Study on Optimal Condition of Adaptive Maximum Torque Per Amp Controlled Induction Motor Drives

Chun-Ki Kwon†

Abstract – Adaptive Maximum Torque Per Amp (Adaptive MTPA) control for induction motor drives seeks to achieve a desired torque with the minimum possible stator current regardless of operating points. This is favorable in terms of inverter operation and nearly optimal in terms of motor efficiency. However, the Adaptive MTPA control was validated only from the viewpoint of tracking a desired torque and was not shown that the desired torque is achieved with minimum possible stator current. This work experimentally demonstrates that optimal condition for Adaptive Maximum Torque Per Amp Control Strategy is achieved regardless of rotor resistance variation.

Keywords: Induction motor model, Optimal control, Adaptive maximum torque per amp (MTPA) control, Maximum torque per amp condition

1. Introduction

Highly efficient operation of induction motor has been studied in the past years [1-6]. Among many efforts to obtain optimal performance, an Adaptive Maximum Torque Per Amp (Adaptive MTPA) control of induction motor drives was proposed [7]. One interesting approach to develop an Adaptive MTPA control is to employ an alternate qd induction machine model (AQDM) rather than the classical qd model (CQDM), such as in [8-11]. The Adaptive MTPA control strategy set forth in [7] has demonstrated the ability to achieve the commanded torque with good accuracy, regardless of temperature variation.

However, efforts have been made to develop many control algorithms for tracking a desired torque, particularly in the case of optimal controls [1-6, 8]. Little study has been conducted on the optimality of the MTPA control that the desired torque was achieved with minimum possible stator current [1-6, 8].

Thus, this work demonstrates its optimality in a way that maximum torque was achieved for a certain stator current, which is also called maximum torque per amp condition. In addition, satisfaction of maximum torque per amp condition is also shown as rotor resistance is varied.

2. Alternate QD Induction Machine (AQDM)

Alternate QD Model (AQDM) included leakage saturation, magnetizing saturation, and distributed system effects in the rotor circuits which CQDM in [8-13] failed to represent over all possible operating conditions due to its constant parameters. Stator and rotor leakage inductance, and the absolute inverse magnetizing inductance are expressed as function of magnitude of magnetizing flux linkage, $\lambda_m$, which is equal to $\sqrt{2} |\lambda_m|$. Stator and rotor leakage inductance, and the absolute inverse magnetizing inductance are denoted as $L_s(\lambda_m)$, $L_r(\lambda_m)$, and $\Gamma_m(\lambda_m)$, respectively. To consider distributed system effects in the rotor circuits, the rotor impedance, $Z_r(j\omega)$ in Laplace form. $Z_r(j\omega)$ is separated into a real and imaginary part, which are denoted $r_r(j\omega)$ and $j\omega L_{r_m}(j\omega)$, respectively.

The steady-state equivalent circuit representing the AQDM in [14] is shown in Fig. 1. Additional details on the AQDM model and its nomenclature are found in [14, 15]. In this work, the functional forms for AQDM parameters are specified as follows:

$$L_s = l_{si} \quad (1)$$
$$L_r(\lambda_m) = l_{r1} + \frac{l_{r2}}{1+(l_{r3}\lambda_m)^{l_{r4}}} \quad (2)$$
$$\Gamma_m(\lambda_m) = m_1 - m_2 \lambda_m + e^{m_2(i_{si}-m_1)} + e^{m_2(i_{si}-m_1)} \quad (3)$$

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Fig. 1. Steady-state equivalent circuit of AQDM model with rotor impedance represented as $r_r(j\omega) + j\omega L_{r_m}(j\omega)$
The parameters in (1)-(4) of AQDM were characterized by applying fitting process to the functional forms of (1)-(4) on the laboratory experimental data taken for a 4-pole, 460 V, 50 Hp, 60 Hz, delta-connected squirrel cage induction motor. The resultant parameters in (1)-(4) of AQDM for the test induction motor are listed in Table 1 and are illustrated in Fig. 2. Detailed procedures of the parameter identification for AQDM is set forth in [14, 16] but is omitted due to limited space.

### 3. Adaptive Maximum Torque per Amp Control Strategy

#### 3.1 Objective and structure

The structure of Adaptive MTPA control strategy is to express root-mean-square magnitude of the stator current $I_s$ and slip frequency $\omega_s$, as functions of the commanded torque and rotor resistance estimate $\hat{r}_r$ as shown in Fig. 3.

Its objective is to produce a desired torque with minimum stator current regardless of rotor resistance. To this end, two properties should be satisfied. One property is the tracking property and the other optimal condition in MTPA control strategy.

#### 3.1.1 Tracking property

The tracking property is to achieve a desired torque at steady states for the current of $I_s^*$, the slip frequency of $\omega_s^*$, and the estimated rotor resistance of $\hat{r}_r$ in the control strategy.

$$\|T_e\left(\omega_s^*, I_s^*, \hat{r}_r\right) - T^*\| < \varepsilon \quad (5a)$$

Herein, $\varepsilon$ is a very small number. As can be seen in Fig. 4, torques at point B and C does not achieve torque command $T^*$, resulting in failure of the tracking property. However, torques at points A and A' satisfies this property.

**Fig. 2.** AQDM parameters for the test induction motor

<table>
<thead>
<tr>
<th>$I_{\theta} (\cdot)$</th>
<th>$r_m (\cdot)$</th>
<th>$Y_e (\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{\theta 1}$</td>
<td>9.06e-4</td>
<td>6.79e0</td>
</tr>
<tr>
<td>$I_{\theta 2}$</td>
<td>6.62e-1</td>
<td>6.26e-1</td>
</tr>
<tr>
<td>$I_{\theta 3}$</td>
<td>4.01e-4</td>
<td>5.03e0</td>
</tr>
<tr>
<td>$I_{\theta 4}$</td>
<td>4.15e-3</td>
<td>1.85e0</td>
</tr>
<tr>
<td>$I_{\theta 5}$</td>
<td>3.75e-1</td>
<td>8.68e-1</td>
</tr>
<tr>
<td>$I_{\theta 6}$</td>
<td>2.59e0</td>
<td>1.29e-1</td>
</tr>
</tbody>
</table>

**Fig. 3.** Structure of adaptive maximum torque per amp control strategy

**Fig. 4.** Optimal condition of (Adaptive) Maximum Torque Per Amp controlled induction machine drive
3.1.2 Optimal condition: MTPA condition

The other is optimal condition in MTPA control strategy at which condition the generated torque $T_r\left( I'_s, \omega_s, \hat{r}_r \right)$ is not only the desired torque but also maximum torque for a given stator current of $I'_s$. This is also called Maximum Torque Per Amp (MTPA) condition which the control strategy was named after. In other words, the desired torque was achieved with minimum stator current by minimizing conduction loss

$$T_r\left( I'_s, \omega_s, \hat{r}_r \right) = \max_{I_s, \omega_s} \left\{ T_r\left( I_s, \omega_s, \hat{r}_r \right) \right\}$$  \hspace{1cm} (5b)

In Fig. 4, the desired torque was achieved at point A' with $I_{s1}$ and at point A with $I_{s2}$. Since $I_{s2}$ is larger than $I_{s1}$, operation with $I_{s1}$ has higher efficiency than the one with $I_{s2}$ due to higher conduction loss. Thus, MTPA condition is satisfied ONLY at A point.

3.2 Derivation of torque equation

To derive Adaptive MTPA control strategy, we can start by expressing the general electromagnetic torque Eq. (6) in the synchronous reference frame in terms of rms magnitude of the applied stator current $I_s$, slip frequency $\omega_s$, and rotor resistance estimate $\hat{r}_r$ as the generated torque in Fig. 3.

$$T_e = \frac{3}{2} P \text{Imag} \left( \frac{Z_{ag}(\lambda_m, \omega_s, r)}{j\omega_s} I_s \right)$$  \hspace{1cm} (13)

where, since $\lambda_m$ is the impedance in parallel with two branches, $j\omega_s/\Gamma_m(\lambda_m)$ and $j\omega_s L_m(\lambda_m) + Z_s(\omega_s)/S$, resulting in

$$Z_{ag}(\lambda_m, \omega_s, r) = \frac{\omega_s}{-j\Gamma_m(\lambda_m) + j\omega_s L_m(\lambda_m) + (r + j\omega_s L_m)}$$  \hspace{1cm} (12)

Thus, substitution of (11) into (10) yields the electromagnetic torque in terms of $\omega_s$, $I_s$, and $r_r$.

$$T_e\left( \omega_s, I_s, r_r \right) = \frac{3}{2} P \text{Imag} \left( \frac{Z_{ag}(\lambda_m, \omega_s, r)}{j\omega_s} I_s \right) I_s$$  \hspace{1cm} (13)

where, since $\lambda_m = \sqrt{2} \left| \hat{I}_m \right|$, $\lambda_m(\omega_s, I_s, r)$ can be computed as

$$\left| \omega_s \hat{\lambda}_m \right| = \sqrt{2} \left| I_s \right| \left| Z_{ag}(\lambda_m, \omega_s, r) \right|$$  \hspace{1cm} (14)

With these two non-linear simultaneous Eqs. (13) and (14), torque can be found when $\omega_s$, $I_s$, and $r_r$ are given. In this work, Newton-Raphson method was utilized to calculate $\hat{\lambda}_m(\omega_s, I_s, r)$ in (14). But any useful nonlinear algebraic equation solver can be used.

3.3 Derivation of AQDM based MTPA Control

3.3.1 Setting up optimization problem

Thus, maximum torque can be obtained by finding optimal slip frequency, $\omega_s$, for the given stator current, $I'_s$, and rotor resistance estimate, $\hat{r}_r$, by applying any optimization technique to (15)

$$\text{maximize} \quad \frac{3}{2} P \text{Imag} \left( \frac{Z_{ag}(\lambda_m, \omega_s, r)}{j\omega_s} I_s \right) I_s$$  \hspace{1cm} (15)

Therein, $\omega_s$ can be any value in real numbers but confined from 0 rad/s to 4 rad/s by our experience in order to avoid finding a local maximizer. Note that maximization of the torque in (13) for a given stator current means the minimization of the current for a given torque. The processing to solve the optimal slip frequency, $\omega_s$, will be repeated for all combinations of $I'_s$ and $r_r$. It is assumed

where the overbar ' − ' indicates complex conjugate.

From the AQDM steady-state equivalent circuit in Fig. 1, $\hat{\lambda}_m$ is expressed as

$$\hat{\lambda}_m(\omega_s, I_s, r) = \frac{Z_{ag}(\lambda_m, \omega_s, r)}{j\omega_s} I_s$$  \hspace{1cm} (11)
that $I_s$ ranges from nearly 0 A to a somewhat over rated current and $r_e$ is selected to vary from 0.01 $\Omega$ to 0.21 $\Omega$.

The $j$−th point of $I_s$ and $k$−th point of $r_e$ will be denoted $I_{s,j}$ and $r_{e,k}$, respectively.

The optimal slip frequency, $\omega_{s,j,k}$, for a given pair of $(I_{s,j}, r_{e,k})$ can be obtained by solving (15) with $I_s$ and $r_e$ replaced by $I_{s,j}$ and $r_{e,k}$.

The resulting optimum slip frequency and the corresponding maximum value of torque for $I_{s,j}$ and $r_{e,k}$ will be denoted $\omega_{s,j,k}$ and $T_{s,j,k}^\star$. These resulting data points are recorded for future data processing and are illustrated in Fig. 5.

### 3.3.2 Curve fitting

The data points \{ $I_{s,j}^\star$, $T_{s,j,k}^\star$, $r_{e,k}^\star$ \} in Fig. 5 (a) are used to construct a stator current control law. Note that $I_s$ is not a function of $r_e$ and so the form of stator current control law can be formulated as

$$
I_s^\star(T_e^\star) = a_1 T_e^\star + a_2 T_e^{\star^h} + a_3 T_e^{\star^h^2}
$$

where $a_1$, $a_2$, $a_3$, $b_1$, and $b_2$ are selected by maximizing the objective fitness function $f_{AMTPA}$ defined as

$$
f_{AMTPA} = \left[ \epsilon + \frac{1}{N_J} \sum_{j=1}^{N_J} \left| I_s^\star - I_{s,j}^\star \right| \right]^{-1}
$$

where $\epsilon$ is a small number ($10^{-3}$) added to the denominator in order to prevent singularities in the unlikely event of a perfect fit, $N_J$ is the number of a set of the stator current command selected, and $I_{s,j}^\star$ is given by (16) with $T_e^\star = T_{s,j}^\star$.

As for the slip frequency $\omega_s^\star$, it is a function of rotor resistance as well as the electromagnetic torque as shown in Fig. 5 (b). The data points \{ $\omega_{s,j,k}^\star$, $T_{s,j,k}^\star$, $r_{e,k}^\star$ \} fit to the functional form

$$
\omega_{s,AMTPA}^\star(T_e^\star, r_e^\star) = d_0 r_e^{-n_1} + d_1 r_e^{n_2} T_e
$$

where $d_0$, $d_1$, $n_1$, and $n_2$ are parameters to be identified. These parameters are identified by maximizing the objective function defined by (19)

$$
f_{AMTPA} = \left[ \epsilon + \frac{1}{N_J} \sum_{j=1}^{N_J} \sum_{k=1}^{N_K} \left| \omega_{s,j,k}^\star - \omega_{s,j,k}^\star \right| \right]^{-1}
$$

where $N_K$ is the number for a set of rotor resistances selected, and $\omega_{s,j,k}^\star$ is given by (18) with $T_e^\star = T_{s,j,k}^\star$ and $r_e^\star = r_{e,k}^\star$. Both (16) and (18) are composed of the AQDM based Adaptive MTPA control law. To obtain coefficients in (16) and (18), any fitting technique could be used. In this work, a genetic algorithm is employed, which was part of the Genetic Optimization System Engineering Tool (GOSET 1.02), a Matlab based toolbox. Details are set forth in [18].

The resulting control laws for $I_s^\star$ and $\omega_s^\star$ for the test machine may be expressed as

$$
I_s^\star(T_e^\star) = 0.102 T_e^\star - 6.410 T_e^{0.011} + 7.790 T_e^{0.132}
$$

and are also depicted, along with the individual data points, in Fig. 5. It can be seen that (20) and (21) fit the calculated data points \{ $I_{s,j}$, $\omega_{s,j}$, $T_{s,j}$ \} in a good accuracy.

### 4. Test Set-up and Configuration for Study on Optimal Condition of Adaptive MTPA Control Strategy

The optimal condition of the proposed Adaptive MTPA control strategy was experimentally investigated. To this
end, a current controlled inverter-fed drive was used to operate the test induction motor. The configuration of the motor drive used in this study is depicted in Fig. 6. Therein, the upper part in Fig. 6 is the power converter topology. The lower part is composed of a speed control block with an anti-windup integrator, synchronous current regulator (SCR), and delta modulator, to determine the switching signal for switching devices, T1~T6.

For this study, the optimal condition of the Adaptive MTPA control strategy was compared with that of the MTPA control strategy whose slip frequency, \( \omega_s \), control be obtained by substituting the value of 0.176 \( \Omega \) for rotor resistance, \( r_r \), in (21), as in Fig. 4. Thus, it results in

\[
\omega_s^{*, MTPA} \left( T_e^* \right) = 1.2707 + 0.0044 \cdot T_e^* 1.15
\]  

(22)

in [7]. Herein, 0.176 \( \Omega \) is the value of the rotor resistance at the temperature of 43 \( ^\circ \)C at which the test induction motor was characterized. For convenience purposes, MTPA control strategy is referred to as Non-Adaptive in that the commanded current and slip frequency are not adaptive to rotor resistance variation.

Additional details on the configuration for the drive and nomenclature are set forth in [16]. Fig. 7 shows the experimental set-up for this work. The induction motor being tested is enclosed in the small chamber and is exactly same as the motor used for the dynamometer.

The experiment focuses on a single operating condition. As shown in Fig. 8, the test induction machine was driven at a speed of 900 rpm and the torque command was set to 150 Nm in both cases. The torque estimator used in this work was shown to be highly accurate when the induction motor is rotating at moderate to high speeds [19-20].

With the experimental setup mentioned above, due to the difficulty in directly measuring the actual rotor resistance of the test motor, the rotor resistance estimator proposed in [21] was utilized and incorporated into the proposed control strategy.

5. Review of Tracking Performance of Adaptive MTPA Control Strategy

To help readers understand the performance of the Adaptive MTPA control strategy, the results demonstrated in [7] are rewritten in this section. In [7], the Adaptive MTPA control strategy was validated by comparing its performance to that of the MTPA control strategy with slip frequency control law of (22).

As in Fig. 9, the performance for the two controls was recorded as the stator surface temperature varies. Therein, the red solid line with dot marks indicates the resultant torque of the Adaptive MTPA control strategy whose slip frequency command is \( \omega_s^{*, MTPA} \) in (21) and the dashed line indicates the resultant torque of the Non-Adaptive MTPA control strategy whose slip frequency command is \( \omega_s^{*, MTPA} \) in (22). As can be seen, Adaptive MTPA control strategy produced the resultant torque closer to the torque command of 150 Nm and larger than resultant torque which the Non-Adaptive MTPA control strategy
6. Study on Optimal Condition of MTPA Control Strategy due to Rotor Temperature Variation

It would be of interest to investigate how MTPA condition is affected by rotor temperature variation.

6.1 MTPA condition of Non-Adaptive MTPA control strategy

Fig. 10 shows that the resultant electromagnetic torques at estimated optimal slip frequency command, \( \omega_s^{* \text{MTPA}} \), in (22) as well as two additional sets of resultant torques taken at 0.9 and 1.1 times of \( \omega_s^{* \text{MTPA}} \). In the initial part of each study as in Fig. 10 (a), when the temperature is low and where rotor resistance is smaller than the rotor resistance used to design Non-adaptive MTPA control strategy, the largest torque and closest to the commanded torque was not produced at \( \omega_s^{* \text{MTPA}} \) but at 0.9 times \( \omega_s^{* \text{MTPA}} \), resulting in failure of MTPA condition. Likewise, at the last part of the studies, the largest torque was again obtained at 1.1 times \( \omega_s^{* \text{MTPA}} \), which also means failure of MTPA condition. However, for some temperature region, maximum torque per ampere condition is actually achieved at the estimated optimal slip frequency command \( \omega_s^{* \text{MTPA}} \). In the middle part of the study as in Fig. 10 (b), when rotor resistance was close to the design value, it can be seen that a slip value of \( \omega_s = \omega_s^{* \text{MTPA}} \) yields the most torque, thus satisfying MTPA condition.

6.2 MTPA condition of adaptive MTPA control strategy

The study on optimal condition, which is Maximum Torque Per Amp condition, of the proposed Adaptive MTPA control strategy was made by comparing its MTPA condition to that of the Non-Adaptive MTPA control strategy to show optimal control of the Adaptive MTPA control strategy. Fig. 11 illustrates the MTPA condition of the two controls as the stator surface temperature increased during the duration of the study. The red solid line with dot marks indicates the estimated torque of the Adaptive MTPA control strategy whose slip frequency command is \( \omega_s^{* \text{AMTPA}} \) given by (21). For Non-Adaptive MTPA control strategy, three sets of estimates over neighborhoods of \( \omega_s^{* \text{MTPA}} \) in (22) have been included to investigate the satisfaction of the Maximum Torque Amp Condition. Therein, the dashed line indicates the estimated torque of the non-adaptive MTPA control strategy whose slip frequency command is \( \omega_s^{* \text{MTPA}} \) given by (22). In the initial part of study, when the temperature is low and where rotor resistance is smaller than the rotor resistance used to design Non-Adaptive MTPA control strategy, the torque estimated at 0.9 times of \( \omega_s^{* \text{MTPA}} \) is largest (of the non-adaptive controls). In the middle part (in time) of the study, when
the rotor resistance was close to the design value, it can be
seen that a slip value of $\omega_s = \omega_{MTPA}$ in (22) yields the
most torque (again, at the Non-Adaptive controls). Finally,
as the studies proceed in time, eventually the largest torque
estimate was obtained using 1.1 times of $\omega_{MTPA}$. On the
other hand, Adaptive MTPA control strategy adjusted the
slip frequency command, $\omega_{AMTPA}$, given by (21) such that
the largest torque are always achieved at $\omega_{AMTPA}$, thus
satisfying MTPA condition.

The observations from Fig. 11 indicate that the Adaptive
MTPA control strategy satisfies MTPA condition regardless
of rotor temperature variation as well as achieve the
desired torque accurately as validated in [7].

7. Conclusion

It was experimentally shown that the Adaptive MTPA
control strategy performs optimally regardless of rotor
resistance variation by showing that the torque produced at
the optimal slip frequency is the largest and desirable at the
same time, indicating that optimal condition (MTPA
condition) of Adaptive MTPA control strategy is satisfied.

As can be seen from experimental results in the previous
section, the Adaptive MTPA control strategy makes true
optimal performance regardless of rotor resistance variation
by reflecting rotor resistance variation in the design of
optimal slip frequency control law.

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Per Amp Control Strategy for Induction Motor

Fig. 11. MTPA conditions along temperature variation when
controlled by Adaptive MTPA control strategy

(a) satisfied (b) satisfied (c) satisfied

with $T_p^{\text{opt}}=150\text{Nm} $
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