}{|\nabla \phi|} \). This yields an evolution equation for \( F \):

\[
\phi_t + F \, |\nabla \phi| = 0 \quad \text{given} \quad \phi(x, t = 0)
\]

(5)

This is the level set equation given by Osher and Sethian [2]. By using this equation, the level set method can handle topological changes naturally.

### 3. Deformable Models Based on Curve Shortening

Alvarez and al [6] described an algorithm for image selective smoothing and edge detection. In this case, the image \( I \) evolves according to

\[
I_t = g(|\nabla (G \ast I)|) \frac{|\nabla I|}{|\nabla I|} \left( \frac{\nabla \psi}{|\nabla \psi|} \right)
\]

(6)

Where \( G \) is a smoothing kernel (for example a Gaussian), and \( g(\omega) = 1/(1 + \omega^p) \) is a non-increasing function which tends to zero as \( \omega \to \infty \). We note that \( |\nabla I| \frac{\nabla \psi}{|\nabla \psi|} \) is equal to \( |\xi| \), where \( \xi \) is the coordinate associated with the direction orthogonal to \( \nabla I \). Thus it diffuses \( I \) in the direction orthogonal to the gradient \( \nabla I \), and does not diffuse in the direction of \( \nabla I \). This means that the image is being smoothed on both side of the edge, with minimal smoothing at the edge itself.

Finally the term \( g(|\nabla (G \ast I)|) \) is used for the enhancement of the edges. If \( |\nabla I| \) is small then the diffusion is strong. If \( |\nabla I| \) is large at a certain point \( (x, y) \), this point is considered as an edge point, and the diffusion is weak. The Eq. (6) represents, of course, an anisotropic diffusion.

Caselles and al [7] proposed a scheme for the detection of object boundaries. The technique is based on active contours evolving in time according to geometric measures of the image. If we assume that the deforming curve \( \gamma \) is given as a level set of a function \( \Phi \), then we can represent the deformation of \( \gamma \) via the deformation of the speed \( F \). The proposed deformation is obtained by modifying the edge detection algorithm. They introduce the same stopping term \( g \) used by Alvarez and al [6] and include two terms. The first one is an inflationary force in the normal direction governed by a real constant \( v \). The second term represents a force attracting the front towards the boundary and which has a stabilizing effect. The evolution equation takes the form

\[
\frac{\partial \phi}{\partial t} = g(|\nabla I|) \left\{ \frac{\nabla \psi}{|\nabla \psi|} \right\} \left( \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + v |\nabla \phi| + \nabla g \cdot \nabla \phi \right)
\]

(7)

The stopping term typically has the form
$$g(|\nabla f|) = \frac{1}{1 + |\nabla f|^p} \quad p = 1 \text{ or } 2$$

$$I = G \ast I$$ is a regularized (smoothed) version of the original image $I$.

We note again that the evolution

$$\frac{\partial \phi}{\partial t} = div(\frac{\nabla \phi}{|\nabla \phi|}) |\nabla \phi| = \kappa |\nabla \phi|$$

(8)

moves in the normal direction with a speed proportional to the curvature $\kappa$ [8].

We remind that the curvature can be written as:

$$\kappa = \frac{\nabla \phi}{|\nabla \phi|} \left( \phi_x^2 \phi_{yy} - 2 \phi_x \phi_y \phi_{xy} + \phi_y^2 \phi_{xx} \right)$$

(9)

### 3.1 Numerical discretization

The geodesic active contour model [7] consists of three terms. We notice that the first term represents a parabolic equation and has diffusive effects, so the use of upwind schemes is inappropriate, and classical central differences are used. The second term is an equation which describes a motion in the direction normal to the front and the third term is a construction of one-side upwind difference in the appropriate direction. The discrete scheme of the Eq. (7) is as follows [9]:

$$\phi_{i,j}^{n+1} = \phi_{i,j}^n +$$

$$\Delta t \left( g_{i,j} \kappa_{i,j}^{n} \left[ (\delta_x \phi_i^{n})^2 + (\delta_y \phi_j^{n})^2 \right]^\frac{1}{2} + \left[ \max(g_{i,j}, 0) \nabla^+ + \min(g_{i,j}, 0) \nabla^- \right] \phi_i^{n} +$$

$$+ \max((g_{i,j}), 0) \delta_x \phi_i^{n} + \min((g_{i,j}), 0) \delta_x \phi_i^{n} +$$

$$+ \max((g_{j,i}), 0) \delta_y \phi_j^{n} + \min((g_{j,i}), 0) \delta_y \phi_j^{n} \right)$$

(10)

where

$$\delta_x \phi_i^{n} = \frac{\phi_{i+1,j}^n - \phi_i^n}{\Delta x}, \quad \delta_y \phi_j^{n} = \frac{\phi_{i,j+1}^n - \phi_{i,j}^n}{\Delta y}$$

$$\delta_x \phi_i^{n}, \quad \delta_y \phi_j^{n}, \quad \delta_x \phi_i^0, \quad \delta_y \phi_j^0$$

are respectively the forward, backward and centered schemes, and

$$\nabla^+ \phi_i^{n} = \left[ \max(\delta_x \phi_i^{n}, 0)^2 + \min(\delta_x \phi_i^{n}, 0)^2 \right]^\frac{1}{2} + \max(\delta_y \phi_j^{n}, 0)^2 + \min(\delta_y \phi_j^{n}, 0)^2)^\frac{1}{2}$$

$$\nabla^- \phi_i^{n}$$

is obtained from $\nabla^+ \phi_i^{n}$ by inverting the signs plus and minus.

For the temporal discretization, we can view the Eq. (7) as a nonlinear evolution operator of the type

$$\frac{\partial}{\partial t} \phi_{i,j} = -L[\phi, i, j]$$

(11)

To achieve higher order accuracy in the temporal discretization, one can use Total Variation Diminishing (TVD) Runge Kutta (RK) methods. These methods guarantee that the total variation of the solution does not increase, so that no new extrema are generated. Using a TVD method would ensure that spurious oscillations caused by the numerical method don’t occur.

An Euler method is taken to advance the solution to time $t^n + \Delta t$,

$$\phi_{i,j}^{n+1} = \phi_{i,j}^n - \Delta t L[\phi^n, i, j, k]$$

(12)

### 3.2 Essentially non-oscillatory methods

In the first order accurate upwind differencing, we approximate the spatial derivatives by the difference operators $\delta^+_x, \delta^-_x, \delta_y, \delta_x$, which are respectively the forward, backward and centered schemes. This scheme can be improved upon by a more accurate approximation for $\phi^+_x = \delta^-_x \phi$ or $\phi^-_x = \delta^+_x \phi$. The essentially non-oscillatory methods ENO are an approach to obtaining high resolution that often give better than second order accuracy in smooth regions, including extrema. The general idea behind ENO is to use a higher order accurate polynomial to reconstruct $\phi$ and to differentiate it to get the approximation to $\phi_x$. Such a polynomial is constructed at each grid point. The key to the algorithm is to choose the neighboring points for the interpolation so that we are not interpolating across steep gradients. The implementation of the Hamilton-Jacobi ENO method is based on the Newton polynomial interpolation via a divided difference table. Given a set of $n$ data points $(x_1, \phi_1), (x_2, \phi_2), \ldots, (x_n, \phi_n)$, the Newton polynomial interpolating these points has the form

$$P_{n-1}(x) = a_0 + a_1(x - x_1) + \cdots + a_{n-1}(x - x_{n-1})$$

(4)

Weighted ENO (WENO) methods combine the results obtained using all possible stencils rather than choosing only one. A weighted combination of the results from all stencils is used, where the weight are based on the magnitudes of the divided differences in such a way that smoother approximations receive greater weight. This is more robust than placing all the weight on a single stencil, since it responds more smoothly to changes in the data. Further details can be found in [10, 11].

### 4. Application of Level Set

We have applied our approach on a variety of both synthetic and real images; experiments results show that we have achieved satisfactory results.

Fig. 1 deals with the segmentation of a synthetic image which has geometrical shapes without noise. The initial
level set function is $\Phi_0(x, y)$ representing a rectangle. The curve is reinitialized every 5 steps and $v = -1$. For the spatial discretization we have used a WENO-5 scheme. We can see that the curve evolves quickly, and after 245 iterations, it surrounds the three geometrical shapes. The curve in 3D represents the distance function.

Fig. 2 represents the image of a star with white Gaussian noise ($\sigma = 0.01$). We have used the “imnoise” Matlab function. The edges of the star are surrounded after 175 iterations.

From Fig. 3, it can be seen that the star in the noisy image is successfully segmented for the variance of the noise $\sigma = \{0.01, 0.1\}$ but it is not the case for $\sigma = 0.5$, where the result is not satisfactory.

Fig. 4 represents an image of two rectangles (one inside the other). The level set methods allow us to surround the two rectangles. It suffices to take several circles as a curve of initialization. In this case, the circles increase and merge. The edges of the two rectangles are reached at 120 iterations.

Fig. 5 deals with a weld radiographic image that includes defects which happen during the welding operation. The edges are obtained at 159 iterations, the results obtained in this case are very satisfactory and we have surrounded the area...
5. Level set without re-initialization

From the practical point of view, the implementation of the level set equation can be quite complicated and expensive, because of the operation of a re-initialization and also because of numerical schemes used to ensure stability of the solution, that is why the research community in the field of image segmentation by level set have turned towards the reduction of time and cost of calculates by offering new models which are stable and fast in execution time. The method proposed by Li [12] which is the level set without re-initialization achieved this aim. The evolution PDE of the level set function in this case can be derived directly from the problem of minimizing a certain energy functional defined on the level set function. These variational methods are known as variational level set methods [13]. Using variational level set method, the re-initialization phase can be embedded in the energy functional and we don't need the re-initialization step.

We have chosen the model proposed by Li et al [12] to apply it to image segmentation. This variational formulation is based on a method of penalty to minimize a certain energy functional.

\[ \varepsilon(\phi) = \mu E(\phi) + \varepsilon_m(\phi) \]  

The Eq. (13) consists in two energies:

5.1 An internal energy term \( E(\phi) \)

\[ E(\phi) = \int_{\Omega} P(|\nabla \phi|) d\Omega \]

This penalty term avoids the re-initialization of the level set function and its deleterious numerical errors. The potential function \( P \) can take two different forms:

- Single well potential for distance regularization

\[ P(s) = \frac{1}{2} (s - 1)^2 \]  

- Double well potential for distance regularization

\[ P(s) = \begin{cases} \frac{1}{(2\pi)^2} (1 - \cos(2\pi s)), & \text{if } s \leq 1 \\ \frac{1}{2} (s - 1)^2, & \text{if } s \geq 1 \end{cases} \]

In a recent paper [14] propose this new potential for penalty term, which is aimed to maintain the signed distance property \( |\nabla \phi| = 1 \) only in a vicinity of the zero level set, while keeping the level set as a constant, with \( |\nabla \phi| = 0 \), at locations far away from the zero level set.

5.2 An external energy term \( \varepsilon_m(\phi) \)

\[ \varepsilon_m(\phi) = \lambda L_\mu(\phi) + \nu A_g(\phi) \]

- \( L_\mu(\phi) = \int_{\Omega} g \delta(\phi)|\nabla \phi| d\Omega \) represents the length term
- \( A_g(\phi) = \int_{\Omega} g H(-\phi) d\Omega \) represents the area term
- \( g = \frac{1}{1 + ||\nabla \phi||^p} \) \( (p = 1 \text{ or } 2) \) is the edge indicator function \( \delta \) is a Dirac function and \( H \) is the Heaviside function.

This term which takes into account the length and the area of the curve drives the motion of the zero level set towards the object boundaries.

In the end, the energy functional \( \varepsilon(\phi) \) is then approximated by \( \varepsilon_m(\phi) \) where we have used two approximations and regularizations of the functions \( H \) and \( \delta \).

\[ H_\alpha(z) = \frac{1}{2} \left( 1 + \frac{1}{\alpha} \arctan \left( \frac{z}{\alpha} \right) \right), \quad \delta_\alpha(z) = \frac{1}{\pi} \frac{\alpha}{\alpha^2 + z^2} \]  

\[ \varepsilon_m(\phi) = \mu \int_{\Omega} P(|\nabla \phi|) d\Omega + \lambda \int_{\Omega} g \delta_\alpha(\phi)|\nabla \phi| d\Omega + \nu \int_{\Omega} g H_\alpha(-\phi) d\Omega \]  

The steepest descent process for minimizing this functional is the following gradient flow:

\[ \frac{\partial \phi}{\partial t} = -\frac{\partial \varepsilon_m}{\partial \phi} \]  

For the potential \( P(s) = \frac{1}{2} (s - 1)^2 \) we obtain

\[ \frac{\partial \phi}{\partial t} = \mu \Delta \phi - \text{div} \left( \frac{\varphi_{\phi}}{|\nabla \phi|} \right) + \lambda \delta_\alpha(\phi) \text{div} \left( \frac{\varphi_{\phi}}{|\nabla \phi|} \right) + \nu g \delta_\alpha(\phi) \]
In the case of the Double-Well Potential defined in (15), we have \( \frac{\partial E(\phi)}{\partial \phi} = -\mu \text{div}(d_p(\nabla \phi) \nabla \phi) \) and the PDE can be expressed as

\[
\frac{\partial \phi}{\partial t} = \mu \text{div}(d_p(\nabla \phi) \nabla \phi) + \lambda \delta_a(\phi) \text{div} \left( g \frac{\nabla \phi}{|\nabla \phi|} \right) \\
+ \upsilon \delta_a(\phi)
\]

(20)

where \( d_p \) is a function defined by \( d_p(s) \triangleq \frac{p'(s)}{s} \), \( \mu > 0 \) is a parameter controlling the effect of penalizing the deviation of \( \phi \) from a signed distance function, \( \lambda > 0 \) and \( \upsilon \) are two constants that weight each term.

This PDE can be solved by applying central difference scheme for spatial partial derivatives and forward difference scheme for temporal partial derivatives. The initialization is not necessarily a signed distance function, but can be arbitrary functions.

6. Results

In this section we applied the method of level set without re-initialization to some radiographic images in order to do a comparison between different methods.

Fig. 6 shows the results of applying the level set method without re-initialization. We can notice that we have the

Fig. 6. Image segmentation using level set method without re-initialization.

Fig. 7. Image Segmentation of radiographic weld image using Single-well potential for distance regularization

Fig. 8. Segmentation of radiographic weld image using Double-well potential for distance regularization.
possibility to use a time step of about 4 – 6 without affecting the stability of the algorithm and we have reduced the execution time because of the elimination of re-initialization step.

Fig. 7 shows the evolution of the contour of the image. The first image shows the initial level set contour in light red. The second figure shows the evolution process of the function of level sets. We can notice that the topology changes after 124 iterations. The third figure represents the level set contour after 310 iterations and an execution time of 7.546045 seconds. The results obtained in this case are not satisfactory. The evolution of contour does not stop at the edges of objects. The low gradients are not surrounded.

Fig. 8 deals with a weld radiographic image that includes defects which could happen during the welding operation. We have used the Double-well potential for distance regularization. The edges are obtained at 310 iterations and an execution time of 10.156553 seconds, the results obtained in this case are very satisfactory and we could surround the area defects.

Fig. 9 which represents a zoom (final step) of the figures 7 and 8 shows that the Double-well potential for distance regularization gives better results than the Single-well potential in order to detect the boundary of weld defect. Fig. 10 represents the application of the level set using Double-well potential on another radiographic image.

7. Conclusion

In this paper, we have presented a mathematical model that allows a fast implementation, adaptive and robust methods for image segmentation; as a comparison we can say that the level set without re-initialization using Double-well potential for distance regularization has several advantages compared to traditional level set and level set without re-initialization using Single-well potential for distance regularization. This model, contrary to the conventional level set, is much faster because we have eliminated the re-initialization step procedure. The second point concerns the edges. Compared to the level set without re-initialization using Single-well potential for distance regularization, this model which uses Double-well potential can be successfully applied to images with low gradients. We can see clearly from the figure 9 that the contour is not cut in two parts. The results obtained were very satisfactory. All objects were surrounded. These results motivated us to adopt it for our application.

References


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