Probabilistic Assessment of Total Transfer Capability Using SQP and Weather Effects


Abstract – This paper presents a probabilistic method to evaluate the total transfer capability (TTC) by considering the sequential quadratic programming and the uncertainty of weather conditions. After the initial TTC is calculated by sequential quadratic programming (SQP), the transient stability is checked by time simulation. Also because power systems are exposed to a variety of weather conditions the outage probability is increased due to the weather condition. The probabilistic approach is necessary to evaluate the TTC, and the Monte Carlo Simulation (MCS) is used to accomplish the probabilistic calculation of TTC by considering the various weather conditions.

Keywords: Total transfer capability, Sequential Quadratic Programming (SQP), Transient stability, Probabilistic approach, Weather condition

1. Introduction

Total transfer capability (TTC) is the largest quantity of electric power that can be transferred over the interconnected transmission networks in a reliable manner while meeting all of the pre- and post-contingency system conditions [1]. The relationship of the TTC and available transfer capability (ATC) is described in the North American Electric Reliability Council (NERC) definition [1].

At the present time, there are two techniques of methods for calculating the TTC, deterministic and probabilistic. The deterministic approaches mainly use the methods such as security constrained optimal power flow (SCOPF), continuation power flow (CPF), linear programming (LP) and repeated power flow (RPF), and have some difficulties in handling the uncertainties of power systems such as the possibility of faults and weather prediction [2–4]. LP is one of fast methods to search the solution for the initial TTC. In order to reduce linearization errors, load flows should be performed periodically. Probabilistic methods have considered the uncertainties of the system performance that could not be addressed in a deterministic way, and have been implemented to evaluate the TTC for various outages [5–7]. As all power system networks and the system components are exposed to nature, they are affected by the weather condition considerably, and the failure rates of transmission lines are increased due to weather conditions. Therefore, the operation of power system could be addressed in a probabilistic approach [8].

This paper presents a probabilistic method to evaluate the TTC by considering the uncertainty of weather conditions. In the TTC evaluation, unlike the previous study of the author [8], optimization method such as SQP is used to calculate the initial TTC. The weather conditions are divided into normal and adverse weather. Because the failure rate in adverse weather condition is considerably larger than that in normal weather, the contingency in power system influences TTC assessment.

2. Determination of TTC Using Deterministic Approach

2.1 Problem formulation

In order to determine the TTC by deterministic method, the mathematical formulation for the TTC evaluation can be expressed as follows:

Maximize \( \lambda \)  
subject to  
\[
P_{G_j} - P_{L_j} - \sum_{j=1}^{n} \|v\|_j (G_{ij} \cos \delta_j + B_{ij} \sin \delta_j) = 0
\]  
\[
Q_{G_i} - Q_{L_i} - \sum_{j=1}^{n} \|v\|_j (G_{ij} \sin \delta_j - B_{ij} \cos \delta_j) = 0
\]
\[
|v|_{\max} \leq |v| \leq |v|_{\max}
\]
\[
S_{\max} \leq S_{\max}
\]
\[
|\delta_{ij}(t) - \delta_{ij}(t)| \leq \delta_{\max}
\]
For calculating the TTC, the injection real and reactive power at source and sink buses are functions of the incremental factor of load and generation in outage $l$, $\lambda_l$. The objective is to maximize incremental factor of load and generation in outage $l$, $\lambda_l$. Eq. (6) means the difference between the critical energy and the transient energy as transient stability constraint. The optimization method enables transfers by increasing the complex load generation and load.

$$\lambda_l : \text{incremental factor of load & generation in outage } l$$

$$P_{gi} = P_{gi0}(1 + \lambda_l k_{gi}) : \text{real power generation at bus } i$$

$$P_{li} = P_{li0}(1 + \lambda_l k_{li}) : \text{real power demand at bus } i$$

$$Q_{gi} = Q_{gi0}(1 + \lambda_l k_{gi}) : \text{reactive power demand at bus } i$$

where,

$P_{gi0}$ : original real power generation at bus $i$

$P_{li0}$, $Q_{gi0}$ : original real/reactive power load at bus $i$

$k_{gi}, k_{li}$ : constants specifying the rate of change in generation and load

$V_i^* : \text{voltage magnitude at bus}$

$V_{im}, V_{imax} : \text{lower and upper limits of voltage magnitude at bus } i$

$S_{ij} : \text{apparent power flow in line } ij$

$S_{ij}^* \text{max} : \text{thermal limit of line } ij$

$\delta_{i0}(t), \delta_{j0}(t) : \text{rotor angles of generator } i, j$

$\delta_{i, \text{max}} : \text{maximum secure relative swing angle}$

2.2 Sequential quadratic programming

Sequential quadratic programming (SQP) is the optimization method for the minimization of the maximum of a set of smooth objective functions subject to equality and inequality constraints and simple bounds on the variables [7]. In order to get the optimal solutions the SQP generates a point satisfying these constraints by solving a strictly convex quadratic program (QP) using a positive definite estimate $H$ of the Lagrangian. And an Armijo-type arc search or line search (monotone, nonmonotone) are used to compute the direction of descent the objective function. Generalized the SQP algorithms are implemented as follows:

**Step 1 Initialization**

i) Initial value of variables $x_0$, step size $t_0$ and search directions $d_k$. If $x_0$ is infeasible for some constraint, substitute a feasible point.

**Step 2 Computation of search**

i) Compute $d_k$, the solution of the strictly convex QP

ii) Compute the step size $t_k$

**Step 3 Updates**

i) Update Hessian matrix of Lagrangian using the Powell modification.

ii) Set $x_{k+1} = x_k + t_k d_k + t_k^2 d_k$

iii) Solve the unconstrained QP problem in $\mu$, eq. (13). Increase $k$ by 1.

$$\min \left[ \sum_j \xi_j \nabla f_j(x_{k+1}) + \xi_k \sum_j \lambda_i \nabla g_j(x_{k+1}) + \sum_j \mu_j \nabla h_j(x_{k+1}) \right]$$

where $\xi_k, \xi_j, \mu_j$, and $\lambda_i$ are the K-T multipliers associated with QP for the objective functions, variable bounds, equality constraints, and inequality constraints respectively.
3. Determination of TTC using Probabilistic Approach

3.1 Weather model with uncertainty

Because power systems are exposed to various weather conditions the failure rate of outdoor components can be increased very significantly during adverse weather periods such as gales, lighting storms, etc. Usually the components that receive the most effects of the weather are transmission lines in the power system. The transmission line can be defined to be in two states that are influenced by weather conditions, normal and adverse weather conditions [12].

3.2 Probabilistic TTC using monte carlo simulation

The sequential Monte Carlo Simulation (MCS) is used to apply the probabilistic approach. The operating characteristic of each component in the system is represented by the two-state model described by up- and down-states, and the operating state of the whole power system can be obtained by considering the state of all components in the system and the uncertainty of weather conditions as shown in Fig. 2. The operating time in the up-state is called time to failure (TTF) and repair time in the down state is called time to repair (TTR). The TTF and the TTR can be expressed by the exponential distribution [12].

\[
T_{TFi} = \frac{1}{\lambda_i} \ln(1-U) \tag{7}
\]

\[
T_{TRi} = \frac{1}{\mu_i} \ln(1-U) \tag{8}
\]

where

\( \lambda_i \) : failure rate of component \( i \)
\( \mu_i \) : repair rate of component \( i \)
\( U \) : uniformly distributed random number

4. Numerical Analysis

4.1 Deterministic Assessment of TTC

In order to show the effectiveness of the proposed algorithm, it has been tested on a 6-bus 7-line system, which is shown in Fig. 4.

In order to show the effectiveness of the proposed method the case is tested as follows:

Case A: Constraints in voltage magnitudes at buses and thermal limits at transmissions not including transient stability

Case B: Constraints in voltage magnitudes at buses and thermal limits at transmissions including transient stability

The sequential quadratic programming (SQP) is used to determine the TTC of each case. Table 1 and Fig. 5 show the result of the TTC and the transient stability in each fault. In table the TTC level of base case, 133.45 MW, means the maximum power that can be transferred from source area to sink area in the base case that no outage happens to the system. When the transient stability is
checked in Case A, the TTC level of line *3-4 fault is equal to 141.98 MW but transient stability is not satisfied in Fig. 5 (b). In Case B, the TTC level of line *3-4 fault is reduced to 107.92 MW while transient stability is satisfied in Fig. 5 (e). It is seen that the TTC level can be determined by the transient stability as well as bus voltage magnitude and line thermal limits. As a result, in case of line fault 4-5 the initial TTC level is equal to 66.59 MW. It is seen that the TTC of the test system not including transient stability is equal to 66.59 MW, the smallest value of TTC levels. The TTC of this test system is determined not by transient stability but by thermal limits in deterministic assessment of TTC.

4.2 Probabilistic assessment of TTC

In order to apply the probabilistic approach considering uncertainty of weather, the weather data are divided into normal and adverse weather for 1 year from Korea Meteorological Administration (KMA), which is in 2012 year. Fig. 6 shows that the weather is divided into the normal and adverse weather states for about 8760 hours, where the number 0 and 1 represent the normal weather and adverse weather conditions, respectively. In adverse weather conditions, the failure rate of a component can be considerably larger than the normal weather condition. This paper assumes that failure rate in the adverse weather is ten times higher than that in the normal weather [8].

Using sequential MCS, The operating state of the system in the normal condition only and the condition that include

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**Table 1. TTC level with and without transient stability**

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<tr>
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<tbody>
<tr>
<td>Base case</td>
<td>133.45</td>
<td>-</td>
<td>133.45</td>
<td>-</td>
</tr>
<tr>
<td>Line *2 - 3 fault</td>
<td>93.66</td>
<td>satisfied (a)</td>
<td>93.66</td>
<td>satisfied (a)</td>
</tr>
<tr>
<td>Line *3 - 4 fault</td>
<td>141.98</td>
<td>not satisfied (b)</td>
<td>107.92</td>
<td>satisfied (e)</td>
</tr>
<tr>
<td>Line *4 - 5 fault</td>
<td>66.59</td>
<td>satisfied (c)</td>
<td>66.59</td>
<td>satisfied (c)</td>
</tr>
<tr>
<td>Line *2 - 5 fault</td>
<td>106.32</td>
<td>satisfied (d)</td>
<td>106.32</td>
<td>satisfied (d)</td>
</tr>
</tbody>
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![Fig. 5. Transient stability.](image-url)
adverse weather condition for 1 year is shown in Fig. 7 and Fig. 8, respectively. The horizontal axis is the operating state of the system for 8760 hour and the vertical axis is the number of surviving components; for example, the number 5 means that two components are faulted among the six 7 components [8]. From Fig. 8, it is seen that the frequency of outage in the condition that include adverse weather is higher than for the normal weather condition. The sequential MCS can be taken to provide the probability distribution function (PDF) of TTC. Considering the normal weather condition, Fig. 9 shows the system can withstand most transient faults in the range between 130 and 140 MW in probability.

Because the outages spread widely without special trend in Fig. 7, the more the TTC value in Fig. 9 moved to large value side, the higher the probability of the outages is.

On the other hand, considering adverse weather in Fig. 10, surviving probability of system is lower than of normal weather. Because many outages occurred at the period affected by adverse weather in Fig. 8, the probability of the

outages is high in lower portion of TTC value in Fig. 10.

If system is operating under the certain adverse weather condition, it is desirable to determine the TTC value in the range between 65 and 70 MW in probability in order to make the system stable. The reason is that the probability of TTC about 66 MW for the case with adverse weather is much higher than compared with the value in the normal weather for 1 year. Especially, it is seen that it is possible to operate power systems at lower TTC during the period such as a special weather statement, if the weather condition is considered in advance.

5. Conclusion

This paper presents a probabilistic method to evaluate the total transfer capability (TTC) by considering the energy margin and the uncertainty of weather conditions. In TTC determination the repeated power flow (RPF) method is used to maximize the incremental factor of load and generation and the transient energy margin method instead of the time simulation such as Runge-Kutta is used.
to check the transient stability. The weather condition that affects the system reliability is considered. As a result of considering weather effect and using probabilistic approach, the TTC for the adverse weather condition is lower than that of the normal weather condition. Especially, it is seen that it is possible to operate power systems at lower TTC during the period such as a special weather statement, if the weather condition is considered in advance.

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References

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