Class E Power Amplifiers using High-Q Inductors for Loosely Coupled Wireless Power Transfer System

Jong-Ryul Yang†, Jinwook Kim* and Young-Jin Park*

Abstract – A highly efficient class E power amplifier is demonstrated for application to wireless power transfer system. The amplifier is designed with an L-type matching at the output for harmonic rejection and output matching. The power loss and the effect of each component in the amplifier with the matching circuit are analyzed with the current ratio transmitted to the output load. Inductors with a quality factor of more than 120 are used in a dc feed and the matching circuit to improve transmission efficiency. The single-ended amplifier with 20 V supply voltage shows 7.7 W output power and 90.8% power added efficiency at 6.78 MHz. The wireless power transfer (WPT) system with the amplifier shows 5.4 W transmitted power and 82.3% overall efficiency. The analysis and measurements show that high-Q inductors are required for the amplifier design to realize highly efficient WPT system.

Keywords: Class E power amplifier, High efficiency, High quality-factor, L-type matching circuit, Loosely coupled wireless power transfer system

1. Introduction

Wireless power transfer (WPT) system supplies electrical power to remote devices, and it offers convenience and safety for using the electrical devices. It is promising technology especially in the mobile devices such as cellular phones and tablets. Because the transmission efficiency of the WPT system is generally lower than that of wired power supply system and the transmitted power through the air is restricted by the government regulation, the transmitted power and transmission efficiency are important factors in the WPT system [1]. A class E power amplifier (PA) is the promising circuit topology of a power transmitter in the WPT system on the HF bands due to high output power and theoretical 100% drain efficiency with a simple design [2]. The highly efficient amplifier based on GaN device was reported in [3], but it is not easy to apply the WPT system in mobile devices due to the cost. Therefore, the efficient and economical amplifier is necessary for the applications.

A class E PA generally consists of a switching transistor, an input matching circuit, an output shunt capacitor, inductors for transistor biasing and dc feeding, an output filter, and an output matching circuit. The output matching circuit of the amplifier is important in the WPT system because impedance mismatches between the PA and coupling coils can decrease the transmission efficiency [4] and the mismatches can also generate an oscillating signal in the amplifier. An L-type output matching circuit is useful for the power transmitter in the WPT system because the circuit can obtain both impedance matching and harmonic rejection [2] with the minimum discrete components in the output of the class E amplifier.

This paper shows the design of the class E PAs operating at 6.78 MHz for the use in the WPT system. Power losses and effects of each component in the amplifier with the matching circuit are analyzed to find elements for improving the efficiency. Based on the analysis, the PAs are designed to have the output matching circuit using high-quality (high-Q) inductors for obtaining harmonic rejection, impedance matching, and high efficiency all together. The high-Q inductor is also used as a dc feeding circuit for decreasing the power loss. The WPT system using the PA is demonstrated for loosely coupled wireless power transfer system.

2. Design and Loss Analysis of the Amplifier

The circuit diagram of the class E zero-voltage switching (ZVS) PA with the matching circuit is shown in Fig. 1(a). The matching circuit can be modified as the equivalent circuit in Fig. 1(b), which is a basic circuit of the class E PA. An ac load resistor $R_s$ in the equivalent circuit, an overall output shunt capacitor $C_{ov}$, and an inductor $L_f$ for output matching circuit can be expressed as in [5]

$$R_s = \frac{8}{\pi^2 + 4} \frac{V_{DD}}{P_v},$$

(1)
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\[
C_o = \frac{8}{\pi (\pi^2 + 4) R_S} \frac{1}{2\pi f} = \frac{P_o}{2\pi f \cdot V_{DD}^2},
\]

(2)

\[
L_f = \frac{Q_L}{2\pi f} R_S = \frac{8Q_L \cdot V_{DD}^2}{2\pi f \cdot (\pi^2 + 4) P_o}
\]

(3)

where \( V_{DD} \) is the dc supply, \( P_o \) is the output power in the \( R_S \), and \( Q_L \) is the loaded quality factor of the \( L_f \)-\( C_f \)-\( R_S \) series resonant circuit. It assumed that the \( Q_L \) is high enough that the current \( i_R \) in the resonant circuit is sinusoidal. Design equations for the capacitor \( C \) and \( C_f \) are

\[
C = \frac{1}{2\pi f \cdot R_L \sqrt{R_S (1 + q_b^2)}} - 1
\]

(4)

\[
q_b = \frac{R_S (1 + q_b^2)}{R_L}
\]

(5)

where \( R_L \) is the output load impedance, which is normally fixed to 50\( \Omega \), and the reactance factor of the impedance to the right of node B in Fig. 1 is

\[
q_b = \frac{\pi (\pi^2 - 4)}{16}.
\]

(6)

The efficiency of the PA can be described with power losses in each component [5]. When it is assumed that the current through the dc feed inductor \( L \) is a nearly constant, the power loss of the inductor \( L \) can be expressed

\[
P_{\text{IL}} = \left( r_L \right) \left( I_{\text{rms}} \right) P_o
\]

(7)

where \( r_L \) is the dc equivalent series resistance (ESR), and \( I_{\text{rms}} \) is the rms value of the inductor current \( i_L \). When the duty cycle is 0.5, the conduction loss in the switching transistor can be expressed

\[
P_{\text{floss}} = \frac{\left( r_{\text{foss}} \right) \left( I_{\text{rms}} \right) P_o}{2(\pi^2 + 4) R_S}
\]

(8)

where \( r_{\text{foss}} \) is the on-resistance of the switching transistor, and \( I_{\text{rms}} \) is the rms value of the switching current \( i_S \). The power loss in the ESR \( r_{\text{foss}} \) of the shunt capacitor \( C_o \) can be expressed as

\[
P_{\text{foss}} = \left( r_{\text{foss}} \right) \left( I_{\text{rms}} \right) P_o
\]

(9)

where \( r_{\text{foss}} \) is the amplitude of the current through \( C_o \), and \( r_{\text{foss}} \) is the amplitude of the current through \( R_L \), \( K \) is the impedance ratio between \( 1/2\pi f C_f \) and \( 1/2\pi f C + R_L \), and the ratio \( |1-K|^2 \) can be expressed as

\[
|1-K|^2 = \frac{C_f^2}{\left(2\pi f \cdot C + R_L\right)^2 + C_f^2 + C^2 + 2CC_f}.
\]

(13)

Fig. 2 shows the ratio \( |1-K|^2 \) which depends on \( Q_o \), \( V_{DD} \), and \( P_{\text{out}} \) at 6.78 MHz. The ratio is a constant for \( Q_o \), which is 0.4614 when the supply voltage is 20 V and the output power is 10 W as shown in Fig. 2(a). Because the ratio is the current rate to transmit the output load from (11) to (13), it can be briefly expressed as

\[
|1-K|^2 = \frac{R_S}{R_f}.
\]

(14)

Fig. 1. Class E zero-voltage-switching power amplifier with an L-type matching circuit: (a) Basic circuit; (b) Equivalent circuit.
When the difference between ESRs of those capacitors is neglected and the ESR is described as $r_{CT}$, the power loss of the capacitors in the matching circuit can be expressed as

$$P_{IR} = \frac{1}{2} \left( \frac{2 \pi f T}{2} \right) P_{R}$$

where $f_T$ is the frequency of the output power $P_{out}$ in the load.

Using (1), (12), and (14), the output power $P_{out}$ in the load can be expressed as

$$P_{out} = \frac{1}{2} \left( \frac{2 \pi f T}{2} \right) R_{o} P_{R}$$

The output power of the class E PA with the matching circuit is the same as that of the general class E PA. The input power of the transistor is given by in [5]

$$P_{in} = gV_{GSm} Q_{g}$$

where $V_{GSm}$ is the peak value of the gate-to-source voltage and $Q_{g}$ is the gate charge at $V_{GSm}$. The power added efficiency (PAE) can be expressed as

$$\eta_{PAE} = \frac{P_{out} - P_{in}}{P_{DC}} = \frac{P_{o} - P_{n}}{P_{o} + P_{loss}} = \eta_{D} - \frac{P_{n}}{P_{o} + P_{loss}}$$

where $P_{DC}$ is the dc power consumption in the amplifier, $P_{in}$ is the overall power loss, and the drain efficiency $\eta_{D}$ is $P_{o} / (P_{o} + P_{loss})$. Fig. 3 shows the calculated drain efficiency and the ratio of the contribution to the $P_{loss}$ in each component of the PA. The main factor of the loss is the matching circuit. The turn-on switching loss is zero due to ZVS operation, and the turn-off switching loss can be considered with a fall time. The average value of the loss associated with the fall time $t_f$ can be expressed as [5]

$$P_{ft} = \frac{1}{2 \pi} \int_{0}^{\pi} i_s v_s d\omega t_f \approx \frac{(2 \pi f T)}{12} P_{o}$$

where $i_s$ and $v_s$ are current and voltage in the switching transistor. Using (1), (12), and (14), the output power $P_{out}$ in the load can be expressed as

$$P_{out} = R_{o} \left| \frac{I_s}{2} \right|^2 = \left| 1 - K \right|^2 R_{o} P_{o} = P_{o}$$

The calculated drain efficiency and power losses in each component of the class E PA with an L-type matching circuit depending on the quality factor of inductors for $r_{ON} = 1.33 \Omega$, $L = 2000 \text{ nH}$, $L_f = 1300 \text{ nH}$, $C_o = 600 \text{ pF}$, $C_f = 440 \text{ pF}$, $C = 10 \text{ nF}$, $t_f = 1 \text{ ns}$, $\left| 1 - K \right|^2 = 0.4614$, and the quality factor of capacitors is 200.
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switching conduction loss by the on-resistance of the transistor, and it cannot be decreased because the discrete transistor is used in the amplifier. Therefore, the losses in the matching circuit and other discrete components, especially inductors as shown in Fig. 3, should be minimized for increasing the efficiency. A high-Q inductor is required to be used in the L-type matching circuit to minimize the losses in the circuit, and a dc feed inductor also has a high quality factor to decrease the power loss in the output.

The initial parameters of the amplifier are designed by using the theoretical method, which is previously shown in this section, and those are optimized by using circuit simulation to obtain the maximum PAE and output power with input and output matching. The previous theoretical design assumes that the inductance of the dc feed inductor is high enough that the current through the inductor is a constant, but the efficiency of PA decreases seriously due to the increase of the ESR of the inductor in this case. Therefore, a small inductor is used in the dc feed path, and the effects of the current variation in the inductor are considered using [6]. Considering the finite dc feed inductor in the amplifier, the current in the off-state of the transistor is affected by the resonant frequency of the inductance of the inductor and the output capacitance. And the inductor could also compensate for the output capacitance of the transistor to extend the operating frequency. Fig. 4 shows the schematic design of the class E power amplifier with the L-type matching circuit. High-Q inductors are made to order by Core Electronics Co., Ltd. The capacitors by Murata Manufacturing Co., Ltd. and the other inductors by Coilcraft are used for input matching and output discrete components. Freescale LDMOS MRFE6VS25NR1, which is more inexpensive than the GaN device, is used as a switching transistor. The size of the fabricated amplifier as shown in the inset of Fig. 4 is 44 x 21 mm² on an FR4 substrate.

3. Measurement Results of the Amplifier

There are three amplifiers using high-Q inductors with different quality factors. Quality factors of the inductors are measured by using [7]. Each inductor is connected on the calibrated PCB, and these factors are obtained from the ratio between inductances and ESRs measured using a vector network analyzer. Inductors with quality factors of more than 190 are used in the dc feed path of the amplifiers. The inductors with quality factors of 125 are used in the L-type matching circuit of the PA #1 and #3, and the quality factor of the inductor used in the network of the PA #2 is 200. The PA #4 is made by the inductors with low quality factors of less than 50 as a control sample. The class E operation is shown with waveforms at the drain of the transistor and the output of the amplifier in Fig. 5. The measurement results are shown in Fig. 6 for the case where the input power is 18.8 dBm and the gate bias of the transistor is 1.4 V, which are the optimum values for high efficiency with over 5 W output power. At 20 V supply voltage, the PAEs of three amplifiers are measured to be 89.8% on the average and maximum 90.8% in the PA #2. The output powers at the same bias condition are 7.1 W on
the average and maximum 7.7 W in the PA #2. The output performances in Fig. 6 are improved as increasing the quality factors of the inductors. The results show that the high-Q inductors should be used for the highly efficient amplifier and the use of inductors of a quality factor of more than 120 is important to make a highly efficient class E power amplifier with around 90% PAE. The measured quality factors of the inductors and output performances at the optimum bias condition are summarized in Table 1.

When the supply voltage is increased from 20 V, the output power increases but the PAE decreases due to the thermal degradation in the output. The stability of the amplifier is important for the system because it could be used to charge the device for a long time. The amplifier continuously operates in the system, and the performances are measured to verify the stability after one hour. As shown in Fig. 7, the PAE and the output power in the PA #2 decrease to 89.3% and 7.5 W with a supply voltage of 20 V.

### Table 1. Measured quality factors of the inductors and output performances of the power amplifiers.

<table>
<thead>
<tr>
<th>@ 6.78 MHz</th>
<th>PA #1</th>
<th>PA #2</th>
<th>PA #3</th>
<th>PA #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-factor of a dc feed inductor</td>
<td>195</td>
<td>195</td>
<td>190</td>
<td>35</td>
</tr>
<tr>
<td>Q-factor in the L-type circuit</td>
<td>125</td>
<td>200</td>
<td>125</td>
<td>35</td>
</tr>
<tr>
<td>$P_{out}$ [W] with 20 V $V_{DD}$</td>
<td>6.8</td>
<td>7.7</td>
<td>6.9</td>
<td>5.5</td>
</tr>
<tr>
<td>PAE [%] with 20 V $V_{DD}$</td>
<td>89.5</td>
<td>90.8</td>
<td>89.1</td>
<td>75.5</td>
</tr>
</tbody>
</table>

![Fig. 7. Thermal degradation of the power amplifier #2 after one hour.](image)

4. Wireless Power Transfer System with the Amplifier

A wireless power transfer network (WPTN) is designed by using [8] and its impedance matching at the transmitter (Tx) is modified to have maximum power transfer efficiency for the case of only one receiving coil by using the source coil. The WPTN is demonstrated with copper wires and a spiral coil on a PCB. The diameter of the copper wire is 0.64 mm and the cross section of the spiral coil is 0.6 x 0.2 mm². A four-turn Tx coil is implemented with a planar circular concentric multi-loop coil. A ten-turn receiver (Rx) coil is made using a planar rectangular spiral coil with a pitch of 1.6 mm. The distance between Tx and Rx coils is 12 mm, which is determined by the thickness of the plastic plate fixed to the coils. The coupling efficiency 88.0% at 6.78 MHz is measured as shown in Fig. 8. The physical parameters and the photograph of the fabricated coils for the network are shown in the inset of Fig. 8. The wireless power transfer system using the amplifier and the WPTN is measured to be 5.4 W transmitted power and 82.3% overall transmission efficiency. The PAE of the amplifier in the system is calculated as 93.5%, which increases by 2.6% compared with the measurement in section 3, using the coupling and the overall transmission efficiencies. The increase of the PAE can be understood by

![Fig. 8. Measured coupling efficiency of the wireless power transfer network and the photograph of the fabricated coils for the network.](image)

![Fig. 9. Output power spectrum of the wireless power transfer system using the amplifier.](image)
obtaining complex conjugate matching between the amplifier and the WPTN. All the harmonics are measured to be less than 60 dBc as shown in Fig. 9 because the WPTN functions as a harmonic rejection filter [4].

5. Conclusion

A 6.78-MHz class E power amplifier using high-Q inductors is realized for loosely coupled wireless power transfer system. An L-type matching circuit is used in the output of the amplifier for obtaining both harmonic rejection and impedance matching. The analysis about the power losses and effects of each component on the overall power loss in the amplifier is described with the current ratio transmitted to the output load in the matching circuit. The efficiency of the amplifier is improved by using high-Q inductors in the matching circuit and a dc feed. The amplifier with inductors of quality factors of approximately 200 achieves 7.7 W output power and 90.8% power added efficiency at 20 V supply voltage. A highly efficient wireless power transfer system is realized using the amplifier and the wireless power transfer network with 88.0% coupling efficiency. The transmitted power is 5.4 W and the overall efficiency is 82.3% in the system.

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References

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